Calculus M211

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The Substitution Rule for Indefinite Integrals

How to find antiderivatives of formulas like

$$\int 2x\sqrt{1+x^2}dx \quad ?$$

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(here we think of dx and du as differentials)

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In other words:

$$dx = \frac{du}{g'(x)}$$

If we change the variable from x to $u = g(x)$ we divide by $g'(x)$!

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Finding the right u is a guessing game. Often multiple choices.

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We could have chosen another u. For example:

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We choose $u = \sqrt{2x+1}$. Then $u' = \frac{1}{2\sqrt{2x+1}} \cdot 2 = \frac{1}{u}$. Then

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$$= \frac{1}{3}u^3 + C = \frac{1}{3}\left(\sqrt{2x+1}\right)^3 + C$$

The Substitution Rule for Indefinite Integrals

$$\int \sqrt{2x+1}dx$$

We choose $u = 2x + 1$. Then $u' = 2$, and hence

$$\int \sqrt{2x+1}dx = \int \sqrt{u}\frac{du}{2} = \frac{1}{2}\int \sqrt{u}\,du$$
$$= \frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}} + C = \frac{1}{3}(2x+1)^{\frac{3}{2}} + C$$

We could have chosen another u. For example:

We choose $u = \sqrt{2x+1}$. Then $u' = \frac{1}{2\sqrt{2x+1}} \cdot 2 = \frac{1}{u}$. Then

$$\int \sqrt{2x+1}dx = \int u\,\frac{du}{1/u} = \int u^2\,du$$
$$= \frac{1}{3}u^3 + C = \frac{1}{3}\left(\sqrt{2x+1}\right)^3 + C = \frac{1}{3}(2x+1)^{\frac{3}{2}} + C$$

The Substitution Rule for Indefinite Integrals

$$\int \frac{x}{\sqrt{1-4x^2}}dx$$

The Substitution Rule for Indefinite Integrals

$$\int \frac{x}{\sqrt{1-4x^2}}dx$$

We choose $u =$

The Substitution Rule for Indefinite Integrals

$$\int \frac{x}{\sqrt{1-4x^2}} dx$$

We choose $u = 1 - 4x^2$.

The Substitution Rule for Indefinite Integrals

$$\int \frac{x}{\sqrt{1-4x^2}} dx$$

We choose $u = 1 - 4x^2$. Then $u' =$

The Substitution Rule for Indefinite Integrals

$$\int \frac{x}{\sqrt{1-4x^2}} dx$$

We choose $u = 1 - 4x^2$. Then $u' = -8x$

The Substitution Rule for Indefinite Integrals

$$\int \frac{x}{\sqrt{1-4x^2}}dx$$

We choose $u = 1 - 4x^2$. Then $u' = -8x$, and hence

$$\int \frac{x}{\sqrt{1-4x^2}}dx$$

The Substitution Rule for Indefinite Integrals

$$\int \frac{x}{\sqrt{1-4x^2}} dx$$

We choose $u = 1 - 4x^2$. Then $u' = -8x$, and hence

$$\int \frac{x}{\sqrt{1-4x^2}} dx = \int \frac{x}{\sqrt{u}} \frac{du}{-8x}$$

The Substitution Rule for Indefinite Integrals

$$\int \frac{x}{\sqrt{1-4x^2}} dx$$

We choose $u = 1 - 4x^2$. Then $u' = -8x$, and hence

$$\int \frac{x}{\sqrt{1-4x^2}} dx = \int \frac{x}{\sqrt{u}} \frac{du}{-8x} = -\frac{1}{8} \int \frac{1}{\sqrt{u}}\, du$$

The Substitution Rule for Indefinite Integrals

$$\int \frac{x}{\sqrt{1-4x^2}} dx$$

We choose $u = 1 - 4x^2$. Then $u' = -8x$, and hence

$$\int \frac{x}{\sqrt{1-4x^2}} dx = \int \frac{x}{\sqrt{u}} \frac{du}{-8x} = -\frac{1}{8} \int \frac{1}{\sqrt{u}}\, du$$

$$= -\frac{1}{8} \cdot 2\sqrt{u} + C$$

The Substitution Rule for Indefinite Integrals

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$$= -\frac{1}{8}\cdot 2\sqrt{u} + C = -\frac{1}{4}\sqrt{1-4x^2} + C$$

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$$\int e^{5x}dx$$

The Substitution Rule for Indefinite Integrals

$$\int \frac{x}{\sqrt{1-4x^2}} dx$$

We choose $u = 1 - 4x^2$. Then $u' = -8x$, and hence

$$\int \frac{x}{\sqrt{1-4x^2}} dx = \int \frac{x}{\sqrt{u}} \frac{du}{-8x} = -\frac{1}{8} \int \frac{1}{\sqrt{u}} du$$

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$$\int e^{5x} dx$$

We choose $u = 5x$.

