Calculus M211

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Indiana University Bloomington

2013

Optimization

We now use calculus to solve practical problems.

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Challenge: convert word problems into mathematical problems

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- understand the problem

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- draw a diagram

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- understand the problem
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- introduce notation

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- understand the problem
- draw a diagram
- introduce notation
- translate the problem to the notation

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We now use calculus to solve practical problems.

Challenge: convert word problems into mathematical problems

- understand the problem
- draw a diagram
- introduce notation
- translate the problem to the notation
- use calculus to solve it

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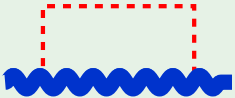

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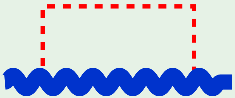

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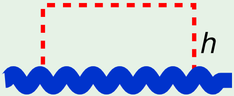

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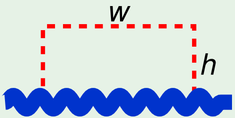

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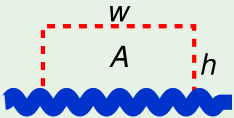

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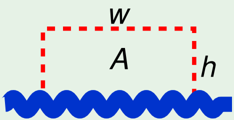

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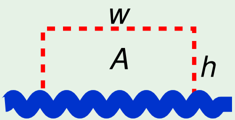

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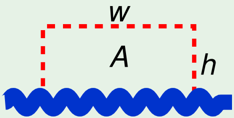

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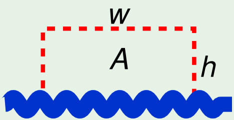

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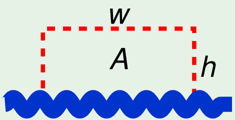

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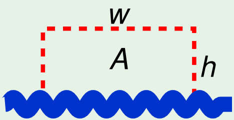

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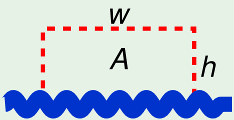

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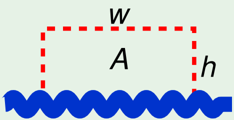

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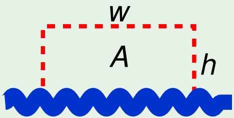

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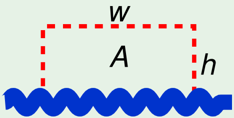

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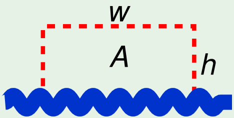

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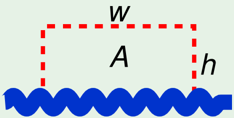

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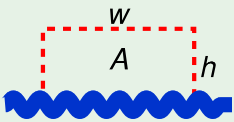

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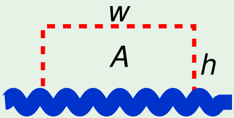

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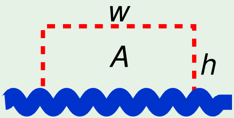

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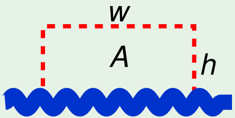

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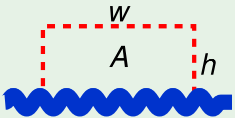

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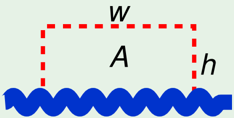

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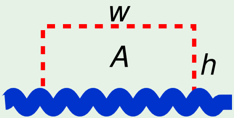

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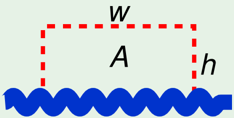

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The dimensions of the field are: 600ft height, 1200ft width.

