Databases

Jörg Endrullis

VU University Amsterdam

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Relational Normal Forms

Overview

1. Functional Dependencies (FDs)
2. Anomalies, FD-based Normal Forms
3. Multivalued Dependencies (MVDs) and 4NF
4. Normal Forms and ER Design
5. Denormalization

Introduction

- **Functional Dependencies (FDs)**
  - are a **generalization of keys**
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The phone number for each instructor is stored multiple times!

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Roughly speaking, BNCF requires that **all FDs are keys.**

- In rare circumstances, a relation might not have an equivalent BCNF form while preserving all its FDs.

The 3NF normal form always exists (and preserves the FDs).

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When an ER model is **well designed**, the resulting derived relational tables will **automatically be in BCNF**.

- Awareness of normal forms can help to detect design errors already in the conceptual design phase.

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Nevertheless, these are **bad design**.

Functional Dependencies

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Whenever two rows of a relation agree in the instructor name `INAME`, they **must** also agree in the `PHONE` column values!

Functional Dependencies

Intuitively, there is a functional dependency

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since the phone number **only depends on the instructor**, not on other course data.

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A **determinant** is a ‘minimal’ functional dependency.

A determinant is like a **partial key:**

- uniquely determines some attributes, but not all in general

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E.g. `INAME` → `TITLE` is **not** satisfied.

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$$\{A_1, \ldots, A_n\} \rightarrow \{B_1, \ldots, B_m\}$$

The **functional dependency (FD)**

$$A_1, \ldots A_n \rightarrow B_1, \ldots B_m$$

holds for a relation R in a database state I if and only if for all tuples $t, u \in I(R)$:

$$\begin{array}{rl} & t.A_1 = u.A_1 \wedge \cdots \wedge t.A_n = u.A_n \\ \Rightarrow & t.B_1 = u.B_1 \wedge \cdots \wedge t.B_m = u.B_m \end{array}$$

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A **key** uniquely determines **all** attributes of its relation.

- There are never two distinct rows with the same key, so the **functional dependency condition is trivially satisfied**.

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We have the following functional dependencies:

- $\mathsf{CRN} \longrightarrow \mathsf{TITLE}, \mathsf{INAME}, \mathsf{PHONE}$

or equivalently:

- $\mathsf{CRN} \longrightarrow \mathsf{TITLE}$
- $\mathsf{CRN} \longrightarrow \mathsf{INAME}$
- $\mathsf{CRN} \longrightarrow \mathsf{PHONE}$

Functional Dependencies

An functional dependency with m attributes on the right

$$A_1, \dots A_n \rightarrow B_1, \dots B_m$$

is **equivalent** to the m FDs:

$$\begin{array}{ccc} A_1, \dots, A_n & \rightarrow & B_1 \\ \vdots & & \vdots \\ A_1, \dots, A_n & \rightarrow & B_m \end{array}$$

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Thus, in the following it suffices to consider FDs with a single column name on the right-hand side.

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Functional dependencies are **constraints** (like keys).

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In this example state, the functional dependency

$$\mathrm{TITLE} \longrightarrow \mathrm{CRN}$$

holds.

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holds. But this is probably **not true in general**!

It is a task of DB design to verify if this is mere coincidence.

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It is a task of DB design to verify if this is mere coincidence.

For the database design process, the only interesting functional dependencies are those that **hold for all possible states.**

Functional Dependencies are Keys

Functional dependencies are a **generalisation of keys**.

$$A_1, \ldots, A_n \text{ is a key of relation } R(A_1, \ldots, A_n, B_1, \ldots, B_m)$$

$$\Longleftrightarrow$$

$$\text{the functional dependency } A_1, \ldots, A_n \rightarrow B_1, \ldots B_m \text{ holds.}$$

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Here `CRN` → `TITLE`, `INAME`, `PHONE`.

Functional Dependencies are Keys

Functional dependencies are partial keys.

The functional dependency

$$A_1, \ldots A_n \rightarrow B_1, \ldots B_m$$

holds for a relation R if $\{\ A_1, \ldots A_n\ \}$ is a key for the relation obtained by restricting R to the columns $\{A_1, \ldots A_n, B_1, \ldots B_m\}$.

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The restriction of the table `COURSES` to $\{$ `INAME`, `PHONE` $\}$ is:

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The attribute `INAME` is a key of this table.

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The **goal of database normalization is to turn FDs into keys**.

The DBMS is then able to enforce the FDs for the user.

Example

The following tables contains books and their authors:

BOOKS

AUTHOR	NO	TITLE	PUBLISHER	ISBN
Elmasri	1	Fund. of DBS	Addison-W.	0805317554
Navathe	2	Fund. of DBS	Addison-W.	0805317554
Silberschatz	1	DBS Concepts	Mc-Graw H.	0471365084
Korth	2	DBS Concepts	Mc-Graw H.	0471365084
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- a book may have multiple authors, one author per row
- attribute `NO` is used to indicate the order of the authors

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- The ISBN uniquely identifies a book. Thus

$$\mathtt{ISBN} \longrightarrow \mathtt{TITLE}, \mathtt{PUBLISHER}$$

  Equivalently
  - $\mathtt{ISBN} \longrightarrow \mathtt{TITLE}$, and
  - $\mathtt{ISBN} \longrightarrow \mathtt{PUBLISHER}$

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- A book may have many authors. Thus

$$\mathtt{ISBN} \longrightarrow \mathtt{AUTHOR}$$

does not hold!