The Substitution Rule for Indefinite Integrals

$$\int \frac{x}{\sqrt{1-4x^2}}dx$$

We choose $u = 1 - 4x^2$. Then $u' = -8x$, and hence

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$$\int e^{5x} dx$$

We choose $u = 5x$. Then $u' = 5$

The Substitution Rule for Indefinite Integrals

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$$= -\frac{1}{8}\cdot 2\sqrt{u} + C = -\frac{1}{4}\sqrt{1-4x^2} + C$$

$$\int e^{5x}dx$$

We choose $u = 5x$. Then $u' = 5$, and hence

$$\int e^{5x}dx = \int e^{u}\,\frac{du}{5}$$

The Substitution Rule for Indefinite Integrals

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We choose $u = 1 - 4x^2$. Then $u' = -8x$, and hence

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$$\int e^{5x}\,dx$$

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$$\int e^{5x} dx$$

We choose $u = 5x$. Then $u' = 5$, and hence

$$\int e^{5x} dx = \int e^u \frac{du}{5} = \frac{1}{5} \int e^u du$$

$$= \frac{1}{5} e^u + C$$

The Substitution Rule for Indefinite Integrals

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We choose $u = 5x$. Then $u' = 5$, and hence

$$\int e^{5x}dx = \int e^{u}\,\frac{du}{5} = \frac{1}{5}\int e^{u}\,du$$
$$= \frac{1}{5}e^{u} + C = \frac{1}{5}e^{5x} + C$$

The Substitution Rule for Indefinite Integrals

A slightly more interesting example:

$$\int x^5 \sqrt{1 + x^2}\, dx$$

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The Substitution Rule for Indefinite Integrals

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The Substitution Rule for Indefinite Integrals

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$$\int x^5 \sqrt{1 + x^2}\, dx$$

We choose $u = 1 + x^2$. Then $u' = 2x$

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We choose $u = 1 + x^2$. Then $u' = 2x$, and hence

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The Substitution Rule for Indefinite Integrals

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We choose $u = 1 + x^2$. Then $u' = 2x$, and hence

$$\int x^5 \sqrt{1 + x^2}\, dx = \int x^5 \sqrt{u}\, \frac{du}{2x}$$

The Substitution Rule for Indefinite Integrals

A slightly more interesting example:

$$\int x^5 \sqrt{1 + x^2}\, dx$$

We choose $u = 1 + x^2$. Then $u' = 2x$, and hence

$$\int x^5 \sqrt{1 + x^2}\, dx = \int x^5 \sqrt{u}\, \frac{du}{2x} = \frac{1}{2} \int x^4 \sqrt{u}\, du$$

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What now?

The Substitution Rule for Indefinite Integrals

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What now? Note that $\quad x^2 = u - 1 \quad$ and $\quad x^4 = (x^2)^2$

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$$\int x^5 \sqrt{1+x^2}\, dx = \frac{1}{2} \int x^4 \sqrt{u}\, du = \frac{1}{2} \int (u-1)^2 \sqrt{u}\, du$$

$$= \frac{1}{2} \int (u^2 - 2u + 1) \sqrt{u}\, du$$

The Substitution Rule for Indefinite Integrals

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$$= \frac{1}{2} \int (u^2 - 2u + 1) \sqrt{u}\, du = \frac{1}{2} \int \left(u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}} \right) du$$

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$$= \frac{1}{2} \left(\frac{2}{7} u^{\frac{7}{2}} - 2 \cdot \frac{2}{5} u^{\frac{5}{2}} + \frac{2}{3} u^{\frac{3}{2}}\right) + C$$

The Substitution Rule for Indefinite Integrals

A slightly more interesting example:

$$\int x^5 \sqrt{1+x^2}\,dx$$

We choose $u = 1 + x^2$. Then $u' = 2x$, and hence

$$\int x^5 \sqrt{1+x^2}\,dx = \int x^5 \sqrt{u}\,\frac{du}{2x} = \frac{1}{2}\int x^4 \sqrt{u}\,du$$

