Calculus M211

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Indiana University Bloomington

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Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus establishes a connection between:

- differentiation calculus, and
- integration calculus

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Differentiation and integration are inverse processes!

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The second part can be written as:
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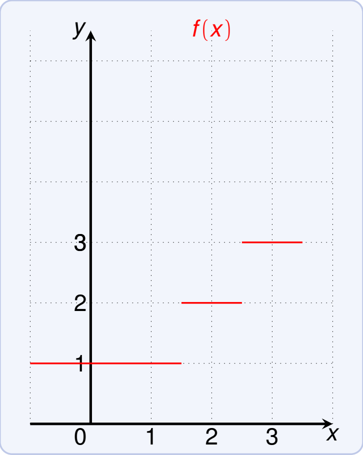

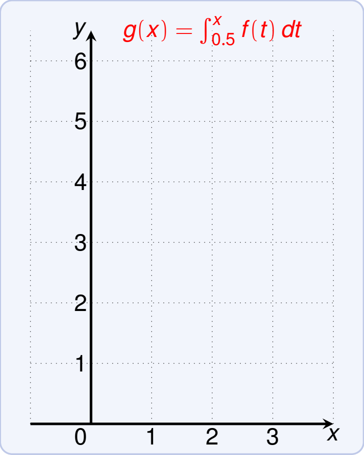

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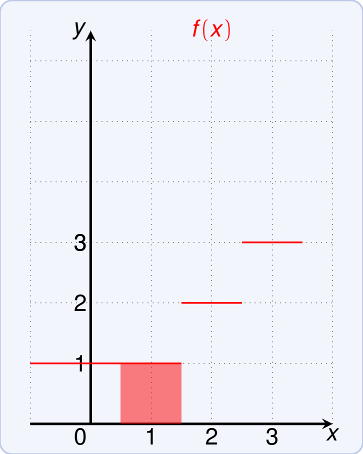

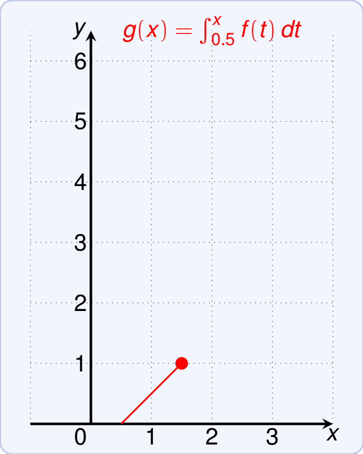

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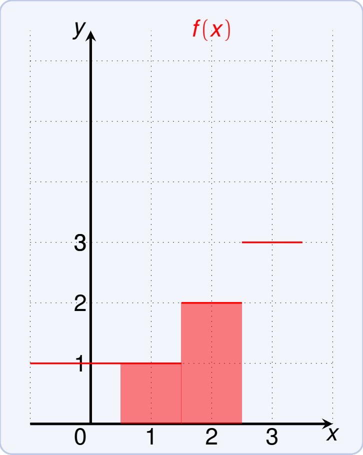

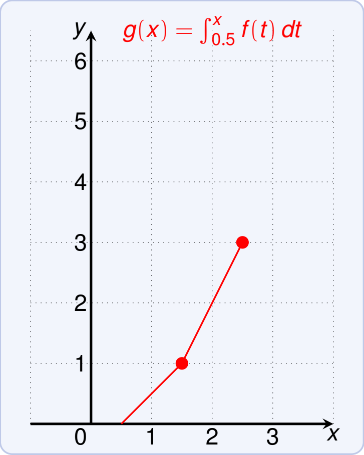

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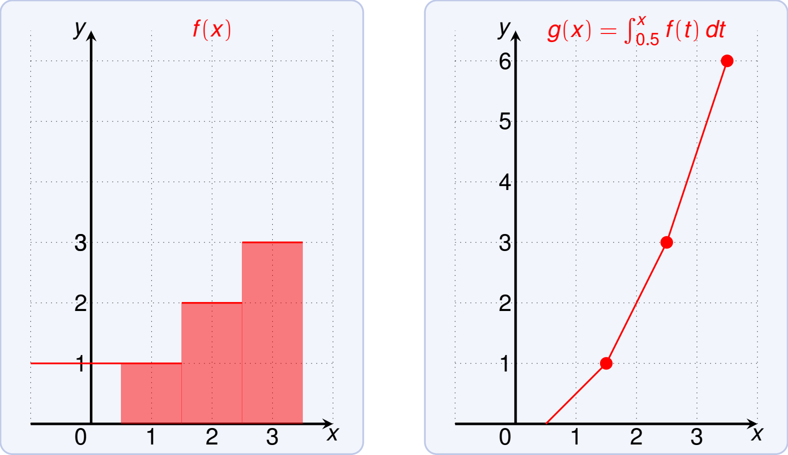