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- One author can write many books, thus

$$\text{AUTHOR} \longrightarrow \text{TITLE}$$

does not hold in general.

Although it happens to hold in the above database state.

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- a book may have multiple authors, one author per row
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- There may be books with the same title but different authors and different publishers. So

  `TITLE`

  determines no other attributes.

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- Every book has only one first (second, third, ...) author. Thus

$$\mathtt{ISBN}, \mathtt{NO} \longrightarrow \mathtt{AUTHOR}$$

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- At first glance, the author of any given book is also uniquely assigned a position in the authorship sequence.

$$\mathtt{ISBN}, \mathtt{AUTHOR} \longrightarrow \mathtt{NO}$$? questionable

However, violated by an author list like Smith & Smith.

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- What about the functional dependency

$$\mathrm{PUBLISHER}, \mathrm{TITLE}, \mathrm{NO} \longrightarrow \mathrm{AUTHOR} \qquad \text{? questionable}$$

Authorship sequence might change in a new edition of a book!

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During database design, **only unquestionable conditions should be used as functional dependencies**.

- Database normalization **alters the table structure** depending on the specified functional dependencies.
  Later hard to change: needs creation/deletion of tables!

Quiz

A table with homework grades:

HOMEWORK_RESULTS

STUD_ID	FIRST	LAST	EX_NO	POINTS	MAX_POINTS
100	Andrew	Smith	1	9	10
101	Dave	Jones	1	8	10
102	Maria	Brown	1	10	10
101	Dave	Jones	2	11	12
102	Maria	Brown	2	10	12

- Which FDs should hold for this table in general?
- Identify FDs that hold in this table but not in general.

Implication of Functional Dependencies

Whenever $A \longrightarrow B$ and $B \longrightarrow C$ hold, then $A \longrightarrow C$ is automatically satisfied.

Note that $\texttt{CRN} \longrightarrow \texttt{PHONE}$ is a consequence of

$$\texttt{CRN} \longrightarrow \texttt{INAME} \quad \text{and} \quad \texttt{INAME} \longrightarrow \texttt{PHONE}$$

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A set of FDs $\{\alpha_1 \longrightarrow \beta_1, \ldots, \alpha_n \longrightarrow \beta_n\}$ **implies** an FD $\alpha \longrightarrow \beta$ if and only if every DB state which satisfies all $\alpha_i \longrightarrow \beta_i, 1 \leqslant i \leqslant n$, also satisfies $\alpha \longrightarrow \beta$.

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  If $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.

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- **Reflexivity:**
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- **Augmentation:**
  If $\alpha \rightarrow \beta$, then $\alpha \cup \gamma \rightarrow \beta \cup \gamma$.

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  If $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$.
- **Augmentation:**
  If $\alpha \rightarrow \beta$, then $\alpha \cup \gamma \rightarrow \beta \cup \gamma$.
- **Transitivity:**
  If $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$.

Implication of Functional Dependencies

The DB designer is normally not interested in all FDs, but only in a **representative FD set** that implies all other FDs.

Armstrong Axioms

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  If $\beta \subseteq \alpha$, then $\alpha \to \beta$.
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  If $\alpha \to \beta$ and $\beta \to \gamma$, then $\alpha \to \gamma$.

Use the Amstrong axioms to show that

$$\begin{aligned} \texttt{ISBN} &\to \texttt{TITLE}, \texttt{PUBLISHER} \\ \texttt{ISBN}, \texttt{NO} &\to \texttt{AUTHOR} \\ \texttt{PUBLISHER} &\to \texttt{PUB\_URL} \end{aligned}$$

implies $\texttt{ISBN} \to \texttt{PUB\_URL}$.

Implication of Functional Dependencies

Simpler way to **check whether** *a* $\rightarrow \beta$ **is implied by an FD set**:

- compute the **cover** α^+ of α, and
- then check if $\beta \subseteq \alpha^+$.

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Cover

The **cover** $\alpha^+_{\mathcal{F}}$ of

- a set of attributes α
- with respect to an FD set $\mathcal{F}$

is the set of all attributes B that are uniquely determined by α:

$$\alpha^+_{\mathcal{F}} := \{\, B \mid \mathcal{F} \text{ implies } \alpha \rightarrow B \,\}$$

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Implication Check

A set of FDs $\mathcal{F}$ implies an FD $\alpha \rightarrow \beta$ if and only if $\beta \subseteq \alpha^+_{\mathcal{F}}$.