What now? Note that $\quad x^2 = u - 1 \quad$ and $\quad x^4 = (x^2)^2$

$$\begin{aligned}
\int x^5 \sqrt{1+x^2}\,dx &= \frac{1}{2}\int x^4 \sqrt{u}\,du = \frac{1}{2}\int (u-1)^2 \sqrt{u}\,du \\
&= \frac{1}{2}\int (u^2 - 2u + 1)\sqrt{u}\,du = \frac{1}{2}\int \left(u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}}\right) du \\
&= \frac{1}{2}\left(\frac{2}{7}u^{\frac{7}{2}} - 2\cdot\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right) + C \\
&= \frac{1}{7}(1+x^2)^{\frac{7}{2}} - \frac{2}{5}(1+x^2)^{\frac{5}{2}} + \frac{1}{3}(1+x^2)^{\frac{3}{2}} + C
\end{aligned}$$

The Substitution Rule for Indefinite Integrals

$$\int \tan x \, dx$$

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First, we note that:

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$$

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The Substitution Rule for Indefinite Integrals

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First, we note that:

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$$

We choose $u = \cos x$.

The Substitution Rule for Indefinite Integrals

$$\int \tan x \, dx$$

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We choose $u = \cos x$. Then $u' =$

The Substitution Rule for Indefinite Integrals

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$$\int \frac{\sin x}{\cos x} dx = \int \frac{\sin x}{u} \, \frac{du}{-\sin x}$$

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$$\int \frac{\sin x}{\cos x} dx = \int \frac{\sin x}{u} \, \frac{du}{-\sin x} = -\int \frac{1}{u} \, du$$
$$= -\ln |u| + C$$

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$$= -\ln|u| + C = -\ln|\cos x| + C$$

Note that

$$-\ln|\cos x| = \ln|\cos x|^{-1} = \ln|\sec x|$$

The Substitution Rule for Indefinite Integrals

$$\int \tan x \, dx$$

First, we note that:

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$$

We choose $u = \cos x$. Then $u' = -\sin x$, and hence

$$\int \frac{\sin x}{\cos x} dx = \int \frac{\sin x}{u} \, \frac{du}{-\sin x} = -\int \frac{1}{u} \, du$$

$$= -\ln |u| + C = -\ln |\cos x| + C$$

Note that

$$-\ln |\cos x| = \ln |\cos x|^{-1} = \ln |\sec x|$$

Thus we can also write:

$$\int \tan x \, dx = \ln |\sec x| + C$$

The Substitution Rule for Definite Integrals

Methods for evaluating a **definite integral using substitution**:

$$\int_a^b f(x)dx$$

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Method 1

The Substitution Rule for Definite Integrals

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- evaluate the indefinite integral $\int f(x)dx$ using substitution
- then use the Fundamental Theorem $\int_a^b f(x)dx = \int f(x)dx\Big]_a^b$

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$$\int_0^4 \sqrt{2x+1}\,dx =$$

The Substitution Rule for Definite Integrals

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$$\int_0^4 \sqrt{2x+1}\,dx = \int \sqrt{2x+1}\,dx\Big]_0^4$$

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$$\int_0^4 \sqrt{2x+1}\,dx = \int \sqrt{2x+1}\,dx\Big]_0^4 = \frac{1}{3}(2x+1)^{\frac{3}{2}}\Big]_0^4$$

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$$\int_0^4 \sqrt{2x+1}\,dx = \int \sqrt{2x+1}\,dx\Big]_0^4 = \frac{1}{3}(2x+1)^{\frac{3}{2}}\Big]_0^4 = 9 - \frac{1}{3}$$

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$$\int_0^4 \sqrt{2x+1}\,dx = \int \sqrt{2x+1}\,dx\Big]_0^4 = \frac{1}{3}(2x+1)^{\frac{3}{2}}\Big]_0^4 = 9 - \frac{1}{3} = \frac{26}{3}$$

The Substitution Rule for Definite Integrals

Methods for evaluating a **definite integral using substitution**:

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- evaluate the indefinite integral $\int f(x)dx$ using substitution
- then use the Fundamental Theorem $\int_a^b f(x)dx = \int f(x)dx\Big]_a^b$

$$\int_0^4 \sqrt{2x+1}\,dx = \int \sqrt{2x+1}\,dx\Big]_0^4 = \frac{1}{3}(2x+1)^{\frac{3}{2}}\Big]_0^4 = 9 - \frac{1}{3} = \frac{26}{3}$$

