Calculus M211

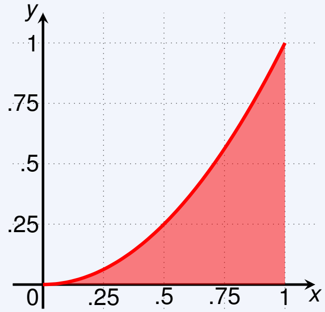
Jörg Endrullis

Indiana University Bloomington

2013

The Area below a Curve

How to compute the area below a curve?



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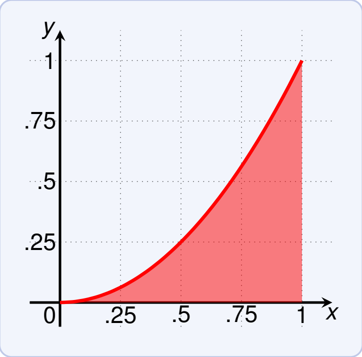

Idea:

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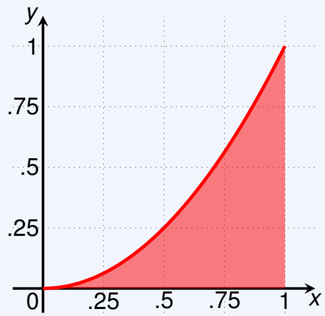

Idea:

- divide the area in vertical strips of equal width

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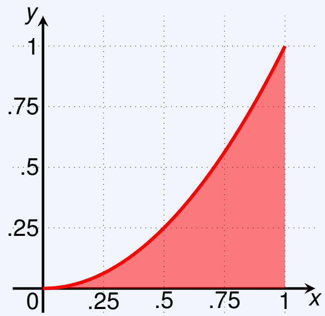
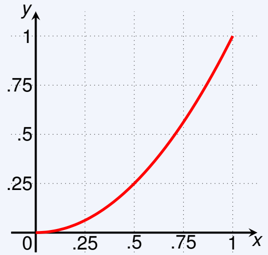


Idea:

- divide the area in vertical strips of equal width
- approximate the area using rectangles

The Area below a Curve

Estimate the area below the curve $f(x) = x^2$ from 0 to 1.



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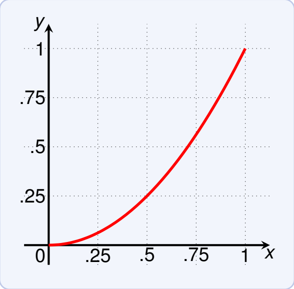

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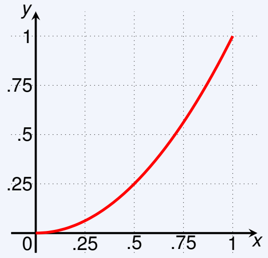

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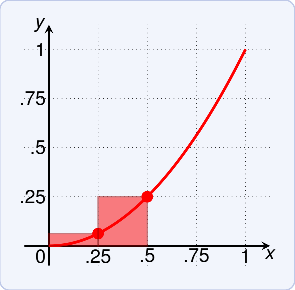

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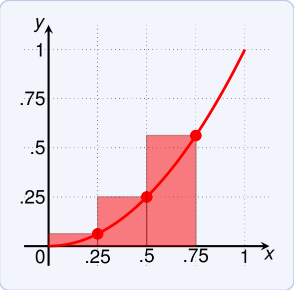

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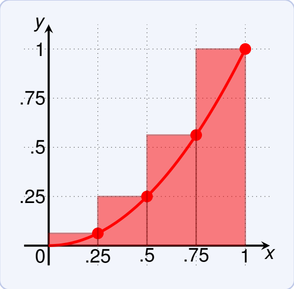

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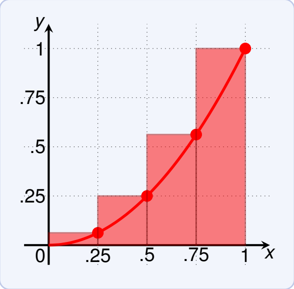