Implication of Functional Dependencies

Cover computation

Input: α (set of attributes)
$\alpha_1 \rightarrow \beta_1, \ldots, \alpha_n \rightarrow \beta_n$ (set of FDs $\mathcal{F}$)
Output: α^+ (the cover of α)

```
$x = \alpha$;
while $x$ did change do
    for all given FD $\alpha_i \rightarrow \beta_i$ do
        if $\alpha_i \subseteq x$ then
            $x = x \cup \beta_i$;    (add attributes in $\beta_i$ to $x$)
        end if
    end for
end while
return $x$;
```

Implication of Functional Dependencies

Compute the cover $\{\mathtt{ISBN}\}^+$ for the following FDs:

$$\begin{aligned} \mathtt{ISBN} &\longrightarrow \mathtt{TITLE}, \mathtt{PUBLISHER} \\ \mathtt{ISBN}, \mathtt{NO} &\longrightarrow \mathtt{AUTHOR} \\ \mathtt{PUBLISHER} &\longrightarrow \mathtt{PUB\_URL} \end{aligned}$$

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4. No further way to extend set x, the algorithm returns

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4. No further way to extend set x, the algorithm returns

   $$\{\texttt{ISBN}\}^+ = \{\texttt{ISBN}, \texttt{TITLE}, \texttt{PUBLISHER}, \texttt{PUB\_URL}\}$$
5. We may now conclude, e.g., $\texttt{ISBN} \rightarrow \texttt{PUB\_URL}$.

How to Determine Keys

Given a set of FDs and the set of all attributes $\mathcal{A}$ of a relation *R*:

$$\alpha \subseteq \mathcal{A} \text{ is key of } R \qquad \Longleftrightarrow \qquad \alpha^{+} = \mathcal{A}$$

That is α is a key if the cover α^{+} contains all attributes.

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We can use FDs to determine all possible keys of *R*.

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Remember: normally, we are interested in **minimal keys** only.

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We can use FDs to determine all possible keys of *R*.

Remember: normally, we are interested in **minimal keys** only.

A key α is **minimal** if every $A \in \alpha$ is **vital**, that is

$$(\alpha - \{A\})^+ \neq \mathcal{A}$$

How to Determine Keys

Finding a Minimal Key

Input: $\mathcal{A}$ (set of all attributes of R)
$\alpha_1 \to \beta_1, \ldots, \alpha_n \to \beta_n$ (set of FDs $\mathcal{F}$)
Output: α (a minimal key of R)

```
$x = \mathcal{A}$;
for all attributes $A \in X$ do
    if $A \in \{x - A\}^+_{\mathcal{F}}$ then
        $x = x - A$;    (remove $A$ from $x$)
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return $x$;
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        $x = x - A$;    (remove $A$ from $x$)
    end if
end for
return $x$;
```

We might get different keys depending on the order in **for all.**

How to Determine Keys

RESULTS

STUD_ID	**EX_NO**	**POINTS**	**MAX_POINTS**
100	1	9	10
101	1	8	10
101	2	11	12

$$\mathcal{F} = \left\{ \begin{array}{rcl} \mathtt{STUD\_ID}, \mathtt{EX\_NO} & \longrightarrow & \mathtt{POINTS} \\ \mathtt{EX\_NO} & \longrightarrow & \mathtt{MAX\_POINTS} \end{array} \right\}$$

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We determine a minimal key for the relation `RESULTS`:

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3. We remove MAX_POINTS since $\{\, \mathtt{EX\_NO} \,\} \subseteq x$:
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3. We remove MAX_POINTS since $\{\,\texttt{EX\_NO}\,\} \subseteq x$:
   $x = \{\,\texttt{STUD\_ID},\ \texttt{EX\_NO}\,\}$
4. Nothing else can be removed. We have a minimal key:

$$\{\,\texttt{STUD\_ID},\ \texttt{EX\_NO}\,\}$$

How to Determine Keys

Finding all Minimal Keys

```
Input:  $A_1, A_2, \ldots, A_n$ (all attributes of $R$) and $\mathcal{F}$ (set of FDs)
  $Results = \varnothing$;
  $Candidates = \{\{A_1\}, \{A_2\}, \ldots, \{A_n\}\}$;
  while $Candidates \neq \varnothing$ do
      choose and remove a smallest $\kappa \in Candidates$;
      if $\kappa^+_{\mathcal{F}} = \{A_1, A_2, \ldots, A_n\}$ then
          if $\kappa$ contains no key in $Results$ then
              $Results = Results \cup \{\kappa\}$;
          end if
      else
          for all $A_i \notin \kappa^+_{\mathcal{F}}$ do
              $\kappa_i = \kappa \cup \{A_i\}$;
              $Candidates = Candidates \cup \{\kappa_i\}$;
          end for
      end if
  end while
  return $Results$;
```

How to Determine Keys

Finding all minimal keys

Find **all** minimal keys the relation R

R				
A	**B**	**C**	**D**	**E**

with the functional dependencies

$$\begin{aligned} \mathsf{A}, \mathsf{D} &\longrightarrow \mathsf{B}, \mathsf{D} \\ \mathsf{B}, \mathsf{D} &\longrightarrow \mathsf{C} \\ \mathsf{A} &\longrightarrow \mathsf{E} \\ \mathsf{C}, \mathsf{D}, \mathsf{E} &\longrightarrow \mathsf{A} \end{aligned}$$

using the algorithm on the previous slide.

