Calculus M211

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The Definite Integral

The **definite integral of** f **from** a **to** b is

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

provided that the limit exists, and has the same value for all possible choices of the **sample points**

$$x_i \text{ from the interval } [a+(i-1)\Delta x,\ a+i\Delta x]$$

where $\Delta x = \frac{b-a}{n}$.

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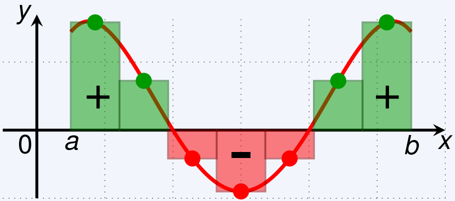

The **Riemann sum** is the sum of the area of rectangles above the x-axis (the green ones) minus the sum of the area of the rectangles below the x-axis (the red ones).

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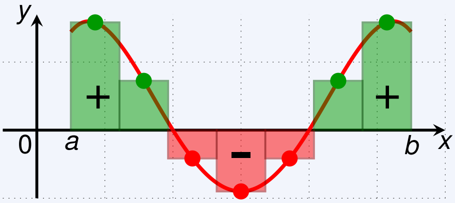

The **Riemann sum** is the sum of the area of rectangles above the x-axis (the green ones) minus the sum of the area of the rectangles below the x-axis (the red ones).

The sample points x_i can be arbitrary from the i-th interval:

- left endpoints, right endpoints or middle of the interval, or
- at maximum (upper sum), or at minimum (lower sum).

Evaluate the Riemann sum for

$$f(x) = 2x - 5$$

from 0 to 6 using 3 strips and right endpoints as sample points.

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Thus the Riemann sum using 3 strips and right endpoints is:

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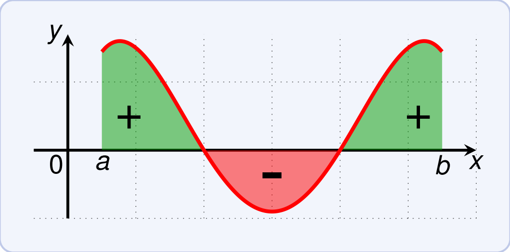
The Definite Integral

The definite integral can be interpreted as the **net area**, that is:

$$\int_a^b f(x)dx = A_1 - A_2$$

where

- A_1 is the area of above the x-axis, below the curve,
- A_2 is the area of below the x-axis, above the curve.



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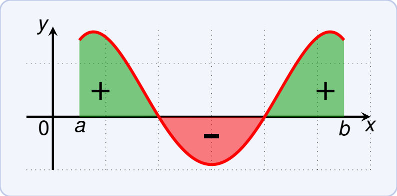

Evaluate the integral

$$\int_0^1 \sqrt{1 - x^2}dx$$

by interpreting it in terms of the area.

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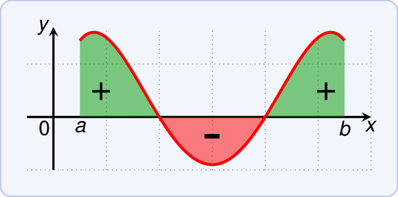

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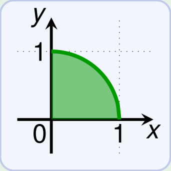

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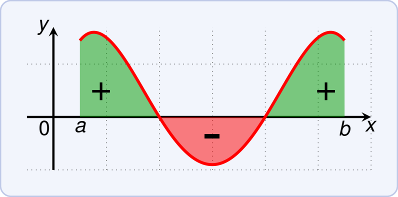

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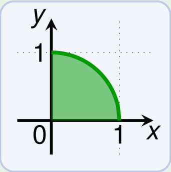

Thus the area is $1/4$ of the area of a circle with radius 1:

$$\int_0^1 \sqrt{1 - x^2} dx = \frac{\pi}{4}$$

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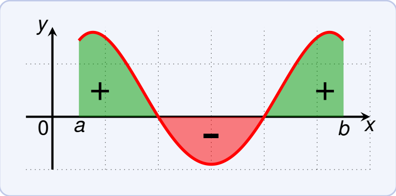

