Calculus M211

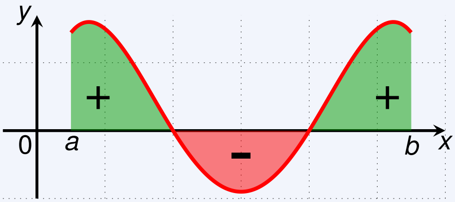
Jörg Endrullis

Indiana University Bloomington

2013

Area Between Curves

The definite integral can be interpreted as the **net area**, that is:



$$\int_a^b f(x)dx$$

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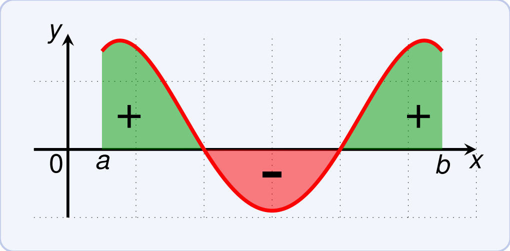

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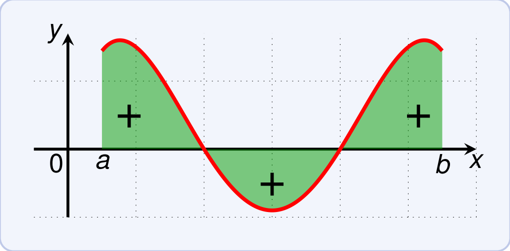
What if we want the area between the curve and the x-axis?



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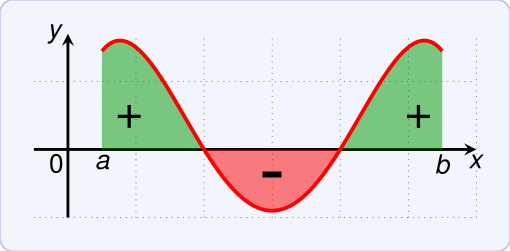

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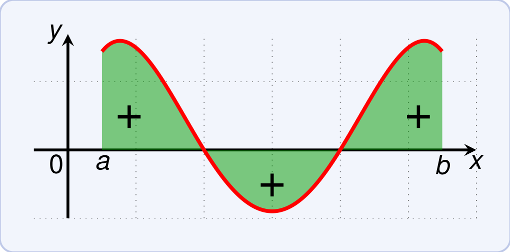

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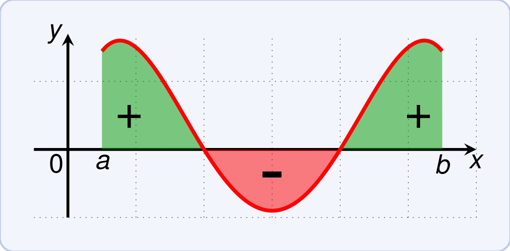

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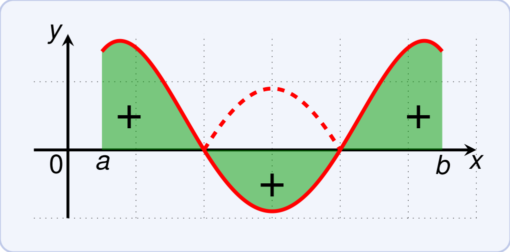

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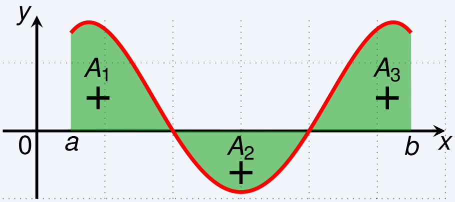

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Then the area between the curve f and the x-axis from a to b is

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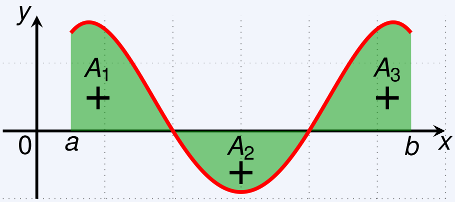

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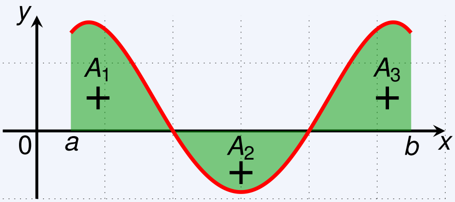

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$$A = \int_a^b |f(x)| dx$$

To evaluate the integral, we split the it into A_1, A_2 and A_3.
Thus we must find the x-intercepts in $[a, b]$!

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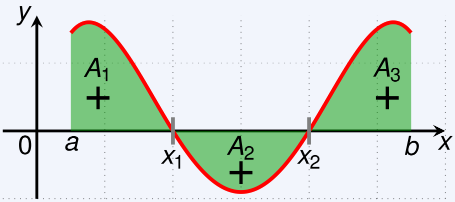

For example, let us consider the diagram above.

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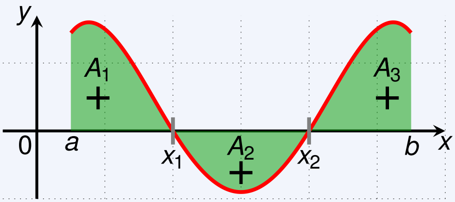

For example, let us consider the diagram above.