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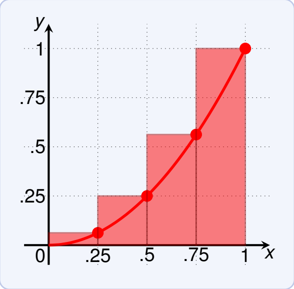

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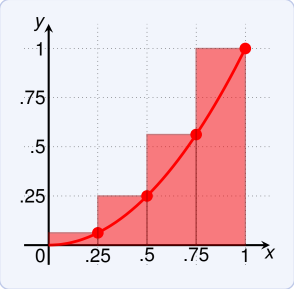

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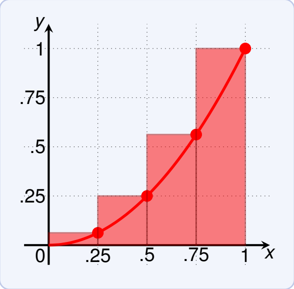

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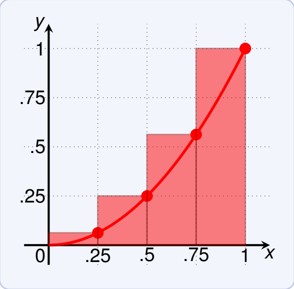

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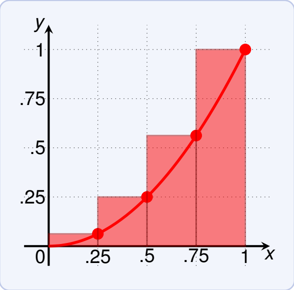

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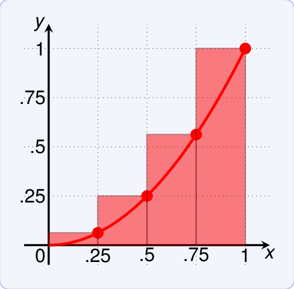

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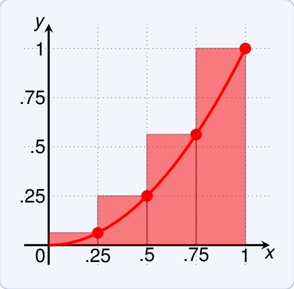

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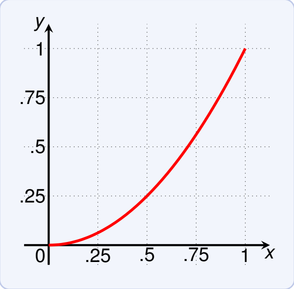

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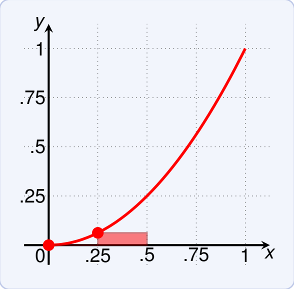

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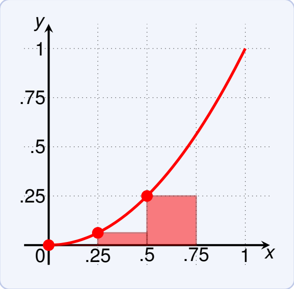

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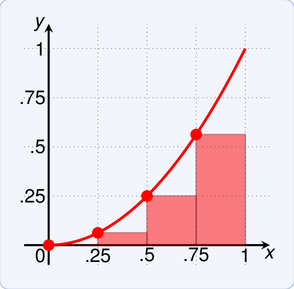

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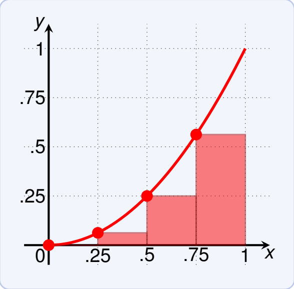

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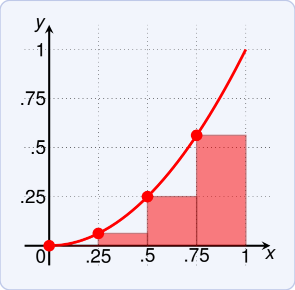

