

Calculus M211

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2013

Indefinite Integrals

The **indefinite integral**

$$\int f(x) dx$$

is a notation for an antiderivative. That is

$$\int f(x) dx = F(x) \quad \text{means} \quad F'(x) = f(x)$$

For example

$$\int x^2 dx = \frac{1}{3}x^3 + C$$

Note the difference between the definite and indefinite integral!

The definite integral $\int_a^b f(x) dx$ is a number.

The indefinite integral $\int f(x) dx$ is a function.

Indefinite Integrals

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The 2nd part of the Fundamental Theorem can be restated as:

If f is a continuous function then

$$\int_a^b f(x) dx = \left[\int f(x) dx \right]_a^b$$

Indefinite Integrals

Table of basic indefinite integrals:

$$\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$$

$$\int cf(x) dx = c \int f(x) dx$$

$$\int k dx = kx + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{x} dx = \ln |x| + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

Indefinite Integrals

Find the general indefinite integral

$$\begin{aligned}\int (10x^4 - 2 \sec^2 x) dx &= \int (10x^4) dx - \int (2 \sec^2 x) dx \\ &= 10 \int x^4 dx - 2 \int \sec^2 x dx \\ &= 10 \frac{1}{5} x^5 - 2 \tan x + C \\ &= 2x^5 - 2 \tan x + C\end{aligned}$$

Find the general indefinite integral

$$\begin{aligned}\int \left(\frac{\cos \phi}{\sin^2 \phi} \right) d\phi &= \int \left(\frac{1}{\sin \phi} \cdot \frac{\cos \phi}{\sin \phi} \right) d\phi \\ &= \int (\csc \phi \cdot \cot \phi) d\phi \\ &= -\csc \phi + C\end{aligned}$$

Indefinite Integrals

Evaluate

$$\begin{aligned}\int_0^3 (x^3 - 6x) dx &= \left[\frac{1}{4}x^4 - 3x^2 \right]_0^3 \\ &= \left(\frac{1}{4}3^4 - 3 \cdot 3^2 \right) - \left(\frac{1}{4}0^4 - 3 \cdot 0^2 \right) \\ &= \frac{81}{4} - 27\end{aligned}$$

Evaluate

$$\begin{aligned}\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} dt &= \int_1^9 \left(2 + \sqrt{t} - \frac{1}{t^2} \right) dt \\ &= \left(2t + \frac{2}{3}t^{\frac{3}{2}} + \frac{1}{t} \right) \Big|_1^9 \\ &= \left(2 \cdot 9 + \frac{2}{3}(\sqrt{9})^3 + \frac{1}{9} \right) - \left(2 + \frac{2}{3} + 1 \right) \\ &= 292/9\end{aligned}$$

Indefinite Integrals: Applications

We can reformulate part 2 of the Fundamental Theorem as:

The integral of a rate of change is the net change.

$$\int_a^b F'(x) dx = F(b) - F(a)$$

Applications:

- ▶ $V(t)$ is the amount of water in a reservoir at time t
- ▶ $V'(t)$ is the rate at which water flows in or out

Then

$$\int_{t_1}^{t_2} V'(t) dt = V(t_2) - V(t_1)$$

is the net change in the amount of water from time t_1 to t_2 .

Indefinite Integrals: Applications

We can reformulate part 2 of the Fundamental Theorem as:

The integral of a rate of change is the net change.

$$\int_a^b F'(x) dx = F(b) - F(a)$$

Applications:

- ▶ $C(t)$ is the concentration of a product of a chemical reaction at time t
- ▶ $C'(t)$ is the rate of reaction

Then

$$\int_{t_1}^{t_2} C'(t) dt = C(t_2) - C(t_1)$$

is the change in concentration of C from time t_1 to t_2 .