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Determinants (Non-trivial, minimal FDs)

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Are the following determinants?

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Are the following determinants?

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Are the following determinants?

- `POINTS`, `MAX_POINTS` for `POINTS`, `MAX_POINTS` ? No
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Determinants

Determinants (Non-trivial, minimal FDs)

The attribute set $A_1, \ldots, A_n$ is called a **determinant** for attribute set $B_1, \ldots, B_m$ if and only if

- the FD $A_1, \ldots, A_n \to B_1, \ldots B_m$ holds, and
- the **lhs is minimal**, i.e., whenever any A_i is removed then $A_1, \ldots, A_{i-1}, A_{i+1}, A_n \to B_1, \ldots B_m$ does *not* hold, and
- the lhs and rhs are distinct, i.e., $\{A_1, \ldots, A_n\} \neq \{B_1, \ldots, B_m\}$.

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Relational Normal Forms

Overview

1. Functional Dependencies (FDs)
2. Anomalies, FD-based Normal Forms
3. Multivalued Dependencies (MVDs) and 4NF
4. Normal Forms and ER Design
5. Denormalization

Consequences of Bad DB Design

Usually a severe sign of **bad DB design** if a table contains an FD (encodes a partial function) that is **not implied by a key.**

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$$\mathtt{INAME} \longrightarrow \mathtt{PHONE}$$

COURSES

CRN	**TITLE**	**INAME**	**PHONE**
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INAME ⟶ PHONE

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This leads to

- **redundant storage of certains facts**

  (here, phone numbers)
- **insert, update, deletion anomalies**

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General FDs are **not** supported by **relational databases.**

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Problem

General FDs are **not** supported by **relational databases.**

The solution is to transform FDs into **key constraints.**
This is what **DB normalization** tries to do.

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Update anomalies

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Deletion anomalies

- When the last course of an instructor is deleted, his/her phone number is lost.

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The relation

$$\texttt{COURSES (}\underline{\texttt{CRN}}\texttt{, TITLE, INAME, PHONE)}$$

with the FDs

$$\begin{aligned} \texttt{CRN} &\rightarrow \texttt{TITLE}, \texttt{INAME}, \texttt{PHONE} \\ \texttt{INAME} &\rightarrow \texttt{PHONE} \end{aligned}$$

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However, the relation `COURSES (`$\underline{\texttt{CRN}}$`, TITLE, INAME)` without the attribute `PHONE` is in BCNF.

Boyce-Codd Normal Form: Examples

Each course meets once per week in a dedicated room:

`CLASS (CRN, TITLE, WEEKDAY, TIME, ROOM)`

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Thus `CLASS` **is in BCNF.**

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Consider the relation

`PRODUCT (NO, NAME, PRICE)`

and the following FDs:

$$\begin{array}{rcl@{\qquad\qquad}rcl}
\mathtt{NO} & \longrightarrow & \mathtt{NAME} & \mathtt{PRICE, NAME} & \longrightarrow & \mathtt{NAME} \\
\mathtt{NO} & \longrightarrow & \mathtt{PRICE} & \mathtt{NO, PRICE} & \longrightarrow & \mathtt{NAME}
\end{array}$$

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- The two left FDs indicate that `NO` is a key.
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Thus the relation `PRODUCT` **is in BCNF**.

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Alternative characterisation of Boyce-Codd Normal Form:

BCNF $\Longleftrightarrow$ every determinant is key

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If a relation R is in BCNF, then…

- Ensuring its key constraints automatically satisfies all FDs. Hence, no additional constraints are needed!
- The **anomalies** (udpate/insertion/deletion) **do not occur.**

Boyce-Codd Normal Form: Quiz

BCNF Quiz

1. Is the relation

   `RESULTS (STUD_ID, EX_NO, POINTS, MAX_POINTS)`

   with the following FDs in BCNF?

   $$\begin{array}{rcl} \texttt{STUD\_ID, EX\_NO} & \longrightarrow & \texttt{POINTS} \\ \texttt{EX\_NO} & \longrightarrow & \texttt{MAX\_POINTS} \end{array}$$

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2. Is the relation

   `INVOICE (INV_NO, DATE, AMOUNT, CUST_NO, CUST_NAME)`

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   $$\begin{aligned} \texttt{INV\_NO} &\longrightarrow \texttt{DATE, AMOUNT, CUST\_NO} \\ \texttt{INV\_NO, DATE} &\longrightarrow \texttt{CUST\_NAME} \\ \texttt{CUST\_NO} &\longrightarrow \texttt{CUST\_NAME} \\ \texttt{DATE, AMOUNT} &\longrightarrow \texttt{DATE} \end{aligned}$$

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Third Normal Form (3NF) is slightly weaker than BCNF:
If a relation is in BCNF, it is automatically in 3NF.

