

# Logic and Modelling

— Natural Deduction in ProofWeb —

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ProofWeb allows to practise natural deduction online:

- ▶ based on the proof assistant Coq
- ▶ the derivations are automatically checked for correctness

# Examples

Example:  $p \vee q, \neg p \vdash q$

Theorem ex1 :  $(A \ \backslash / \ B) \rightarrow \sim A \rightarrow B.$

Proof.

imp\_i H.

imp\_i HnA.

dis\_e (A \ / B) HA HB.

ass H.

fls\_e.

neg\_e (A).

ass HnA.

ass HA.

ass HB.

Qed.

# Examples

Example:  $p \rightarrow (q \rightarrow r) \vdash q \rightarrow (p \rightarrow r)$

Theorem ex2 : (A -> (B -> C)) -> (B -> (A -> C)).

Proof.

imp\_i H.

imp\_i HB.

imp\_i HA.

imp\_e B.

imp\_e A.

ass H.

ass HA.

ass HB.

Qed.

# Examples

**Example:**  $(p \rightarrow q) \rightarrow r \vdash p \rightarrow (q \rightarrow r)$

Theorem ex3 : ((A -> B) -> C) -> (A -> (B -> C)).

Proof.

imp\_i H.

imp\_i HA.

imp\_i HB.

imp\_e (A -> B).

ass H.

imp\_i HA'.

ass HB.

Qed.

# Examples

Example:  $p \vee (q \wedge r) \vdash p \vee q$

Theorem ex4 : (A \/\ (B /\ C)) -> (A \/\ B).

Proof.

imp\_i H.

f\_dis\_e H HA HBC.

f\_dis\_i1 HA.

dis\_i2.

f\_con\_e1 HBC.

Qed.

# Examples

Example:  $a \vee b, a \rightarrow c, \neg d \rightarrow \neg b \vdash c \vee d$

Theorem ex5 :  $(A \vee B) \rightarrow (A \rightarrow C) \rightarrow (\sim D \rightarrow \sim B) \rightarrow (C \vee D)$ .

Proof.

imp\_i HAoB. imp\_i HAC. imp\_i HDB.

f\_dis\_e HAoB HA HB.

dis\_i1.

imp\_e A.

ass HAC.

ass HA.

dis\_i2.

PBC HD.

neg\_e (B).

imp\_e ( $\sim D$ ).

ass HDB. ass HD. ass HB.

Qed.

# Examples

**Example:**  $(a \rightarrow b) \wedge (b \rightarrow a) \vdash (a \wedge b) \vee (\neg a \wedge \neg b)$

Theorem ex6 :  $(A \rightarrow B) \rightarrow (B \rightarrow A)$   
 $\rightarrow ((A \wedge B) \vee (\sim A \wedge \sim B))$ .

Proof.

imp\_i HAB. imp\_i HBA.

dis\_e (A  $\vee$   $\sim$ A) HA HnA.

LEM.

dis\_i1.

con\_i.

ass HA.

imp\_e A.

ass HAB. ass HA.

dis\_i2.

con\_i.

ass HnA.

MT (A).

ass HBA. ass HnA.

Qed.



# Examples

**Example:**  $a \wedge (b \vee c) \vdash (a \wedge b) \vee (a \wedge c)$

Theorem ex7 : (A /\ (B \/ C))  
-> ((A /\ B) \/ (A /\ C)).

Proof.

imp\_i H.

dis\_e (B \/ C) HB HC.

f\_con\_e2 H.

dis\_i1.

con\_i.

f\_con\_e1 H.

ass HB.

dis\_i2.

con\_i.

f\_con\_e1 H.

ass HC.

Qed.

# Examples

**Example:**  $\vdash ((p \rightarrow q) \rightarrow p) \rightarrow p$

Theorem ex8 : ((A -> B) -> A) -> A.

Proof.

imp\_i H.

dis\_e (A \ / ~A) HA HnA.

LEM.

ass HA.

imp\_e (A -> B).

ass H.

imp\_i HA.

fls\_e.

f\_neg\_e HnA HA.

Qed.