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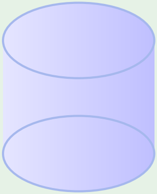

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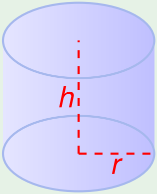

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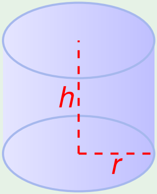

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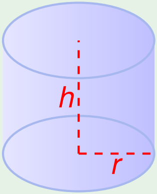

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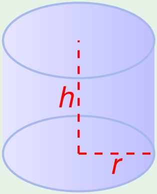

Introducing notation:

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- let r be the radius

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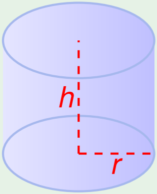

Introducing notation:

- let h be the height
- let r be the radius
- let V be the volume

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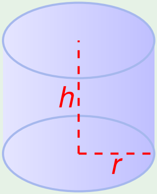

Introducing notation:

- let h be the height
- let r be the radius
- let V be the volume
- let A be the surface area

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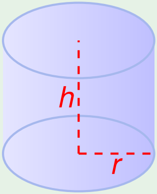

Introducing notation:

- let h be the height
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$V =$

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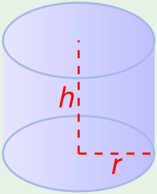

$V = \pi r^2 h$

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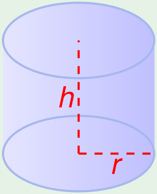

Introducing notation:

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$$V = \pi r^2 h = 1$$

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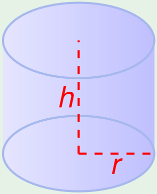

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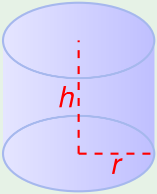

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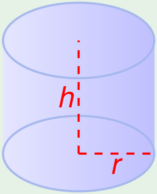

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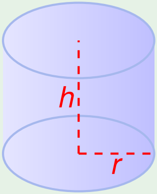

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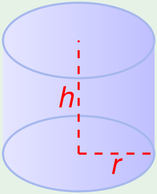

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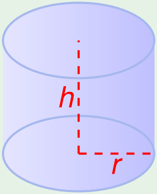

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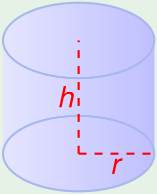

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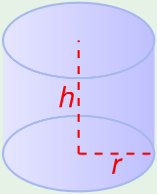

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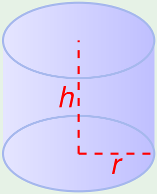

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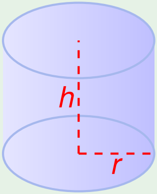

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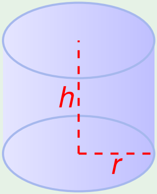

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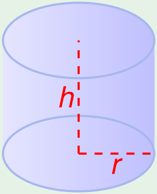

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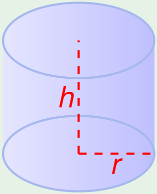

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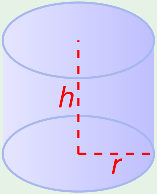

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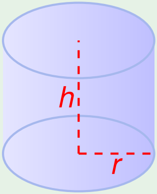

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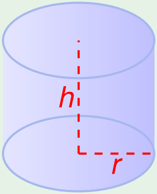

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$$A'(r) = 0 \iff r = 1/\sqrt[3]{2\pi} \text{ is the only critical number}$$

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However, $A(1/\sqrt[3]{2\pi})$ must be the **absolute minimum** since:

- A is decreasing, $A'(r) < 0$, for all $r < 1/\sqrt[3]{2\pi}$,
- A is increasing, $A'(r) > 0$, for all $r > 1/\sqrt[3]{2\pi}$.