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In short, we can say:

BCNF $\iff$ for every non-trivial FD:

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Alternative characterisation of 3NF:

3NF $\iff$ **every determinant of a non-key attribute is a key**

Third Normal Form Quiz

3NF vs BCNF

BOOKINGS

COURT	START_TIME	END_TIME	RATE
1	9:30	11:00	SAVER
2	9:30	12:00	PREMIMUM-A
1	12:00	14:00	STANDARD

The table contains bookings for one day at a tennis club:

- there are courts 1 (hard court) and 2 (grass court)
- the rates are
  - `SAVER` for member bookings of court 1
  - `STANDARD` for non-member bookings of court 1
  - `PREMIMUM-A` for member bookings of court 2
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Third Normal Form Quiz

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Quiz:

- Find a representative FDs set.
- Is the table in BCNF? Is the table in 3NF?

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If a table *R* is not in BCNF, we can **split** it into two tables.

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Table Decomposition

If the FD $A_1, \ldots, A_n \rightarrow B_1, \ldots B_m$ violates BCNF:

- create a new relation $S(\underline{A_1}, \ldots, \underline{A_n}, B_1, \ldots, B_m)$ and
- remove $B_1, \ldots, B_m$ from the original relation R.

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If the FD $A_1, \ldots, A_n \rightarrow B_1, \ldots B_m$ violates BCNF:

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The FD `INAME` → `PHONE` is the reason why table

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We can reconstruct the original table as follows:

```
CREATE VIEW COURSES (CRN, TITLE, INAME, PHONE)
    AS
SELECT I.CRN, I.TITLE, I.INAME, P.PHONE
  FROM INTSTRUCTORS I, PHONEBOOK P
 WHERE I.INAME = P.INAME
```

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"Lossy" decomposition

Original table
(key A, B, C)

A	B	C
a_{11}	b_{11}	c_{11}
a_{11}	b_{11}	c_{12}
a_{11}	b_{12}	c_{11}

Decomposition

R_1

A	B
a_{11}	b_{11}
a_{11}	b_{12}

R_2

A	C
a_{11}	c_{11}
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"Reconstruction"
$R_1 \bowtie R_2$

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All splits initiated by the **table decomposition method** for transforming relations into BCNF satisfy the condition of the decomposition theorem.

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All splits initiated by the **table decomposition method** for transforming relations into BCNF satisfy the condition of the decomposition theorem.

It is **always possible** to transform a relation into BCNF by lossless splitting (if necessary, split repeatedly).

Splitting Relations

Not every lossless split is reasonable!

STUDENTS

SSN	**FIRST_NAME**	**LAST_NAME**
111-22-3333	John	Smith
123-45-6789	Maria	Brown

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We may now store instructors and phone numbers without any affiliation to courses.

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The **correct solution** is to **eliminate** **`AGE`** from the table and to **define a view** which contains all columns plus the **computed** column `AGE` (invoking a SQL stored procedure).

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Whether or not to split depends on the intended application:

- A table of ZIP codes might be of interest on its own.

  E.g. it this were a database for a mailing company.

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2. **Minimise**: For each $A_1, \ldots, A_n \rightarrow B$ and each $i = 1, \ldots, n$
    - Compute the cover $\{A_1, \ldots, A_{i-1}, A_{i+1}, \ldots, A_n\}^+_{\mathcal{F}}$. If the result contains B, replace $\mathcal{F}$ by

$$\begin{aligned}\mathcal{F}' = (&\mathcal{F} - \{A_1, \ldots, A_n \rightarrow B\}) \\ &\cup \{A_1, \ldots, A_{i-1}, A_{i+1}, \ldots, A_n \rightarrow B\}\end{aligned}$$

Keep repeating until all left-hand sides are minimal.

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   Keep repeating until all left-hand sides are minimal.
3. **Remove implied FDs**: For each FD $\alpha \rightarrow B$
   - Compute the cover $\alpha^+_{\mathcal{F}'}$, where $\mathcal{F}' = \mathcal{F} - \{\alpha \rightarrow B\}$.
     If the cover contains B continue with $\mathcal{F} \leftarrow \mathcal{F}'$.

3NF Synthesis Algorithm

Compute the canonical set of FDs for

$$
\begin{aligned}
\mathsf{A}, \mathsf{B}, \mathsf{C} &\longrightarrow \mathsf{D}, \mathsf{E}\\
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4. For any two relations R_1, R_2 constructed in steps 2,3, if the schema of R_1 is contained in the schema R_2, discard R_1.

3NF Synthesis Algorithm: Example

Use the 3NF synthesis algorithm to normalise the relation

`R (A,B,C,D,E,F)`

with the following canonical functional dependencies:

$$\mathtt{A} \longrightarrow \mathtt{D}$$

$$\mathtt{B} \longrightarrow \mathtt{C}$$

$$\mathtt{B} \longrightarrow \mathtt{D}$$

$$\mathtt{D} \longrightarrow \mathtt{E}$$

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Sometimes we can enforce non-key constraints as follows:

- create a **materialised view** that contains the non-key FD (a selection of the columns of the FD)
- define the key constraint on the materialised view
  - updates to the table will cause updates to the view through
  - constraint checking is index-based, hence efficient

Summary

- Tables should **not** contain FDs other than those implied by the keys (i.e., all tables should be in **BCNF**).

  Such violating FDs indicate the combination of pieces of information which should be stored separately (presence of an embedded function). This leads to redundancy.