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$$\int_0^3 (x-1)dx$$

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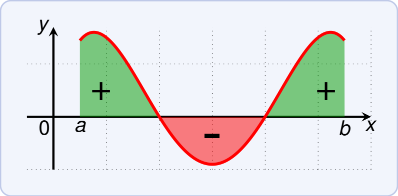

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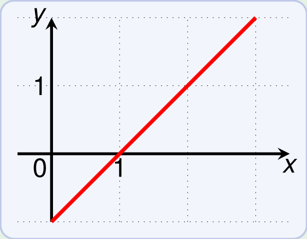

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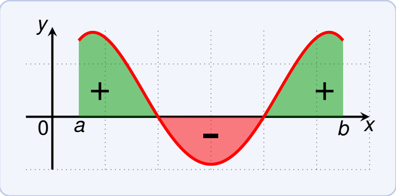

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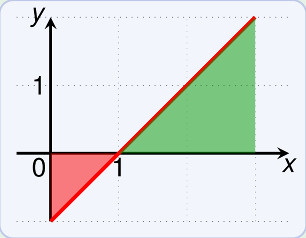

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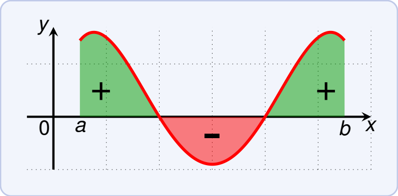

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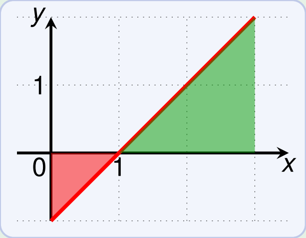

Thus the integral is:

$$\int_0^3 (x-1)dx = \frac{1}{2}(2\cdot 2) - \frac{1}{2}(1\cdot 1) = 1.5$$

The Definite Integral

The integral is a number.

The variable name x does not influence the integral:

$$\int_a^b f(x)dx = \int_a^b f(t)dt = \int_a^b f(r)dr$$

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However, most of the functions we work with are:

If

- f is continuous on $[a, b]$, or
- f has only a finite number of jump discontinuities,

then f is integrable on $[a, b]$, that is, the $\int_a^b f(x)dx$ exist.

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If f is integrable on $[a, b]$, then the limit

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

gives the same value no matter how we choose the sample points x_i from the i-th interval.

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Thus for simplicity we can choose the right end points.

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This simplifies the definition of the definite integral:

If f is integrable on $[a, b]$, then

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta$.

Computing with Sums

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$$\sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

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Evaluate the definite integral of f from 0 to 6 is

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \qquad \text{where } f(x) = 2x - 5$$

using right endpoints for the sample points x_i.

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- the i-th interval is $[0 + (i-1)\Delta x,\ 0 + i\Delta x]$
- the right endpoints are $x_i = i\Delta x$

The Definite Integral

Evaluate the definite integral of f from 0 to 6 is

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \qquad \text{where } f(x) = 2x - 5$$

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$$= \frac{6}{n}\left(\frac{6}{n} n(n+1) - 5n\right)$$

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$$\int_a^b f(x)dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{6n+6}{n} = 6$$

Properties of the Definite Integral

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Assume $\int_0^{10} f(x)dx = 17$ and $\int_0^{8} f(x)dx = 12$, find $\int_8^{10} f(x)dx$.

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Use the properties of integrals to evaluate:

$$\begin{aligned}\int_0^1 (4 + 3x^2)dx &= \int_0^1 4\ dx + \int_0^1 3x^2 dx \\ &= 4 + 3\int_0^1 x^2 dx\end{aligned}$$

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We have already seen that

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The function e^{-x^2} is decreasing on $[0, 1]$.

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Thus on $[0, 1]$: maximum is

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Thus on $[0,1]$: maximum is $f(0) = 1$

Comparison Properties of the Definite Integral

- If $f(x) \geq 0$ for all $a \leq x \leq b$, then
$$\int_a^b f(x)dx \geq 0$$
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$$\int_a^b f(x)dx \ \geq \ \int_a^b g(x)dx$$
- If $m \leq f(x) \leq M$ for all $a \leq x \leq b$, then
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$$e^{-1}(1-0) = e^{-1} \quad \leq \quad \int_0^1 e^{-x^2}dx \quad \leq \quad 1 = 1(1-0)$$