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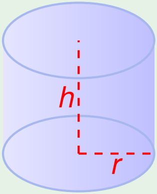

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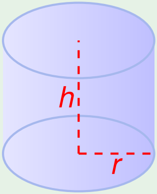

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Then $h = 1/(\pi r^2)$

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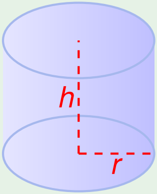

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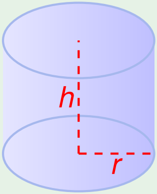

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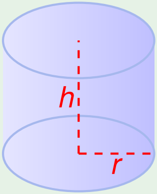

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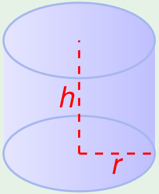

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Then $h = 1/(\pi r^2) = \sqrt[3]{2\pi}^2/\pi = \sqrt[3]{4\pi^2/\pi^3} = 2/\sqrt[3]{2\pi} = 2r$

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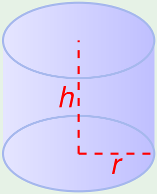

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Then $h = 1/(\pi r^2) = \sqrt[3]{2\pi}^2/\pi = \sqrt[3]{4\pi^2/\pi^3} = 2/\sqrt[3]{2\pi} = 2r$

Hence radius $r = 1/\sqrt[3]{2\pi}$ and height $h = 2r$ minimizes the cost.

Optimization

The argument we have used on the last slide is the following:

First Derivative Test for Absolute Extreme Values

Let f be continuous, defined on an open or closed interval.
Let c be a critical number of f.

- If $f'(x) > 0$ for all $x < c$, and $f'(x) < 0$ for all $x > c$, then $f(c)$ is the absolute maximum of f.
- If $f'(x) < 0$ for all $x < c$, and $f'(x) > 0$ for all $x > c$, then $f(c)$ is the absolute minimum of f.

Optimization

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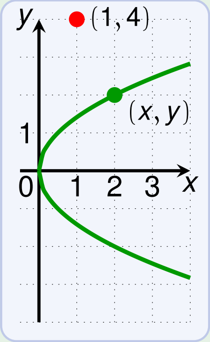

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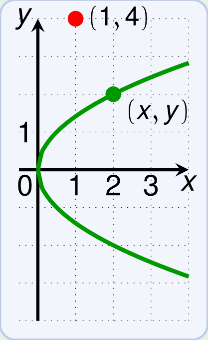

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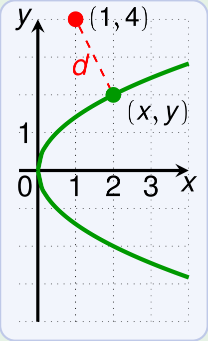

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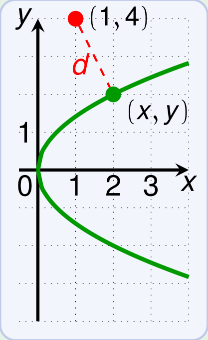

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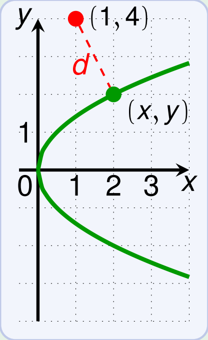

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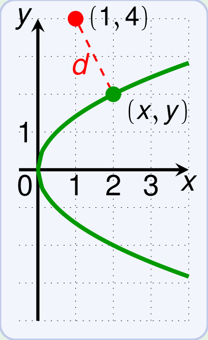

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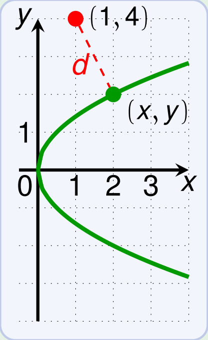

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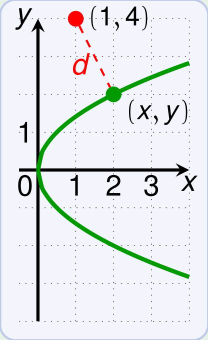

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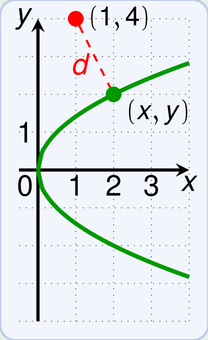