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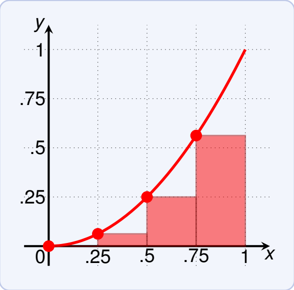

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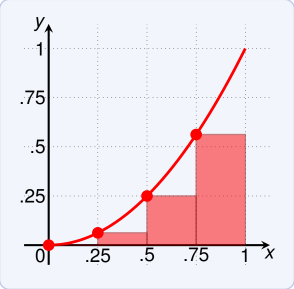

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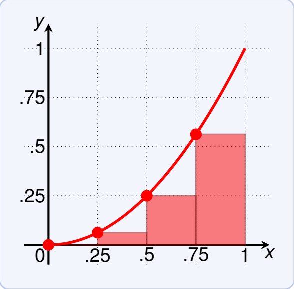

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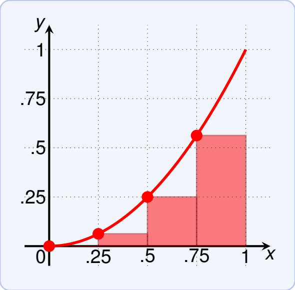

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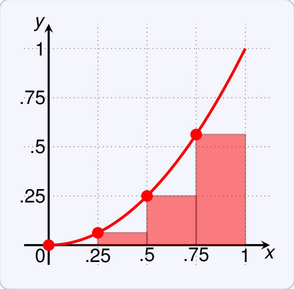

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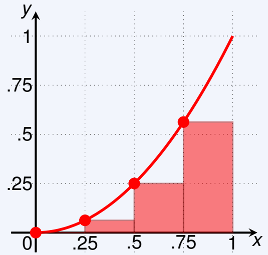

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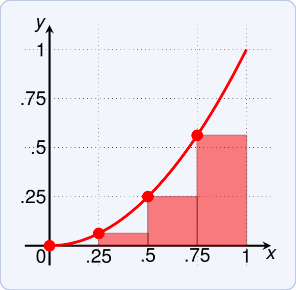

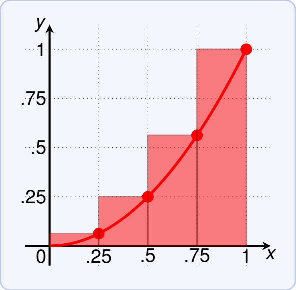

$$0.21875 = L_4 \quad < \quad A \quad < \quad R_4 = 0.46875$$

We have obtained an estimation of A:

- we can improve the estimation by taking more strips

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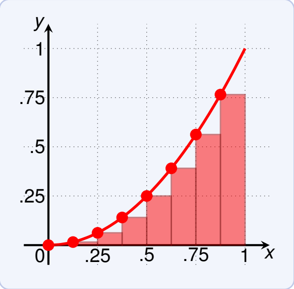

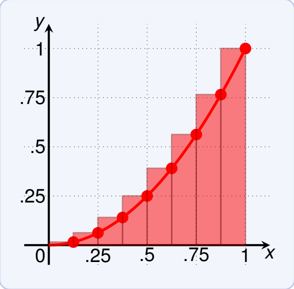

$$0.21875 = L_4 \quad < \quad A \quad < \quad R_4 = 0.46875$$
$$0.2734375 = L_8 \quad < \quad A \quad < \quad R_8 = 0.3984375$$

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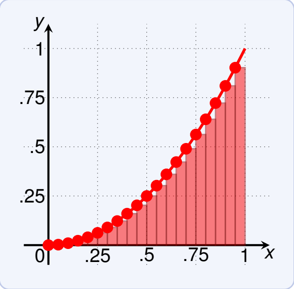

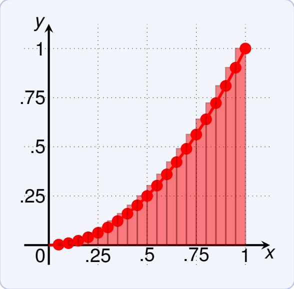

$$
\begin{array}{rcccl}
0.21875 = L_4 & < & A & < & R_4 = 0.46875 \\
0.2734375 = L_8 & < & A & < & R_8 = 0.3984375 \\
0.3087500 = L_{20} & < & A & < & R_{20} = 0.3587500
\end{array}
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$$= \frac{1}{n}\cdot\frac{1}{n^2}(1^2 + 2^2 + 3^2 + \ldots + n^2)$$

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$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

The Area below a Curve

Estimate the area below the curve $f(x) = x^2$ from 0 to 1.