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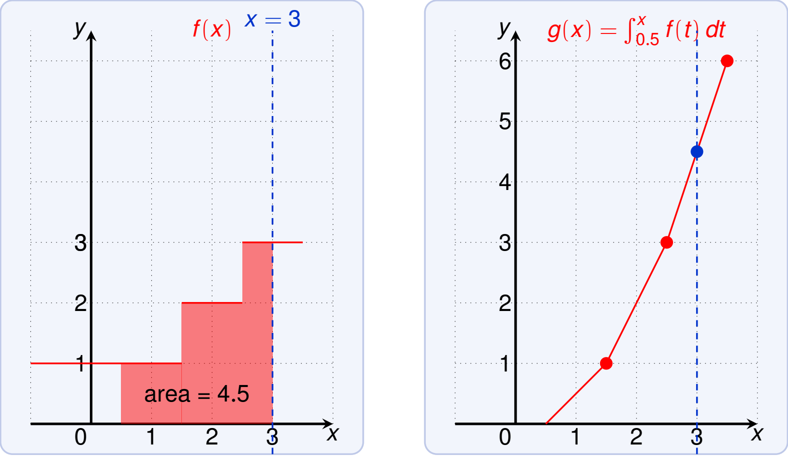

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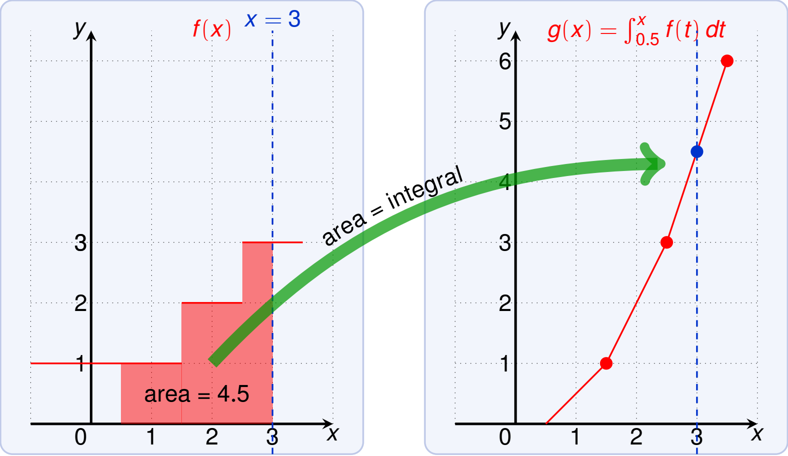

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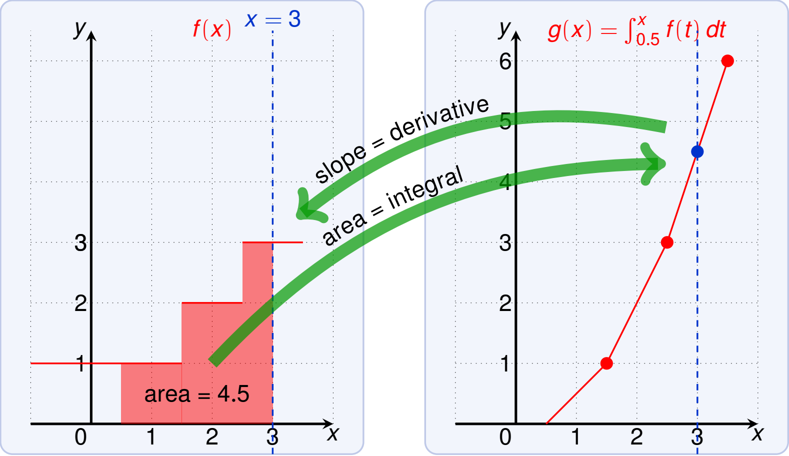