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If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

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We choose $u =$

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We choose $u = 2x + 1$. Then $u' = 2$

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$$\int_0^4 \sqrt{2x+1}\,dx = \int_{u(0)}^{u(4)} \sqrt{u}\,\frac{du}{2} = \frac{1}{2}\int_1^9 \sqrt{u}\,du$$

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&= -\frac{1}{5}\left(-\frac{1}{u}\right)\Big]_{-2}^{-7} = -\frac{1}{5}\left(-\frac{1}{-7} - \left(-\frac{1}{-2}\right)\right) \\
&= -\frac{1}{5}\left(\frac{1}{7} - \frac{1}{2}\right) = -\frac{1}{5}\left(\frac{2}{14} - \frac{7}{14}\right) = \frac{1}{14}
\end{aligned}$$

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The Substitution Rule: Exercises

$$\int e^{-x}\,dx \qquad\qquad \text{take } u =$$

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The Substitution Rule: Exercises

$$\int e^{-x}\,dx \qquad \text{take } u = -x$$

$$\int x^3(2+x^4)^5\,dx \qquad \text{take } u = 2 + x^4$$

$$\int x^2\sqrt{x^3+1}\,dx \qquad \text{take } u = x^3 + 1$$

$$\int \frac{1}{(1-6t)^4}\,dt \qquad \text{take } u = 1 - 6t$$

$$\int \cos^3\phi\,\sin\phi\,dt \qquad \text{take } u = \cos\phi$$

$$\int \frac{\sec^2(\frac{1}{x})}{x^2}\,dt \qquad \text{take } u =$$

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The Substitution Rule: Exercises

$$\int (x+1)\sqrt{2x+x^2}\,dx \quad \text{take } u =$$

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$$\int (x+1)\sqrt{2x+x^2}\,dx \quad \text{take } u = 2x + x^2\text{, then } u' = 2(1+x)$$

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The Substitution Rule: Exercises

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$$\int \frac{(\ln x)^2}{x}\,dt \qquad \text{take } u = \ln x$$

$$\int (x^3+3x)(x^2+1)\,dt \quad \text{take } u = x^3 + 3x\text{, then } u' = 3(x^2+1)$$

The Substitution Rule: Exercises

Evaluate

$$\int x \sin(x^2)\, dx$$

The Substitution Rule: Exercises

Evaluate

$$\int x \sin(x^2)\, dx$$

We take $u =$

The Substitution Rule: Exercises

Evaluate

$$\int x \sin(x^2)\, dx$$

We take $u = x^2$

The Substitution Rule: Exercises

Evaluate

$$\int x \sin(x^2)\, dx$$

We take $u = x^2$, then $u' =$

The Substitution Rule: Exercises

Evaluate

$$\int x \sin(x^2)\, dx$$

We take $u = x^2$, then $u' = 2x$

The Substitution Rule: Exercises

Evaluate

$$\int x \sin(x^2)\, dx$$

We take $u = x^2$, then $u' = 2x$ and

$$\int x \sin(x^2)\, dx$$

The Substitution Rule: Exercises

Evaluate

$$\int x \sin(x^2)\, dx$$

We take $u = x^2$, then $u' = 2x$ and

$$\int x \sin(x^2)\, dx = \int x \sin u \,\frac{du}{2x}$$

The Substitution Rule: Exercises

Evaluate

$$\int x \sin(x^2)\, dx$$

We take $u = x^2$, then $u' = 2x$ and

$$\begin{aligned} \int x \sin(x^2)\, dx &= \int x \sin u \, \frac{du}{2x} \\ &= \frac{1}{2} \int \sin u \, du \end{aligned}$$

The Substitution Rule: Exercises

Evaluate

$$\int x \sin(x^2)\, dx$$

We take $u = x^2$, then $u' = 2x$ and

$$\begin{aligned}
\int x \sin(x^2)\, dx &= \int x \sin u \,\frac{du}{2x} \\
&= \frac{1}{2}\int \sin u \, du \\
&= -\frac{1}{2}\cos u + C
\end{aligned}$$