We now let the number of strips go to infinity: $\lim_{n\to\infty} R_n$

The formula for the area R_n with n strips is:

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Hence, we have

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Hence, we have

$$\lim_{n\to\infty} R_n = \lim_{n\to\infty}\frac{1}{6}\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right) = \frac{2}{6}$$

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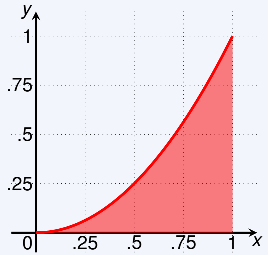


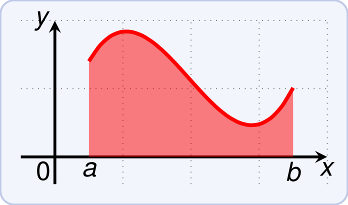
We define the area A to be the limit of these approximations

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} L_n = \frac{1}{3}$$

The Area below a Curve

Now let's look at a general curve above the x-axis:

The area below the curve of a function f on an interval $[a, b]$.



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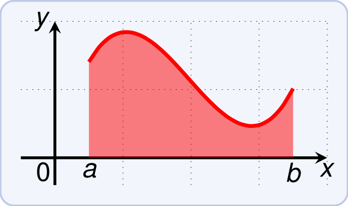

We use n rectangles:

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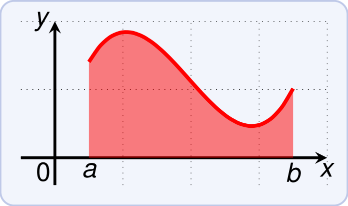

We use n rectangles:

- the width of the interval is $b - a$

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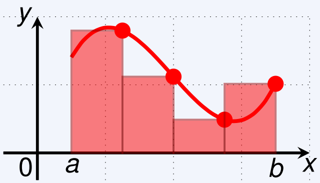

We use n rectangles:

- the width of the interval is $b - a$
- the width of each strip is $\Delta x =$

The Area below a Curve

Now let's look at a general curve above the x-axis:

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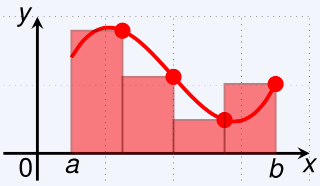


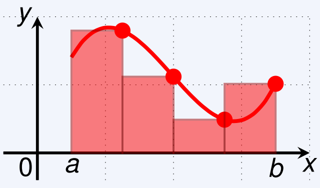
We use n rectangles:

- the width of the interval is $b - a$
- the width of each strip is $\Delta x = (b - a)/n$

The Area below a Curve

Now let's look at a general curve above the x-axis:

The area below the curve of a function f on an interval $[a, b]$.



We use n rectangles:

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- the interval for the i-th strip is

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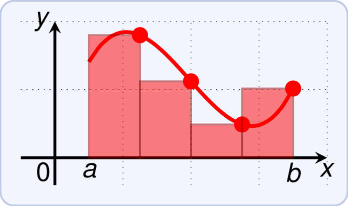

We use n rectangles:

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- the width of each strip is $\Delta x = (b - a)/n$
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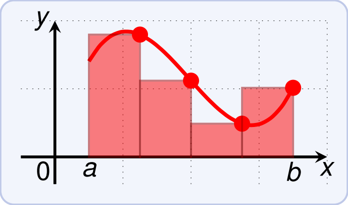

We use n rectangles: $\Delta x = (b - a)/n$

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Now let's look at a general curve above the x-axis:

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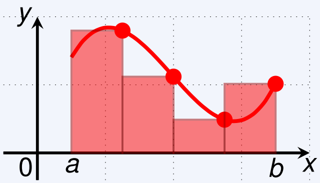

