

Logic and Modelling

— Natural Deduction in ProofWeb —

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ProofWeb allows to practise natural deduction online:

- ▶ based on the proof assistant Coq
- ▶ the derivations are automatically checked for correctness

Examples

Example: $p \vee q, \neg p \vdash q$

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Theorem ex1 : $(A \vee B) \rightarrow \sim A \rightarrow B.$

Examples

Example: $p \vee q, \neg p \vdash q$

Theorem ex1 : (A \ / B) -> ~A -> B.

Proof.

imp_i H.

imp_i HnA.

dis_e (A \ / B) HA HB.

ass H.

fls_e.

neg_e (A).

ass HnA.

ass HA.

ass HB.

Qed.

Examples

Example: $p \rightarrow (q \rightarrow r) \vdash q \rightarrow (p \rightarrow r)$

Theorem ex2 : $(A \rightarrow (B \rightarrow C)) \rightarrow (B \rightarrow (A \rightarrow C))$.

Examples

Example: $p \rightarrow (q \rightarrow r) \vdash q \rightarrow (p \rightarrow r)$

Theorem ex2 : (A -> (B -> C)) -> (B -> (A -> C)).

Proof.

imp_i H.

imp_i HB.

imp_i HA.

imp_e B.

imp_e A.

ass H.

ass HA.

ass HB.

Qed.

Examples

Example: $(p \rightarrow q) \rightarrow r \vdash p \rightarrow (q \rightarrow r)$

Theorem ex3 : $((A \rightarrow B) \rightarrow C) \rightarrow (A \rightarrow (B \rightarrow C))$.

Examples

Example: $(p \rightarrow q) \rightarrow r \vdash p \rightarrow (q \rightarrow r)$

Theorem ex3 : ((A -> B) -> C) -> (A -> (B -> C)).

Proof.

imp_i H.

imp_i HA.

imp_i HB.

imp_e (A -> B).

ass H.

imp_i HA'.

ass HB.

Qed.

Examples

Example: $p \vee (q \wedge r) \vdash p \vee q$

Theorem ex4 : $(A \setminus / (B \ / \ C)) \rightarrow (A \setminus / B) .$

Examples

Example: $p \vee (q \wedge r) \vdash p \vee q$

Theorem ex4 : (A \/\ (B /\ C)) -> (A \/\ B).

Proof.

imp_i H.

f_dis_e H HA HBC.

f_dis_i1 HA.

dis_i2.

f_con_e1 HBC.

Qed.

Examples

Example: $a \vee b, a \rightarrow c, \neg d \rightarrow \neg b \vdash c \vee d$

Theorem ex5 : $(A \vee B) \rightarrow (A \rightarrow C) \rightarrow (\sim D \rightarrow \sim B) \rightarrow (C \vee D)$.

Examples

Example: $a \vee b, a \rightarrow c, \neg d \rightarrow \neg b \vdash c \vee d$

Theorem ex5 : $(A \vee B) \rightarrow (A \rightarrow C) \rightarrow (\sim D \rightarrow \sim B) \rightarrow (C \vee D)$.

Proof.

imp_i HAoB. imp_i HAC. imp_i HDB.

f_dis_e HAoB HA HB.

dis_i1.

imp_e A.

ass HAC.

ass HA.

dis_i2.

PBC HD.

neg_e (B).

imp_e ($\sim D$).

ass HDB. ass HD. ass HB.

Qed.

Examples

Example: $(a \rightarrow b) \wedge (b \rightarrow a) \vdash (a \wedge b) \vee (\neg a \wedge \neg b)$

Theorem ex6 : $(A \rightarrow B) \rightarrow (B \rightarrow A)$
 $\rightarrow ((A \wedge B) \vee (\sim A \wedge \sim B)) .$

Examples

Example: $(a \rightarrow b) \wedge (b \rightarrow a) \vdash (a \wedge b) \vee (\neg a \wedge \neg b)$

Theorem ex6 : $(A \rightarrow B) \rightarrow (B \rightarrow A)$
 $\rightarrow ((A \wedge B) \vee (\sim A \wedge \sim B))$.

Proof.

imp_i HAB. imp_i HBA.

dis_e (A \vee \sim A) HA HnA.

LEM.

dis_i1.

con_i.

ass HA.

imp_e A.

ass HAB. ass HA.

dis_i2.

con_i.

ass HnA.

MT (A).

ass HBA. ass HnA.

Qed.

Examples

Example: $a \wedge (b \vee c) \vdash (a \wedge b) \vee (a \wedge c)$

Theorem ex7 : $(A \wedge (B \vee C))$
 $\rightarrow ((A \wedge B) \vee (A \wedge C)) .$

Examples

Example: $a \wedge (b \vee c) \vdash (a \wedge b) \vee (a \wedge c)$

Theorem ex7 : (A /\ (B \/ C))
-> ((A /\ B) \/ (A /\ C)).

Proof.

imp_i H.

dis_e (B \/ C) HB HC.

f_con_e2 H.

dis_i1.

con_i.

f_con_e1 H.

ass HB.

dis_i2.

con_i.

f_con_e1 H.

ass HC.

Qed.

Examples

Example: $\vdash ((p \rightarrow q) \rightarrow p) \rightarrow p$

Theorem ex8 : $((A \rightarrow B) \rightarrow A) \rightarrow A.$

Examples

Example: $\vdash ((p \rightarrow q) \rightarrow p) \rightarrow p$

Theorem ex8 : ((A -> B) -> A) -> A.

Proof.

imp_i H.

dis_e (A \ / ~A) HA HnA.

LEM.

ass HA.

imp_e (A -> B).

ass H.

imp_i HA.

fls_e.

f_neg_e HnA HA.

Qed.