The area between the curve and the x-axis from a to b is

$$A = \int_a^b |f(x)|dx \; = \left|\int_a^{x_1} f(x)\,dx\right| + \left|\int_{x_1}^{x_2} f(x)\,dx\right| + \left|\int_{x_2}^{b} f(x)\,dx\right|$$

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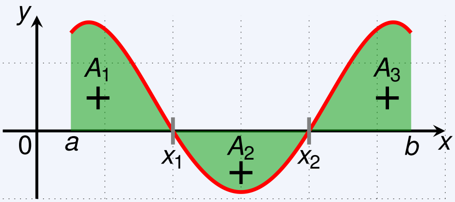

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Note that we split the integral from a to the first x-intercept, from the first to the second x-intercept,...

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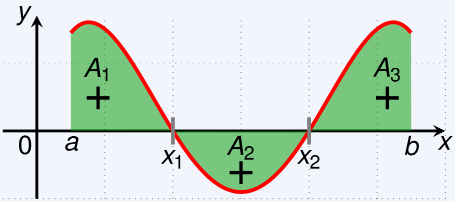

Let f be continuous on $[a, b]$, and let

- $x_1 < x_2 < \ldots < x_n$ be all x-intercepts in $[a, b]$,
- define $x_0 = a$ and $x_{n+1} = b$

Then the area between the curve f and the x-axis from a to b is

$$A = \int_a^b |f(x)| dx = \sum_{i=0}^{n} \left| \int_{x_i}^{x_{i+1}} f(x)\, dx \right|$$

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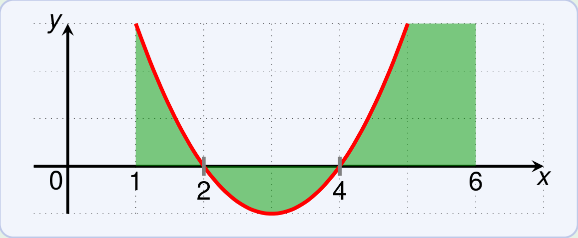

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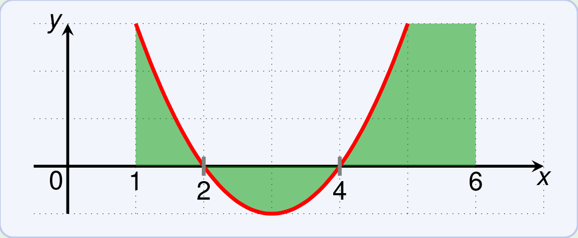

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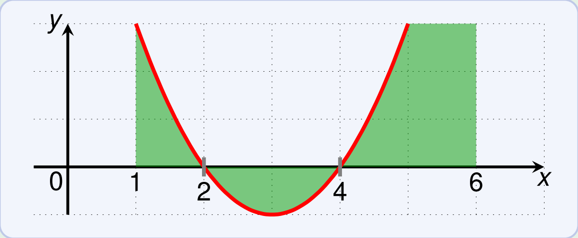

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&= \frac{28}{3}
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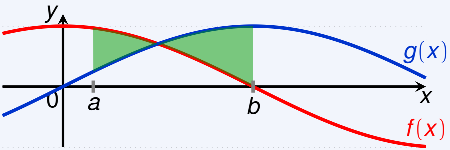
The area between two curves $f(x)$ and $g(x)$ from a to b is:

$$A = \int_a^b |f(x) - g(x)| dx$$



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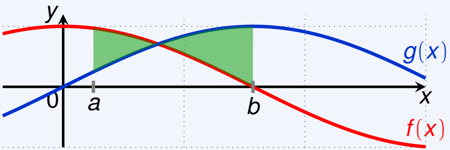
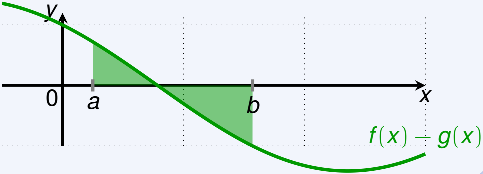


The area between f and g = area between $f - g$ and the x-axis.



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$$= |(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}) - (-1 - 0)| + |(-0 - 1) - (-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}})|$$

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$$A = \left| \int_0^{\pi/4} f(x) dx \right| + \left| \int_{\pi/4}^{\pi/2} f(x) dx \right| = |F(x)]_0^{\pi/4}| + |F(x)]_{\pi/4}^{\pi/2}|$$

$$= |(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}) - (-1 - 0)| + |(-0 - 1) - (-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}})|$$

$$= | - \frac{2}{\sqrt{2}} + 1| + | - 1 + \frac{2}{\sqrt{2}}|$$

Area Between Curves

Find the area of the region bounded by the curves

$$y = \sin x \qquad y = \cos x \qquad x = 0 \qquad x = \pi/2$$

Area is equal to the area between $\sin x - \cos x$ and the x-axis:

$$A = \int_0^{\pi/2} |\sin x - \cos x| dx$$

We have to find the x-intercepts in the interval $[0, \pi/2]$:

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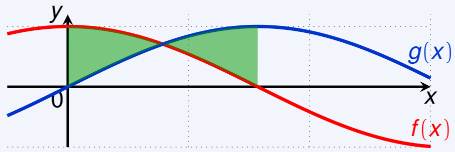

$$A = 2\sqrt{2} - 2$$