Calculus M211

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Indiana University Bloomington

2013

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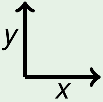

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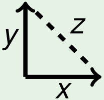

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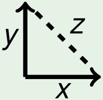

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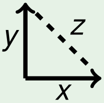

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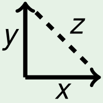

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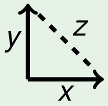

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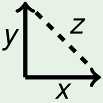

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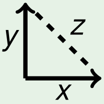

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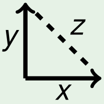

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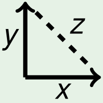

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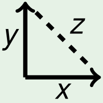

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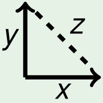

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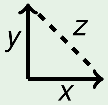

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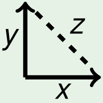

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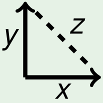

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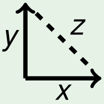

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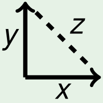

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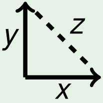

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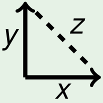

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Their distance increases with $\sqrt{13}$ miles per hour.

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A bacteria culture is growing under ideal conditions and doubling every hour. If the initial population is 100 bacteria,

How many bacteria will there be after half an hour?

The formula for the population is of the form $P(t) = 100 \cdot e^{kt}$.

Let's determine k. After 1h we have 200 bacteria, thus

$$200 = 100 \cdot e^{k \cdot 1} \implies 2 = e^k \implies k = \ln 2$$

Thus $P(t) = 100 \cdot e^{t \ln 2} = 100 \cdot 2^t$.

After half an hour we have $100 \cdot 2^{\frac{1}{2}} \approx 141$ bacteria.

At what rate will the population be increasing at that point?

The rate of growth is $P'(t) = 100 \cdot \ln 2 \cdot 2^t$.

After half an hour the rate of growth is $100 \cdot \ln 2 \cdot 2^{\frac{1}{2}}$.

When will the bacteria population reach 1000?

$$1000 = 100 \cdot e^{t \ln 2} \iff 10 = e^{t \ln 2} \iff \ln 10 = t \ln 2$$

Thus after $t = \ln 10 / \ln 2 \approx 3.3$ hours.

Let $v(t)$ in m/s be the velocity of a particle moving along a line.

Review - Midterm Exam 3

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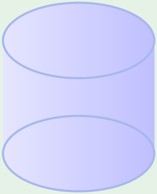

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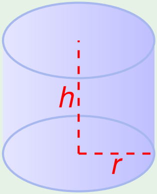

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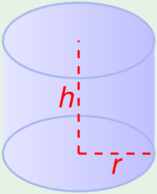

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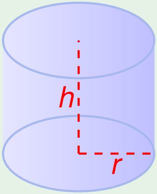

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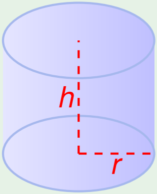

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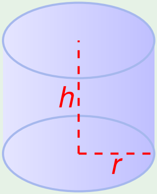

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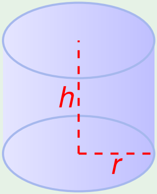

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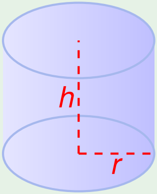

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$V =$

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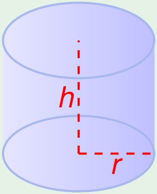

$V = \pi r^2 h$

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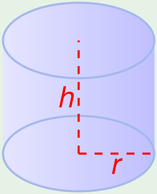

Introducing notation:

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$$V = \pi r^2 h = 1$$

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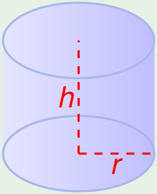

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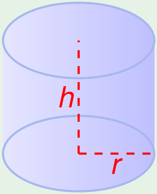

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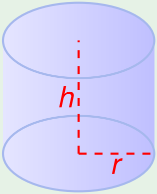

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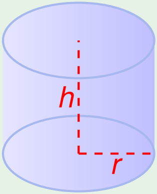

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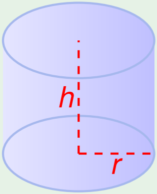

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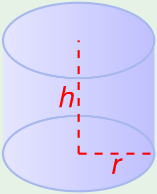