We use n rectangles: $\Delta x = (b - a)/n$

The area of the rectangles oriented at right-endpoints is:

$R_n =$

The Area below a Curve

Now let's look at a general curve above the x-axis:

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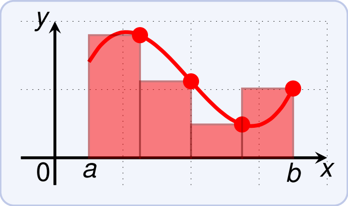

We use n rectangles: $\Delta x = (b - a)/n$

The area of the rectangles oriented at right-endpoints is:

$R_n = \Delta x \cdot f(a + 1\Delta x)$

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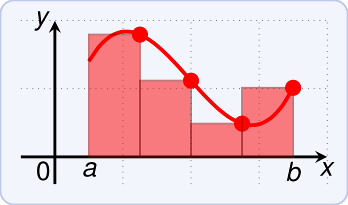
Now let's look at a general curve above the x-axis:

The area below the curve of a function f on an interval $[a, b]$.



We use n rectangles: $\Delta x = (b - a)/n$

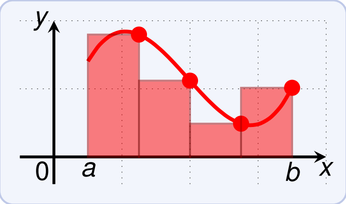
The area of the rectangles oriented at right-endpoints is:

$R_n = \Delta x \cdot f(a + 1\Delta x) + \Delta x \cdot f(a + 2\Delta x)$

The Area below a Curve

Now let's look at a general curve above the x-axis:

The area below the curve of a function f on an interval $[a, b]$.



We use n rectangles: $\Delta x = (b - a)/n$

The area of the rectangles oriented at right-endpoints is:

$$R_n = \Delta x \cdot f(a + 1\Delta x) + \Delta x \cdot f(a + 2\Delta x) + \ldots + \Delta x \cdot f(a + n\Delta x)$$

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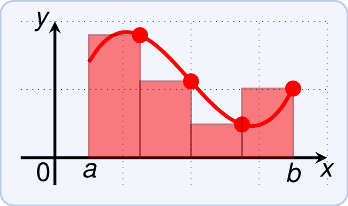

We use n rectangles: $\Delta x = (b - a)/n$

The area of the rectangles oriented at right-endpoints is:

$$\begin{aligned} R_n &= \Delta x \cdot f(a + 1\Delta x) + \Delta x \cdot f(a + 2\Delta x) + \ldots + \Delta x \cdot f(a + n\Delta x) \\ &= \Delta x\big(f(a + 1\Delta x) + f(a + 2\Delta x) + \ldots + f(a + n\Delta x)\big) \end{aligned}$$

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Now let's look at a general curve above the x-axis:

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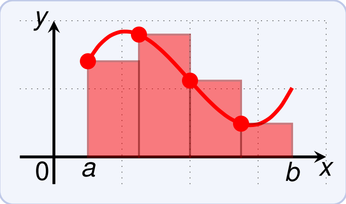

We use n rectangles: $\Delta x = (b - a)/n$

The area of the rectangles oriented at right-endpoints is:

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The area of the rectangles oriented at left-endpoints is:

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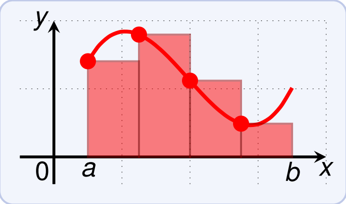

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The area of the rectangles oriented at left-endpoints is:

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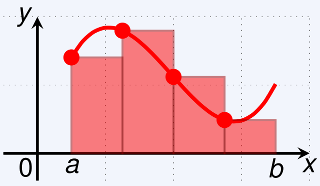

We use n rectangles: $\Delta x = (b - a)/n$

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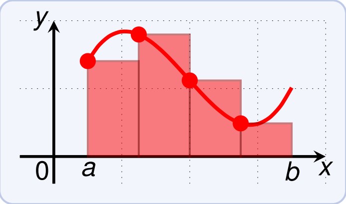