Indefinite Integrals: Applications

We can reformulate part 2 of the Fundamental Theorem as:

The integral of a rate of change is the net change.

$$\int_a^b F'(x) dx = F(b) - F(a)$$

Applications:

- ▶ $C(x)$ is the costs of producing x units of some product
- ▶ $C'(x)$ is the marginal costs

Then

$$\int_{x_1}^{x_2} C'(x) dx = C(x_2) - C(x_1)$$

is the increase in costs when the production is increased from x_1 to x_2 .

Indefinite Integrals: Applications

We can reformulate part 2 of the Fundamental Theorem as:

The integral of a rate of change is the net change.

$$\int_a^b F'(x) dx = F(b) - F(a)$$

Applications:

We consider an object moving in a straight line:

- ▶ $s(t)$ is the position function
- ▶ $v(t) = s'(t)$ is the velocity

Then

$$\int_{t_1}^{t_2} v(t) dt = s(t_2) - s(t_1)$$

is the net change of the position, the **displacement**, from time t_1 to t_2 .

Indefinite Integrals: Applications

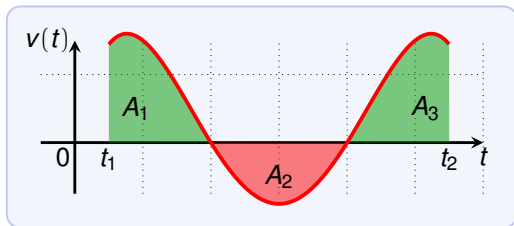
We consider an object moving in a straight line:

- ▶ $v(t)$ is the velocity

Then

$$\int_{t_1}^{t_2} |v(t)| dt$$

is the **total distance** the object traveled during the time interval.



$$\text{displacement} = \int_{t_1}^{t_2} v(t) dt = A_1 - A_2 + A_3$$

$$\text{total distance} = \int_{t_1}^{t_2} |v(t)| dt = A_1 + A_2 + A_3$$

Indefinite Integrals: Applications

We consider an object moving in a straight line:

- ▶ $a(t)$ is the acceleration
- ▶ $v(t)$ is the velocity

Then

$$\int_{t_1}^{t_2} a(t) dt = v(t_2) - v(t_1)$$

is the net change in velocity from t_1 to t_2 .

Indefinite Integrals: Applications

We consider an object moving in a straight line with velocity

$$v(t) = t^2 - t - 6 \text{ m/s}$$

Find the displacement and distance traveled during $1 \leq t \leq 4$.

The displacement is

$$\begin{aligned}\int_1^4 v(t) dt &= \left(\frac{1}{3}t^3 - \frac{1}{2}t^2 - 6t \right) \Big|_1^4 \\ &= \left(\frac{1}{3}4^3 - \frac{1}{2}4^2 - 6 \cdot 4 \right) - \left(\frac{1}{3}1^3 - \frac{1}{2}1^2 - 6 \cdot 1 \right) \\ &= \left(\frac{64}{3} - \frac{16}{2} - 24 \right) - \left(\frac{1}{3} - \frac{1}{2} - 6 \right) = -\frac{9}{2}\end{aligned}$$

The displacement is -4.5m .

Indefinite Integrals: Applications

We consider an object moving in a straight line with velocity

$$v(t) = t^2 - t - 6 \text{ m/s}$$

Find the displacement and distance traveled during $1 \leq t \leq 4$.

The total distance traveled is $\int_1^4 |v(t)| dt$

We need to find the x -intercepts of $v(t)$ in $[1, 4]$:

$$v(t) = (t + 2)(t - 3) = 0 \quad \iff \quad t = -2 \text{ or } t = 3$$

Thus we have:

$$\begin{aligned} \int_1^4 |v(t)| dt &= \left| \int_1^3 v(t) \right| + \left| \int_3^4 v(t) \right| \\ &= \left| \left(\frac{1}{3}v^3 - \frac{1}{2}t^2 - 6t \right) \Big|_1^3 \right| + \left| \left(\frac{1}{3}v^3 - \frac{1}{2}t^2 - 6t \right) \Big|_3^4 \right| \\ &= |-22/3| + |17/6| = 61/6 \end{aligned}$$

The total distance traveled is $61/6$ m.