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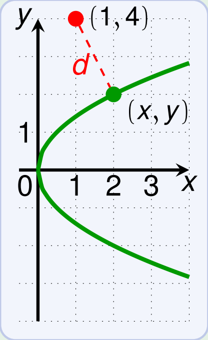

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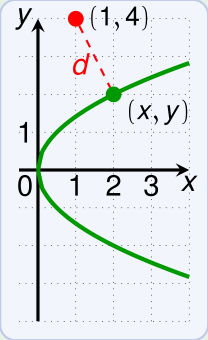

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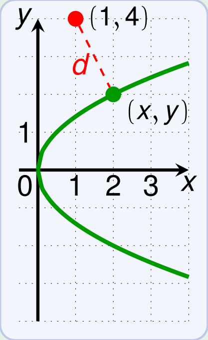

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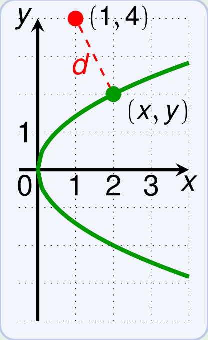

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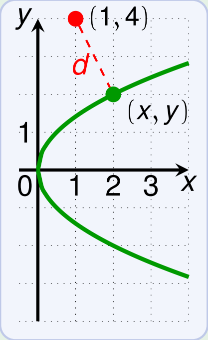

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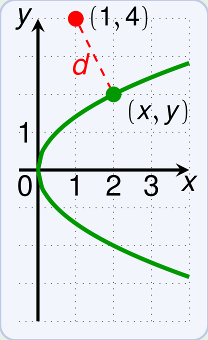

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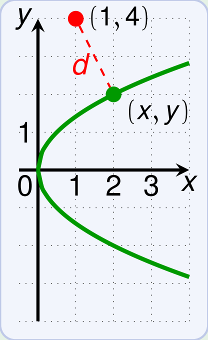

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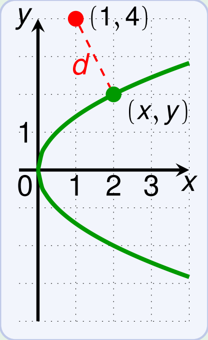

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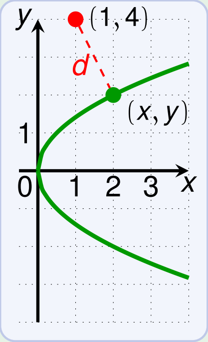

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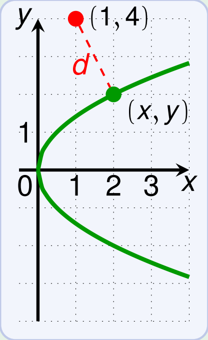

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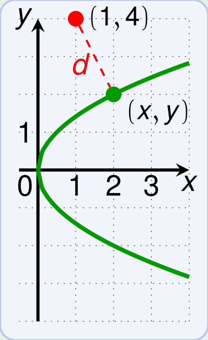

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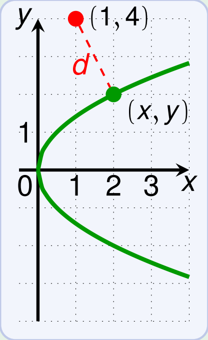

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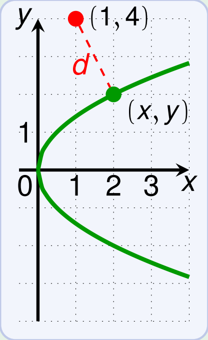

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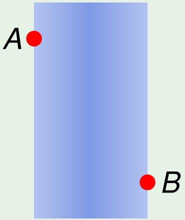

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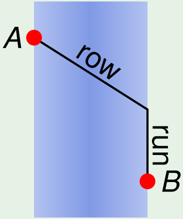