We use n rectangles: $\Delta x = (b - a)/n$

The area of the rectangles oriented at right-endpoints is:

$$\begin{aligned} R_n &= \Delta x \cdot f(a + 1\Delta x) + \Delta x \cdot f(a + 2\Delta x) + \ldots + \Delta x \cdot f(a + n\Delta x) \\ &= \Delta x \big(f(a + 1\Delta x) + f(a + 2\Delta x) + \ldots + f(a + n\Delta x) \big) \end{aligned}$$

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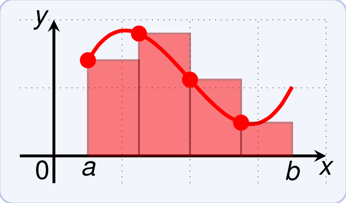

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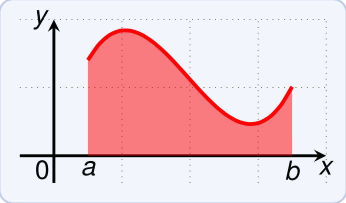
The Area below a Curve

The **area** A under the graph of a continuous function f whose graph lies above the x-axis is the limit:

$$\begin{aligned} A &= \lim_{n\to\infty} R_n \\ &= \lim_{n\to\infty} \left[\Delta x \big(f(a+1\Delta x) + f(a+2\Delta x) + \ldots + f(a+n\Delta x) \big)\right] \end{aligned}$$

where $\Delta x = (b-a)/n$.



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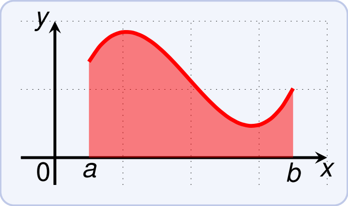

For continuous f this limit always exists, and is the same as

$$\lim_{n\to\infty} L_n = \lim_{n\to\infty} \left[\Delta x\big(f(a+0\Delta x) + \ldots + f(a+(n-1)\Delta x)\big)\right]$$

The Area below a Curve

Recall that the interval of the *i*-th strip is:

$$I_i = [a + (i-1)\Delta x,\ a + i\Delta x]$$

The Area below a Curve

Recall that the interval of the *i*-th strip is:

$$I_i = [a + (i-1)\Delta x,\ a + i\Delta x]$$

$$R_n = \Delta x \big(f(x_1) + f(x_2) + \ldots + f(x_n) \big)$$

where x_i is the right endpoint of the interval I_i

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where x_i is the right endpoint of the interval I_i

$$L_n = \Delta x\big(f(x_1) + f(x_2) + \ldots + f(x_n)\big)$$

where x_i is the left endpoint of the interval I_i

The Area below a Curve

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$$I_i = [a + (i-1)\Delta x,\ a + i\Delta x]$$

$$R_n = \Delta x\big(f(x_1) + f(x_2) + \ldots + f(x_n)\big)$$

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$$L_n = \Delta x\big(f(x_1) + f(x_2) + \ldots + f(x_n)\big)$$

where x_i is the left endpoint of the interval I_i

For continuous curve f above the x-axis we have:

$$A = \lim_{n\to\infty} \Big[\Delta x\big(f(x_1) + f(x_2) + \ldots + f(x_n)\big)\Big]$$

independent of what **sample points** x_i we take from I_i**.**

The Area below a Curve

Recall that the interval of the *i*-th strip is:

$$I_i = [a + (i-1)\Delta x,\ a + i\Delta x]$$

$$R_n = \Delta x\big(f(x_1) + f(x_2) + \ldots + f(x_n)\big)$$

where x_i is the right endpoint of the interval I_i

$$L_n = \Delta x\big(f(x_1) + f(x_2) + \ldots + f(x_n)\big)$$

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For continuous curve f above the x-axis we have:

$$A = \lim_{n\to\infty} \left[\Delta x\big(f(x_1) + f(x_2) + \ldots + f(x_n)\big)\right]$$

independent of what **sample points** x_i we take from I_i.