The Substitution Rule: Exercises

Evaluate

$$\int x \sin(x^2)\, dx$$

We take $u = x^2$, then $u' = 2x$ and

$$\begin{aligned}
\int x \sin(x^2)\, dx &= \int x \sin u \, \frac{du}{2x} \\
&= \frac{1}{2} \int \sin u \, du \\
&= -\frac{1}{2} \cos u + C \\
&= -\frac{1}{2} \cos x^2 + C
\end{aligned}$$

The Substitution Rule: Exercises

Evaluate

$$\int x^2 e^{x^3}\, dx$$

The Substitution Rule: Exercises

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$$\int x^2 e^{x^3}\, dx$$

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$$\int x^2 e^{x^3}\, dx$$

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The Substitution Rule: Exercises

Evaluate

$$\int x^2 e^{x^3} \, dx$$

We take $u = x^3$, then $u' =$

The Substitution Rule: Exercises

Evaluate

$$\int x^2 e^{x^3} \, dx$$

We take $u = x^3$, then $u' = 3x^2$

The Substitution Rule: Exercises

Evaluate

$$\int x^2 e^{x^3}\,dx$$

We take $u = x^3$, then $u' = 3x^2$ and

$$\int x^2 e^{x^3}\,dx$$

The Substitution Rule: Exercises

Evaluate

$$\int x^2 e^{x^3}\, dx$$

We take $u = x^3$, then $u' = 3x^2$ and

$$\int x^2 e^{x^3}\, dx = \int x^2 e^u \frac{du}{3x^2}$$

The Substitution Rule: Exercises

Evaluate

$$\int x^2 e^{x^3}\, dx$$

We take $u = x^3$, then $u' = 3x^2$ and

$$\begin{aligned}\int x^2 e^{x^3}\, dx &= \int x^2 e^u \, \frac{du}{3x^2} \\ &= \frac{1}{3}\int e^u \, du\end{aligned}$$

The Substitution Rule: Exercises

Evaluate

$$\int x^2 e^{x^3}\, dx$$

We take $u = x^3$, then $u' = 3x^2$ and

$$\begin{aligned} \int x^2 e^{x^3}\, dx &= \int x^2 e^u \,\frac{du}{3x^2} \\ &= \frac{1}{3}\int e^u\, du \\ &= \frac{1}{3} e^u + C \end{aligned}$$

The Substitution Rule: Exercises

Evaluate

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We take $u = x^3$, then $u' = 3x^2$ and

$$\begin{aligned}\int x^2 e^{x^3}\,dx &= \int x^2 e^u\,\frac{du}{3x^2}\\ &= \frac{1}{3}\int e^u\,du\\ &= \frac{1}{3}e^u + C\\ &= \frac{1}{3}e^{x^3} + C\end{aligned}$$

Evaluate

$$\int \frac{\sin 2x}{1+\cos^2 x}\,dx$$

The Substitution Rule: Exercises

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$$\int \frac{\sin 2x}{1+\cos^2 x}\,dx$$

We recall that $\sin 2x = 2\sin x \cos x$

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We take $u =$

The Substitution Rule: Exercises

Evaluate

$$\int \frac{\sin 2x}{1+\cos^2 x}\,dx$$

We recall that $\quad \sin 2x = 2\sin x\cos x$

We take $u = 1 + (\cos x)^2$

The Substitution Rule: Exercises

Evaluate

$$\int \frac{\sin 2x}{1 + \cos^2 x}\,dx$$

We recall that $\sin 2x = 2\sin x \cos x$

We take $u = 1 + (\cos x)^2$.

Then $u' =$

The Substitution Rule: Exercises

Evaluate

$$\int \frac{\sin 2x}{1+\cos^2 x}\,dx$$

We recall that $\quad \sin 2x = 2\sin x\cos x$

We take $u = 1 + (\cos x)^2$.

Then $u' = 2\cos x(-\sin x)$

The Substitution Rule: Exercises

Evaluate

$$\int \frac{\sin 2x}{1 + \cos^2 x}\, dx$$

We recall that $\quad \sin 2x = 2 \sin x \cos x$

We take $u = 1 + (\cos x)^2$.

Then $u' = 2 \cos x(-\sin x) = -\sin 2x$

The Substitution Rule: Exercises

Evaluate

$$\int \frac{\sin 2x}{1+\cos^2 x}\,dx$$

We recall that $\sin 2x = 2\sin x\cos x$

We take $u = 1 + (\cos x)^2$.