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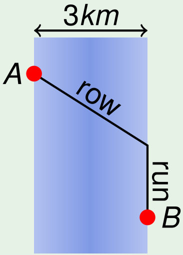

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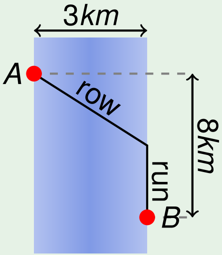

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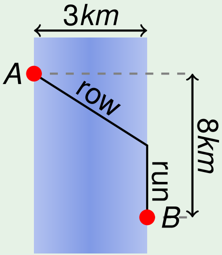

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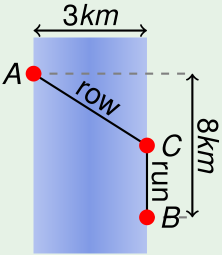

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- let C be the landing point

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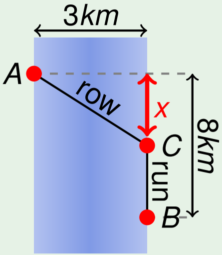

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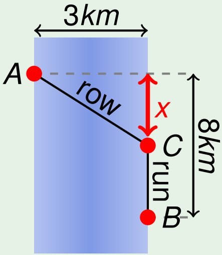

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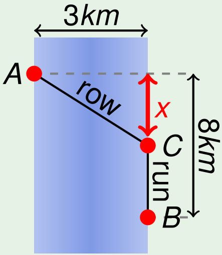

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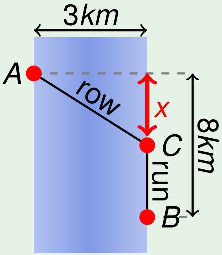

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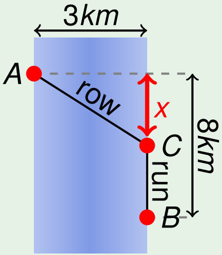

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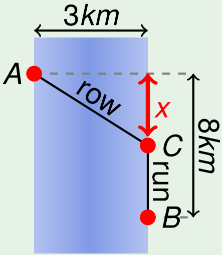

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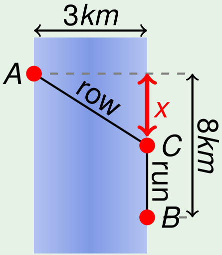

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The time for rowing is and running:

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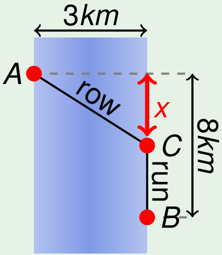

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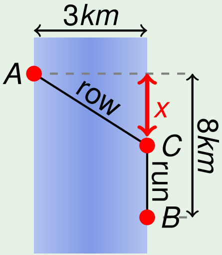

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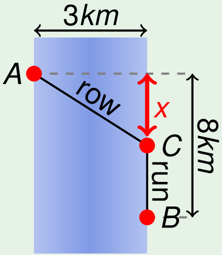

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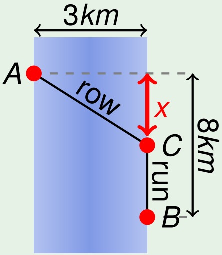

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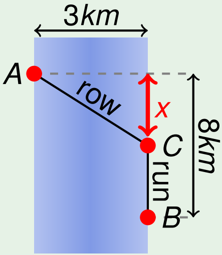

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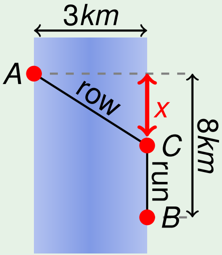

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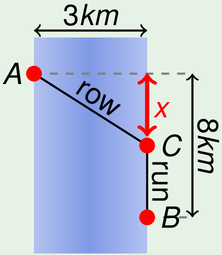

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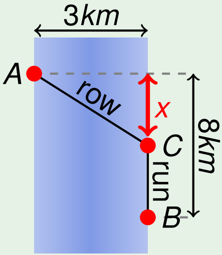