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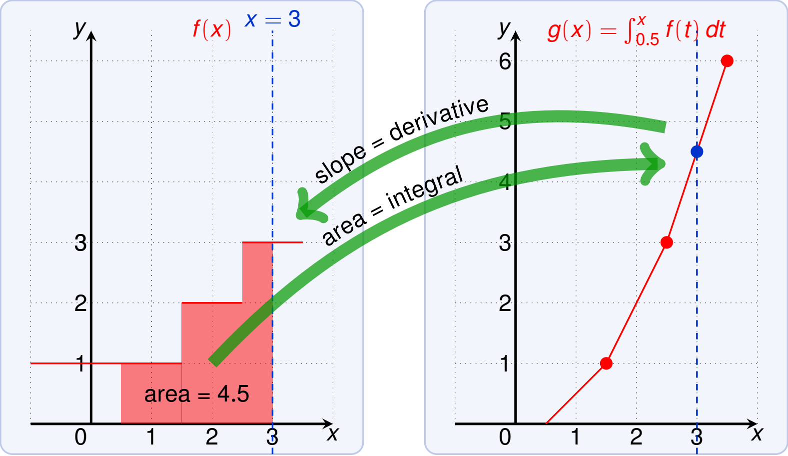

Observe: $g'(x) = f(x)$ except where f is not continuous.

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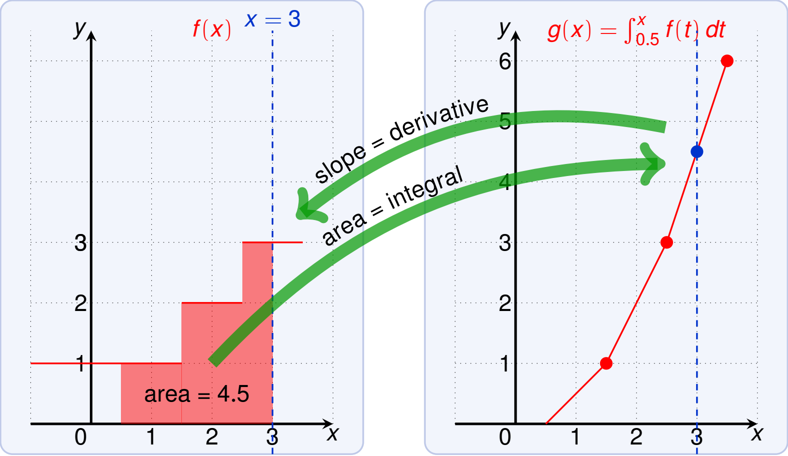

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The slope (derivative) is the inverse of taking the area (integral).

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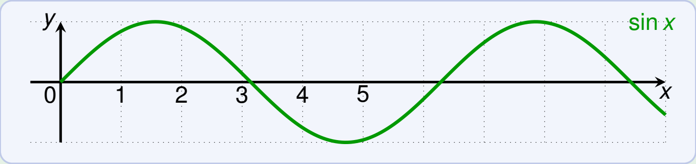
$\int_0^x \sin x \, dx =$



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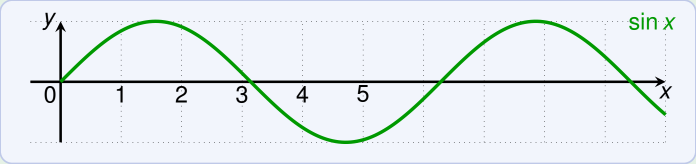

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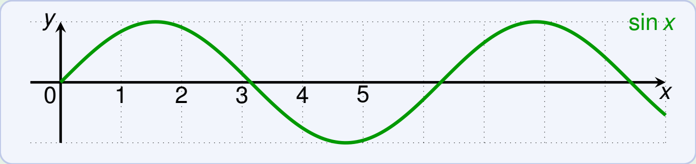

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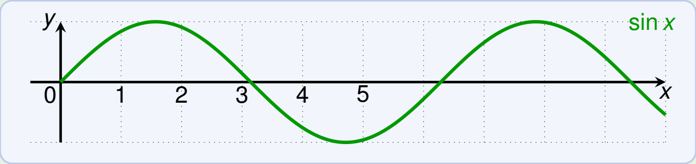

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By the Fundamental Theorem of Calculus, part 1:

$$g'(x) = \sqrt{1 + x^2}$$

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We could have used any antiderivative $F(x) = e^x + C$!

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Alternative notation

$$F(x)\Big|_a^b \quad = \quad \Big[F(x)\Big]_a^b \quad = \quad F(x)\Big]_a^b$$

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from 0 to 1.

From 0 to 1 the curve is above the x-axis. Thus area = integral.

An antiderivative of f is $F(x) = \frac{1}{3}x^3$.

By the Fundamental Theorem, the area is:

$$A = \int_0^1 x^2\,dx = \frac{1}{3}x^3\Big]_0^1 = \frac{1}{3}1^3 - \frac{1}{3}0^3$$

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Does this make sense?

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The calculation is wrong since $\frac{1}{x^2}$ is not continuous on $[-1, 3]$!

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$$\int_{-1}^{3} \frac{1}{x^2} dx \quad \text{does not exist}$$

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