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  - Normalization to BCNF might not preserve FDs.
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- Sometimes it may make sense to avoid a split (and thus to violate BCNF).

  The DB designer has to carefully resolve such scenarios, incorporating application or domain knowledge.

Relational Normal Forms

Overview

1. Functional Dependencies (FDs)
2. Anomalies, FD-based Normal Forms
3. Multivalued Dependencies (MVDs) and 4NF
4. Normal Forms and ER Design
5. Denormalization

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However, there are **further types of constraints** which are also useful to during DB design.

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MVDs lead to the **Fourth Normal Form (4NF)**.

Multivalued Dependencies

The following table shows for each employee:

- knowledge of programming languages
- knowledge of programming DBMSs

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ENAME	**PROG_LANG**	**DBMS**
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Note: table may only be decomposed if `PROG_LANG` and `DBMS` are **independent**; otherwise **loss of information**.

E.g. it may not be decomposed if the semantics of the table is that the employee knows the interface between the language and the database.

Multivalued Dependencies

The **multivalued dependency (MVD)**

$$\texttt{ENAME} \twoheadrightarrow \texttt{PROG\_LANG}$$

means that the **set of values** in column `PROG_LANG` associated with every `ENAME` is **independent of all other columns.**

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That is, the table contains an

embedded function from `ENAME` to **sets of** `PROG_LANG`

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Formally, `ENAME` $\twoheadrightarrow$ `PROG_LANG` holds if: whenever two tuples agree on `ENAME`, one can exchange their `PROG_LANG` values and the resulting tupes are in the same table.

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This expresses the **independence** of `PROG_LANG` for a given `ENAME` from the rest of the table columns.

Multivalued Dependencies

Multivalued Dependency

A **multivalued dependency (MVD)**

$$A_1, \ldots, A_n \twoheadrightarrow B_1, \ldots, B_m$$

is satisfied in a DB state *I* if and only if

- for all tuples t, u in $I(R)$ with $t.A_i = u.A_i, 1 \leqslant i \leqslant n$,

  there are two further tuples t', u' in $I(R)$ such that

  1. t' agrees with t except that $t'.B_i = u.B_i, 1 \leqslant i \leqslant m$, and
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  2. u' agrees with u except that $u'.B_i = t.B_i, 1 \leqslant i \leqslant m$.

The condition means that the values of the B_i are swapped:

$$t\ \boxed{a_1, \ldots, a_n,\ b_1, \ldots, b_m,\ c_1, \ldots, c_k} \qquad t'\ \boxed{a_1, \ldots, a_n,\ b'_1, \ldots, b'_m,\ c_1, \ldots, c_k}$$

$$u\ \boxed{a_1, \ldots, a_n,\ b'_1, \ldots, b'_m,\ c'_1, \ldots, c'_k} \qquad u'\ \boxed{a_1, \ldots, a_n,\ b_1, \ldots, b_m,\ c'_1, \ldots, c'_k}$$

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More general:

For a relation $R(A_1, \ldots, A_n, B_1, \ldots, B_m, C_1, \ldots, C_k)$,
the following multivalued dependencies are equivalent

- $A_1, \ldots, A_n \twoheadrightarrow B_1, \ldots, B_m$
- $A_1 \ldots, A_n \twoheadrightarrow C_1, \ldots, C_k$

Swapping the B_j values in two tuples is the same as swapping the values for all other columns (the A_i values are identical, so swapping them has no effect).

Multivalued Dependencies

If the FD $A_1, \dots A_n \to B_1, \dots B_m$ holds, the corresponding MVD

$$A_1, \dots, A_n \twoheadrightarrow B_1, \dots, B_m$$

is trivially satisfied.

The FD means that if tuples t, u agree on the A_i then also on the B_j.
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Deduction rules to derive **all** implied FDs/MVDs

- The three Armstrong Axioms for FDs.
- If $\alpha \twoheadrightarrow \beta$ then $\alpha \twoheadrightarrow \gamma$, where γ are all remaining columns.
- If $\alpha_1 \twoheadrightarrow \beta_1$ and $\alpha_2 \supseteq \beta_2$ then $\alpha_1 \cup \alpha_2 \twoheadrightarrow \beta_1 \cup \beta_2$.
- If $\alpha \twoheadrightarrow \beta$ and $\beta \twoheadrightarrow \gamma$ then $\alpha \twoheadrightarrow (\gamma - \beta)$.
- If $\alpha \rightarrow \beta$, then $\alpha \twoheadrightarrow \beta$.
- If $\alpha \twoheadrightarrow \beta$ and $\beta' \subseteq \beta$ and there is γ with $\gamma \cap \beta = \emptyset$ and $\gamma \rightarrow \beta'$, then $\alpha \rightarrow \beta'$.

Fourth Normal Form

Fourth Normal Form (4NF)

A relation is in **Fourth Normal Form (4NF)** if every MVD

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is

- either trivial, or
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Since every FD is also an MVD, 4NF is stronger than BCNF.
That is, if a relation is in 4NF, it is automatically in BCNF.

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However, it is not very common that 4NF is violated, but BCNF is not.