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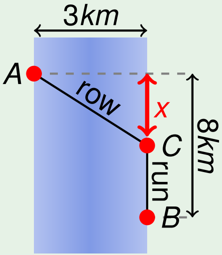

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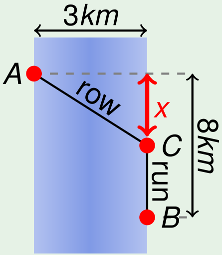

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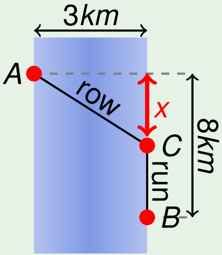

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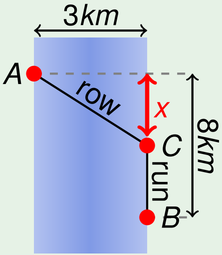

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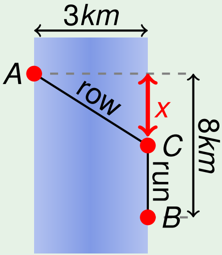

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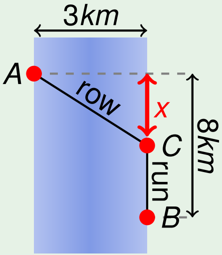

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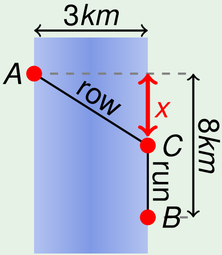

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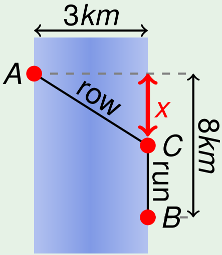

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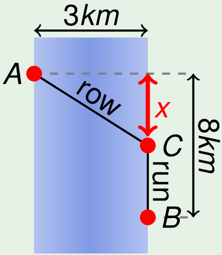

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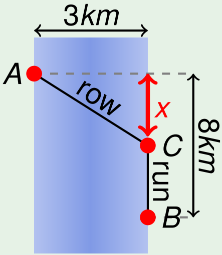

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Thus landing $9/\sqrt{7}$km downstream is the fastest.

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Find the area of the largest rectangle that can be inscribed in a semi-circle circle of radius r.

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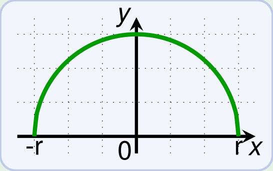

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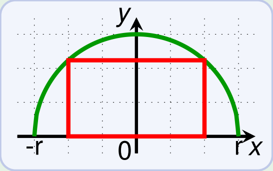

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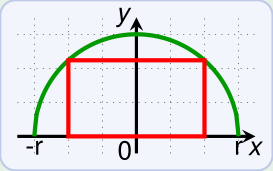

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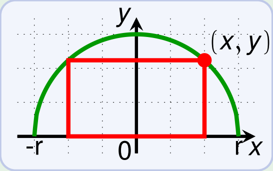

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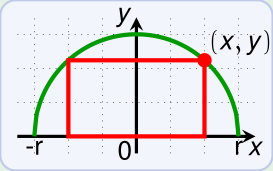

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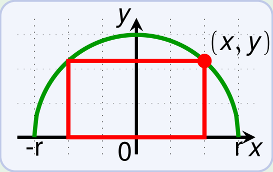

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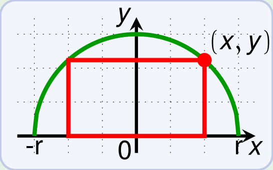

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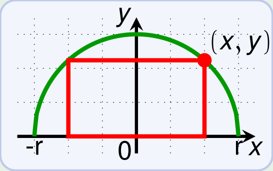

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- let (x, y) be the upper right corner of the rectangle
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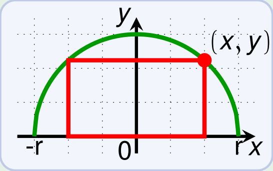
The area is $A(x) = 2xy = 2x\sqrt{r^2 - x^2}$

Optimization

Find the area of the largest rectangle that can be inscribed in a semi-circle circle of radius r.