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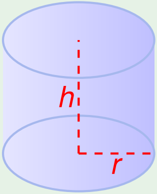

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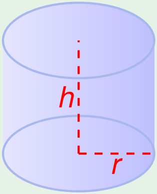

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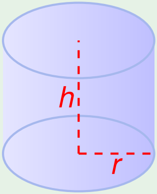

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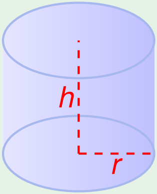

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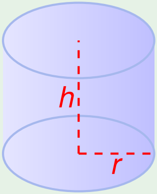

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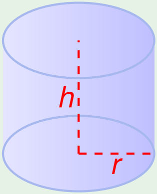

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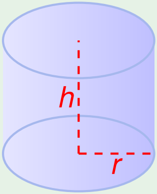

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Cannot use Closed Interval Method since $(0, \infty)$ is not closed.

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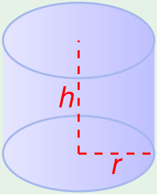

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Cannot use Closed Interval Method since $(0, \infty)$ is not closed.

However, $A(1/\sqrt[3]{2\pi})$ must be the **absolute minimum** since:

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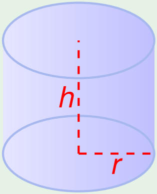

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Review - Midterm Exam 3

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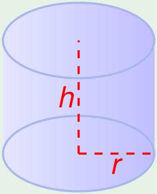

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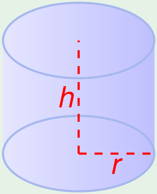

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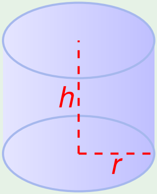

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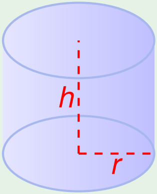

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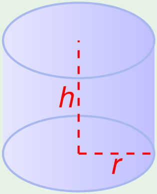

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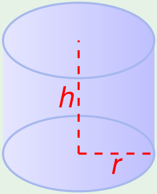

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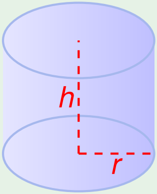

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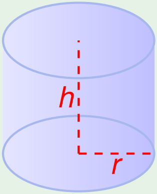

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Hence radius $r = 1/\sqrt[3]{2\pi}$ and height $h = 2r$ minimizes the cost.

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Review - Midterm Exam 3

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We have to find the x-intercepts in the interval $[0, \pi/2]$:

$$\sin x - \cos = 0 \iff \sin x = \cos x \iff x = \pi/4$$

Antiderivative of $f(x) = \sin x - \cos x$ is $F(x) = -\cos x - \sin x$:

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Review - Midterm Exam 3

Find the area of the region bounded by the curves

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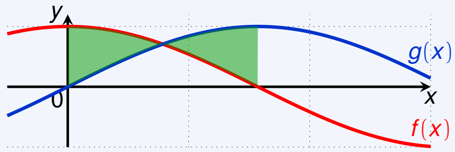

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Review - Midterm Exam 3

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$$\implies 2x^2 = 1 \implies x = \sqrt{1/2}$$

Thus the slope is -1 at point $(\sqrt{1/2}, \sqrt{1/2})$.

Review - Midterm Exam 3

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Review - Midterm Exam 3

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Review - Midterm Exam 3

We consider the function $f(x)$ with

$$f(x) = \frac{1 - x^3}{1 + x^3} \quad f'(x) = \frac{6x^2}{(1 + x^3)^2} \quad f''(x) = \frac{12x(2x^3 - 1)}{(1 + x^3)^3}$$

Find all

- horizontal, vertical asymptotes,
- the left and right limits at vertical asymptotes,
- points with horizontal tangents and local extrema
- on which intervals is f increasing/decreasing?
- on which intervals is f concave up/down?
- inflection points

Then sketch the graph of $f(x)$.

Review - Midterm Exam 3

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Review - Midterm Exam 3

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Review - Midterm Exam 3

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So the critical numbers are

- $x = 0$, then $f'(x)$ undefined
- $x = \frac{15}{8}$, then $f'(x) = 0$

Review - Midterm Exam 3

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Review - Midterm Exam 3

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Review - Midterm Exam 3

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Hence the equation has a solution $x = c$.