Fourth Normal Form

The relation

`EMP_KNOWLEDGE (ENAME, PROG_LANG, DBMS)`

is an example of a relation that is in BCNF, but not in 4NF.

The relation has no non-trivial FDs.

Other Constraints

Multiple choice test

The following relation encodes the correct solution to a typical multiple choice test:

ANSWERS

QUESTION	ANSWER	TEXT	CORRECT
1	A	...	Y
1	B	...	N
1	C	...	N
2	A	...	N
2	B	...	Y
2	C	...	N

Using keys to enforce other constraints

The constraint is not an FD, MVD, or JD:

> "*Each question can only have one correct answer.*"

- Can you suggest a transformation of table `ANSWERS` such that the above constraint is already implied by a key?

Relational Normal Forms

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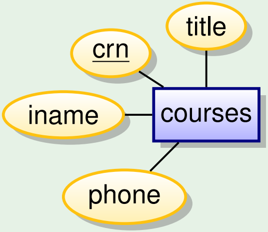

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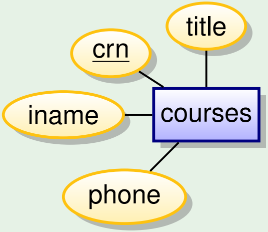

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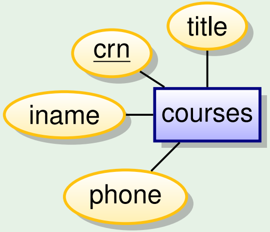

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- Also in the ER model, FDs between attributes of an entity should be implied by a key constraint.

Examples

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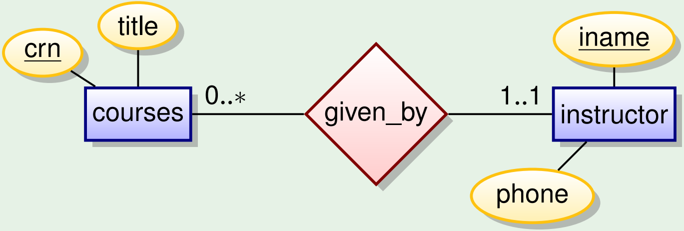
ER entity split

In this case, the instructor is an independent entity:



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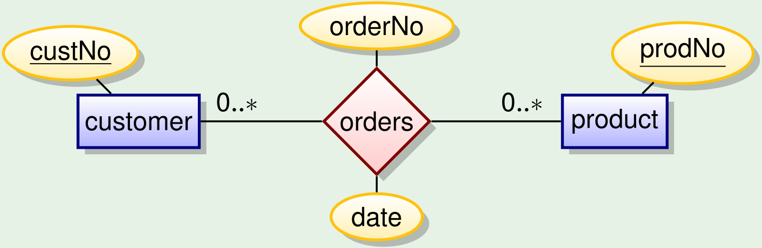
Functional dependencies between **attributes of a relationship** always violate BCNF.

Violation of BCNF on the ER level



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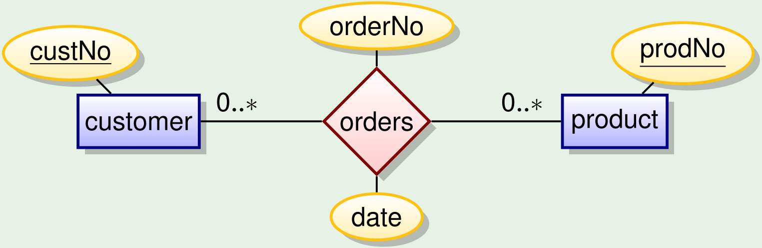

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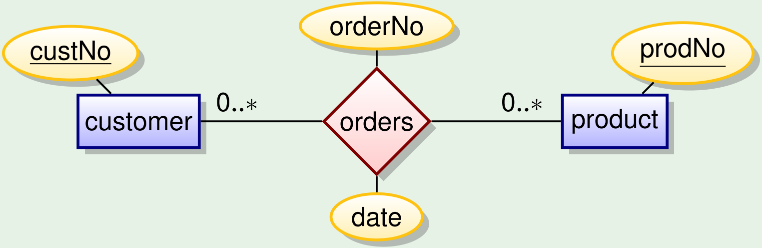

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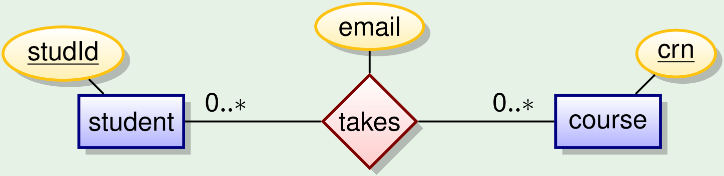
This shows that the concept "order" is an independent entity.