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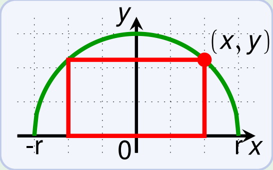

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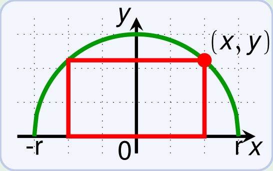

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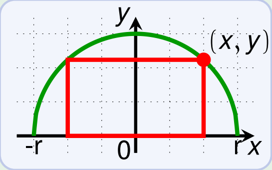

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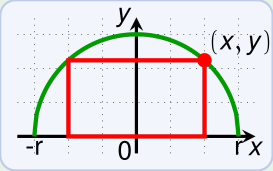

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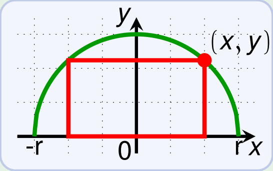

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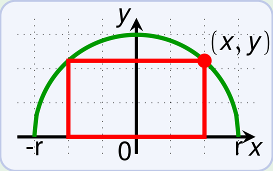

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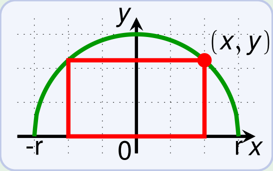

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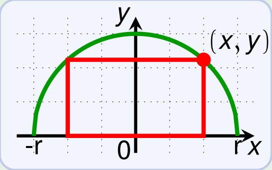

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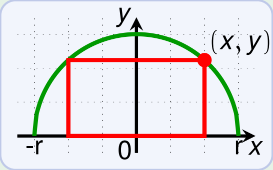

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$$A(r/\sqrt{2}) =$$

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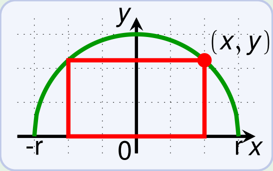

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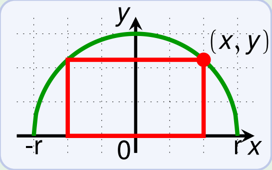

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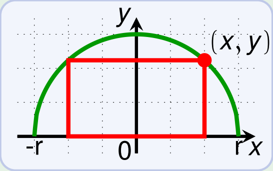

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A store sells 100 blu-ray players per week for 200$ each. A market survey shows that for each 10$ discount, the store would sell 40 more players per week. The store buys the players at a price of 150$ per piece.

Optimization

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$$s(x) =$$

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$$s(x) = 100 + 40 \cdot \frac{x}{10}$$

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Introducing notation:

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$$s(x) = 100 + 40 \cdot \frac{x}{10} = 100 + 4x$$

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&= -4x^2 + 100x + 5000
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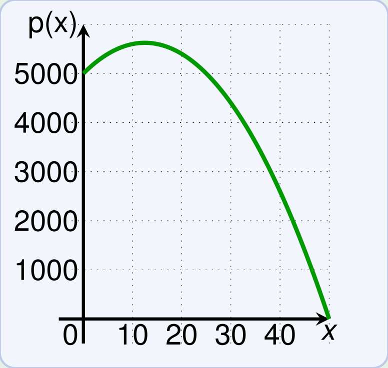
By the Closed Interval Method, 12.5$ discount for maximal profit.

Optimization

A store sells 100 blu-ray players per week for 200$ each. A market survey shows that for each 10$ discount, the store would sell 40 more players per week. The store buys the players at a price of 150$ per piece.

What selling price would maximize the profit of the store?



By the Closed Interval Method, 12.5$ discount for maximal profit.