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Then $u' = 2 \cos x(-\sin x) = -\sin 2x$ and

$$\int \frac{\sin 2x}{1 + \cos^2 x}\, dx = \int \frac{\sin 2x}{u} \frac{du}{-\sin 2x}$$

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Evaluate

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$$= -\int \frac{1}{u}\,du$$

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$$\begin{aligned}\int \frac{\sin 2x}{1+\cos^2 x}\, dx &= \int \frac{\sin 2x}{u}\, \frac{du}{-\sin 2x} \\ &= -\int \frac{1}{u}\, du \\ &= -\ln|u| + C\end{aligned}$$

The Substitution Rule: Exercises

Evaluate

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We recall that $\quad \sin 2x = 2\sin x \cos x$

We take $u = 1 + (\cos x)^2$.

Then $u' = 2\cos x(-\sin x) = -\sin 2x$ and

$$\begin{aligned}\int \frac{\sin 2x}{1+\cos^2 x}\,dx &= \int \frac{\sin 2x}{u}\,\frac{du}{-\sin 2x}\\ &= -\int \frac{1}{u}\,du\\ &= -\ln|u| + C\\ &= -\ln|1+(\cos x)^2| + C\end{aligned}$$

The Substitution Rule: Exercises

Evaluate

$$\int_0^1 \cos(\pi x/2)\, dx$$

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We take $u =$

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We take $u = \pi x/2$, then $u' = \pi/2$

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We take $u = \pi x/2$, then $u' = \pi/2$ and

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The Substitution Rule: Exercises

Evaluate

$$\int_0^1 \cos(\pi x/2)\, dx$$

We take $u = \pi x/2$, then $u' = \pi/2$ and

$$\int_0^1 \cos(\pi x/2)\, dx = \int_{u(0)}^{u(1)} \cos(u)\, \frac{du}{\pi/2}$$

The Substitution Rule: Exercises

Evaluate

$$\int_0^1 \cos(\pi x/2)\, dx$$

We take $u = \pi x/2$, then $u' = \pi/2$ and

$$\begin{aligned}\int_0^1 \cos(\pi x/2)\, dx &= \int_{u(0)}^{u(1)} \cos(u)\, \frac{du}{\pi/2} \\ &= \frac{2}{\pi} \int_0^{\pi/2} \cos(u)\, du\end{aligned}$$

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Evaluate

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We take $u = \pi x/2$, then $u' = \pi/2$ and

$$\begin{aligned}\int_0^1 \cos(\pi x/2)\, dx &= \int_{u(0)}^{u(1)} \cos(u)\, \frac{du}{\pi/2} \\ &= \frac{2}{\pi} \int_0^{\pi/2} \cos(u)\, du \\ &= \frac{2}{\pi} \Big(\sin u \Big]_0^{\pi/2} \Big)\end{aligned}$$

The Substitution Rule: Exercises

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\int_0^1 \cos(\pi x/2)\, dx &= \int_{u(0)}^{u(1)} \cos(u)\, \frac{du}{\pi/2} \\
&= \frac{2}{\pi} \int_0^{\pi/2} \cos(u)\, du \\
&= \frac{2}{\pi} \Big(\sin u \Big]_0^{\pi/2} \Big) \\
&= \frac{2}{\pi} \left(\sin(\pi/2) - \sin(0) \right)
\end{aligned}$$

The Substitution Rule: Exercises

Evaluate

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We take $u = \pi x/2$, then $u' = \pi/2$ and

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&= \frac{2}{\pi} \left(\sin u \right]_0^{\pi/2} \Big) \\
&= \frac{2}{\pi} \left(\sin(\pi/2) - \sin(0) \right) \\
&= \frac{2}{\pi}
\end{aligned}$$

Symmetry

Symmetry can sometimes help to simplify integrals!

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Suppose f is continuous on $[-a, a]$:

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The function f is

$f(-x)$

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$$f(-x) = \frac{\sin(-x)/\cos(-x)}{1 + (-x)^2 + (-x)^4}$$

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The function f is

$$f(-x) = \frac{\sin(-x)/\cos(-x)}{1 + (-x)^2 + (-x)^4} = \frac{-\sin x/\cos x}{1 + x^2 + x^4} = -f(x)$$

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The function f is odd since

$$f(-x) = \frac{\sin(-x)/\cos(-x)}{1 + (-x)^2 + (-x)^4} = \frac{-\sin x/\cos x}{1 + x^2 + x^4} = -f(x)$$

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