Examples

Violations of BCNF might also be due to the **wrong placement of an attribute.**

Questionable attribute placement



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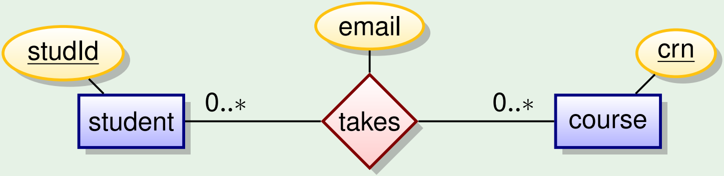

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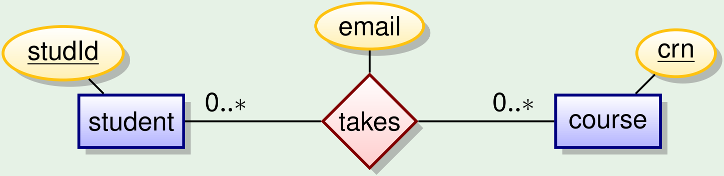

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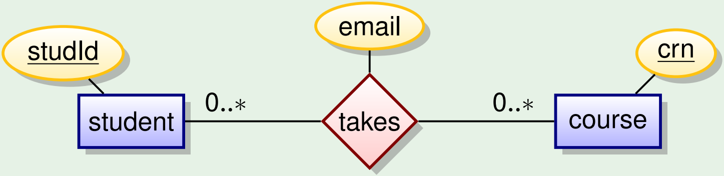

- The relationship is translated into

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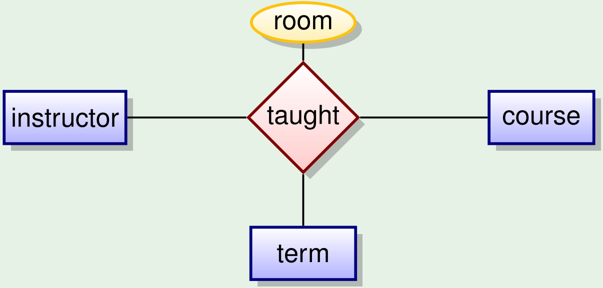
- Then the FD `STUD_ID` → `EMAIL` violates BCNF.
- Obviously, email should be an attribute of Student.

Examples

If an attribute of a ternary relationship depends only on two of the entities, this violates BCNF.

Ternary relationship



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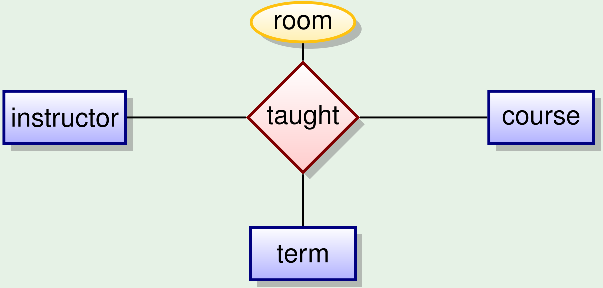

If every course is taught only once per term, then attribute room depends only on term and course (but not instructor).

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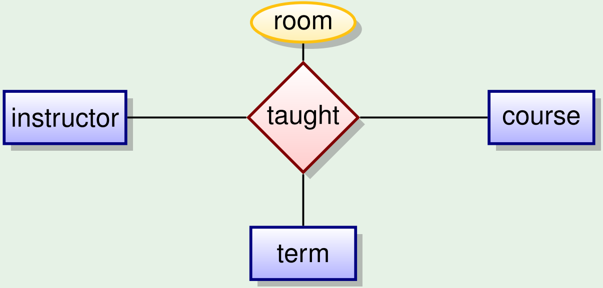

If every course is taught only once per term, then attribute room depends only on term and course (but not instructor).

Then the FD TERM, COURSE → ROOM violates BCNF.

Normalization: Summary

Relational normalization is about:

- **Avoiding redundancy.**
- **Storing separate facts (functions) separately.**
- **Transforming general integrity constraints** into constraints that are supported by the DBMS: **keys.**

- Relational normalization theory is mainly based on FDs, but there are other types of constraints (e.g., MVDs).

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For example, if an application extensively access the phone number of instructors, performance-wise it may make sense to add column `PHONE` to table `COURSES`.

COURSES			
<u>CRN</u>	**TITLE**	**INAME**	**PHONE**

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COURSES

CRN	**TITLE**	**INAME**	**PHONE**

This **avoids the otherwise required joins** (on attribute `INAME`) between tables `COURSES` and `PHONEBOOK`.

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  - a more complicated application logic (e.g., the need for triggers)
  - and the risk that a faulty application will turn the DB inconsistent
- Denormalization may not only be used to avoid joins:
  - Complete **separate, redundant tables** may be created (increasing the potential for parallel operations).
  - Columns may be added which **aggregate** information in other columns/rows.

Relational Normal Forms: Objectives

After completing this chapter, you should be able to

- work with **functional dependencies** (FDs),
    - define what they are
    - detect them in database schemas
    - decide implication, determine keys
- explain insert, update, and delete **anomalies,**
- understand, explain and use **BCNF**
    - test a given relation for BCNF, and
    - transform a relation into BCNF
- understand, explain and use **3NF**
    - test a given relation for 3NF, and
    - transform a relation into 3NF
- understand, explain **MVDs** and **4NF**
- detect **normal form violations** on the level of ER,
- explain when and how to **denormalize** a DB schema