The limit is the same no matter what x_i we choose from I_i!

The Area below a Curve

Recall that the interval of the i-th strip is:

$$I_i = [a + (i-1)\Delta x,\ a + i\Delta x]$$

$$R_n = \Delta x\big(f(x_1) + f(x_2) + \ldots + f(x_n)\big)$$

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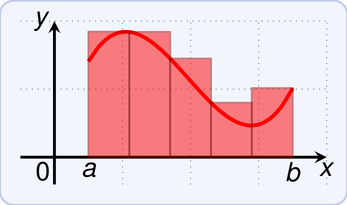
A famous choice of sample points are upper and lower sums. . .

The Area below a Curve



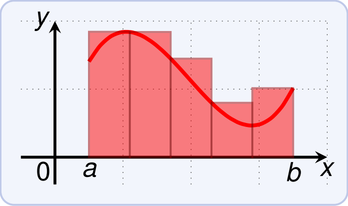
upper sum U_5

The **upper sum** is

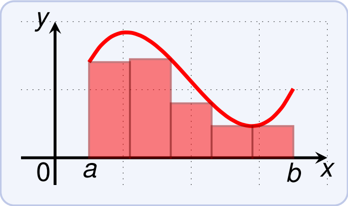
$$U_n = \Delta x \big(f(x_1) + f(x_2) + \ldots + f(x_n) \big)$$

where x_i is chosen from I_i such that $f(x_i)$ is the maximum on I_i

The Area below a Curve



upper sum U_5



lower sum D_5

The **upper sum** is

$$U_n = \Delta x\big(f(x_1) + f(x_2) + \ldots + f(x_n)\big)$$

where x_i is chosen from I_i such that $f(x_i)$ is the maximum on I_i

The **lower sum** is

$$D_n = \Delta x\big(f(x_1) + f(x_2) + \ldots + f(x_n)\big)$$

where x_i is chosen from I_i such that $f(x_i)$ is the minimum on I_i

Sigma Notation

We can use the **sigma notation** to write sums more compactly:

$$\sum_{i=1}^{n} h(i) \quad = \quad h(1) + h(2) + \ldots + h(n)$$

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$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

The Area below a Curve

The area under a curve f above the x-axis from a to b is:

$$A = \lim_{n\to\infty} \left(\sum_{i=1}^{n} \Delta x \cdot f(x_i) \right)$$

where:

- $\Delta x = (b-a)/n$ is the width of the strips,
- $I_i = [a+(i-1)\Delta x,\ a+i\Delta x]$ is the interval of the i-th strip,
- x_i is the sample point from the i-th interval I_i.

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- left endpoint $x_i = a + (i-1)\Delta x$ of the interval
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Usual choices for x_i are

- left endpoint $x_i = a + (i-1)\Delta x$ of the interval
- right endpoint $x_i = a + i\Delta x$ of the interval
- middle $x_i = a + (i - \frac{1}{2})\Delta x$ of the interval

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- middle $x_i = a+(i-\frac{1}{2})\Delta x$ of the interval
- upper sum: $f(x_i)$ is the maximum on the interval I_i

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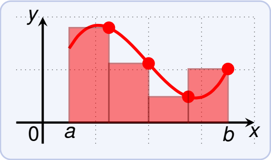
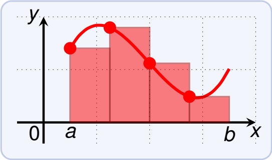
- middle $x_i = a+(i-\frac{1}{2})\Delta x$ of the interval
- upper sum: $f(x_i)$ is the maximum on the interval I_i
- lower sum: $f(x_i)$ is the minimum on the interval I_i

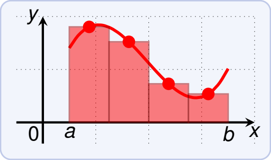
The Area below a Curve



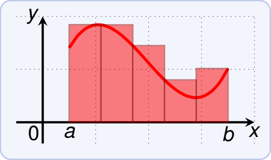
right-endpoints with 5 strips



left-endpoints with 5 strips



midpoints with 5 strips



upper sum with 5 strips

lower sum with 5 strips