Calculus M211

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Antiderivatives / Integrals

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  If $n \geq 0$, then this is valid for any interval.
  If $n < 0$ & $n \neq -1$, then valid for intervals not containing 0.

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So the general antiderivative of f is:

$$F(x) = \begin{cases} \ln x + C_1 & \text{for } x > 0 \\ \ln(-x) + C_2 & \text{for } x < 0 \end{cases}$$

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$$f(x) = x^4 + x^3 - 2x^2 + Cx + D$$

To ensure $f(0) = 4$ and $f(1) = 1$, we need to find C and D:

$f(0) = D$

Antiderivatives / Integrals

Find f if

$$f''(x) = 12x^2 + 6x - 4$$

and $f(0) = 4$ and $f(1) = 1$.

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Therefore the function f we are looking for is:

$$f(x) = x^4 + x^3 - 2x^2 - 3x + 4$$

Antiderivatives / Integrals

A particle moves in a straight line and has

- acceleration $a(t) = 6t + 4$
- initial velocity is $v(0) = -6$cm/s
- initial displacement is $s(0) = 9$cm

Find the position function $s(t)$.

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As $s(0) = 9$cm/s, it follows that $D =$

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$$v(t) = 3t^2 + 4t + C$$

As $v(0) = -6$cm/s, it follows that $C = -6$.

The position function is an antiderivative of the velocity:

$$s(t) = t^3 + 2t^2 - 6t + D$$

As $s(0) = 9$cm/s, it follows that $D = 9$.

Thus the position function is:

$$s(t) = t^3 + 2t^2 - 6t + 9 \quad \text{in cm}$$

Antiderivatives / Integrals

Near the surface of the earth, the gravitational force produces a downward acceleration of approximately $9.8\text{m}/\text{s}^2$ (or $32\text{ft}/\text{s}^2$).

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A ball is thrown upward with a speed of 48ft/s from the edge of cliff 432ft above ground. When does the ball reach its maximum height? When does it hit the ground?

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Let $s(t)$ be the distance above the ground, and $v(t)$ the velocity:

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Let $s(t)$ be the distance above the ground, and $v(t)$ the velocity:

$$a(t) =$$

Antiderivatives / Integrals

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Let $s(t)$ be the distance above the ground, and $v(t)$ the velocity:

$$a(t) = -32$$

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Let $s(t)$ be the distance above the ground, and $v(t)$ the velocity:

$$a(t) = -32$$
$$v(t) = -32t + C$$

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Let $s(t)$ be the distance above the ground, and $v(t)$ the velocity:

$$a(t) = -32$$

$$v(t) = -32t + C \qquad v(0) = C = 48$$

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$$s(t) = -16t^2$$

Near the surface of the earth, the gravitational force produces a downward acceleration of approximately $9.8\text{m}/\text{s}^2$ (or $32\text{ft}/\text{s}^2$).

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Let $s(t)$ be the distance above the ground, and $v(t)$ the velocity:

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$$v(t) = -32t + C \qquad v(0) = C = 48$$

$$s(t) = -16t^2 + 48t$$

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$$v(t) = -32t + C \qquad v(0) = C = 48$$

$$s(t) = -16t^2 + 48t + D \qquad s(0) = D = 432$$

Antiderivatives / Integrals

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The ball reaches the maximal height when

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The ball reaches the maximal height when

$$v(t) = 0$$

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The ball reaches the maximal height when

$$v(t) = 0 = -32t + 48$$

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The ball reaches the maximal height when

$$v(t) = 0 = -32t + 48, \text{ that is, after } t = 1.5 \text{ seconds}$$

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$v(t) = 0 = -32t + 48$, that is, after $t = 1.5$ seconds

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$$
\begin{aligned}
a(t) &= -32 \\
v(t) &= -32t + C \qquad v(0) = C = 48 \\
s(t) &= -16t^2 + 48t + D \qquad s(0) = D = 432
\end{aligned}
$$

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We reject the negative solution, and find $t = 3/2 + 3/2 \cdot \sqrt{13}$.