Calculus M211

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2013

Curve Sketching

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- find **local maxima and minima**

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- find intervals of **increase** $f'(x) > 0$ and **decrease** $f'(x) < 0$
- find **local maxima and minima**
- determine **concavity** on intervals and **points of inflection**
  - $f''(x) > 0$ concave upward
  - $f''(x) < 0$ concave downward
  - inflections points where $f''(x)$ changes the sign

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  - f' changes from $+$ to $-$ at $c \implies$ maximum
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Then sketch the curve:

- draw asymptotes as thin dashed lines
- mark intercepts, local extrema and inflection points
- draw the curve taking into account:
  - increase / decrease, concavity and asymptotes

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The second derivative is:

$$f''(x) = \frac{-4(x^2-1)^2 - (-4x)\cdot 2(x^2-1)\cdot 2x}{(x^2-1)^4}$$

$$= \frac{-4(x^2-1)+16x^2}{(x^2-1)^3} = \frac{12x^2+4}{(x^2-1)^3}$$

$12x^2 + 4 > 0 \quad$ for all x

$f''(x) > 0 \iff (x^2-1)^3 > 0 \iff x^2 - 1 > 0 \iff |x| > 1$

- concave upward on $(-\infty, -1) \cup (1, \infty)$
- concave downward on

Curve Sketching

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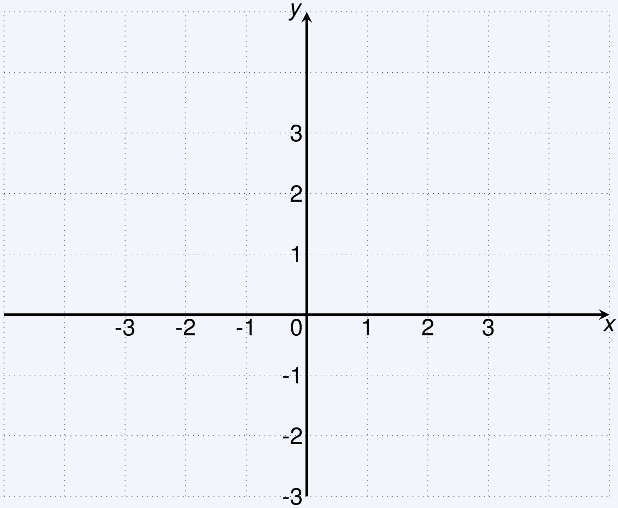
- concave upward on $(-\infty, -1) \cup (1, \infty)$
- concave downward on $(-1, 1)$
- inflection points: none (-1 and 1 not in the domain)

Curve Sketching

Sketch the curve of $f(x) = \frac{2x^2}{x^2-1}$.



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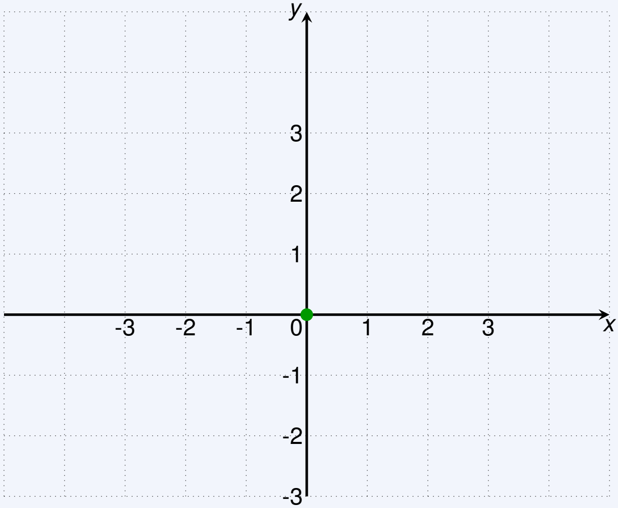

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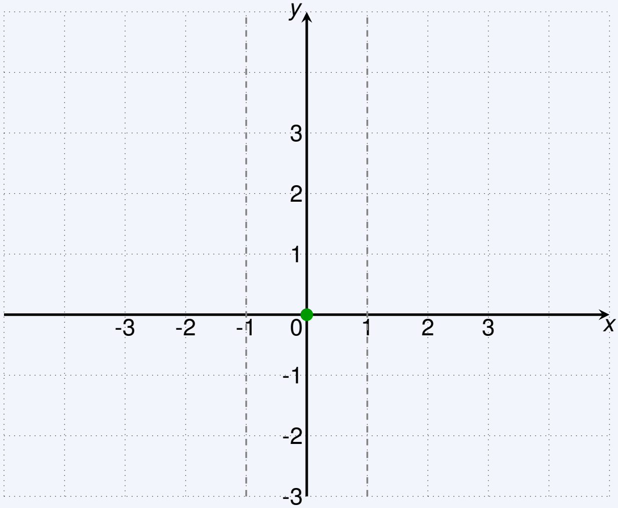

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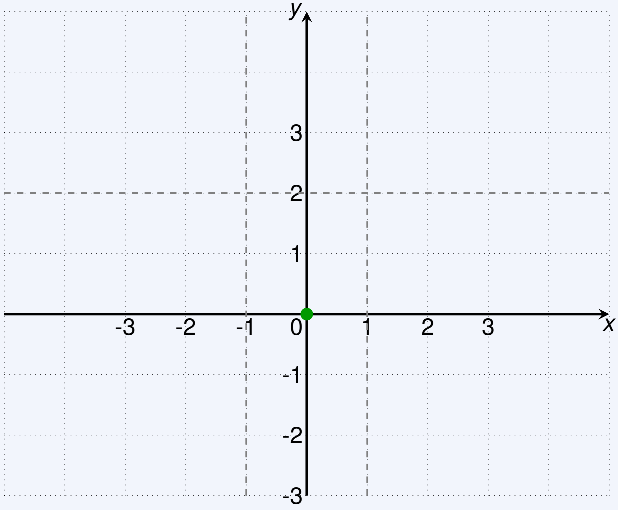

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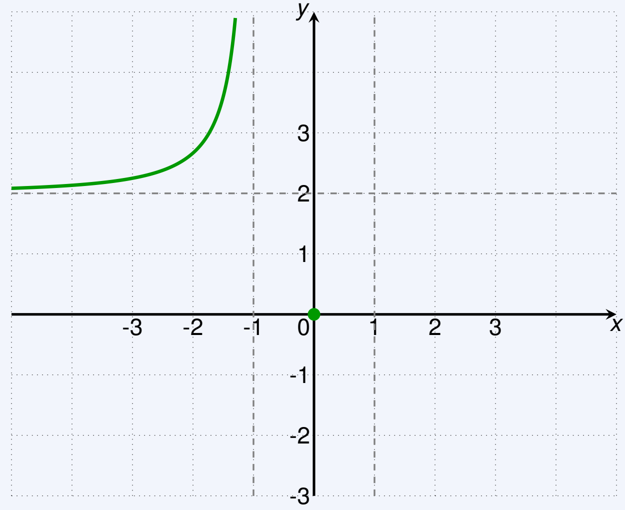

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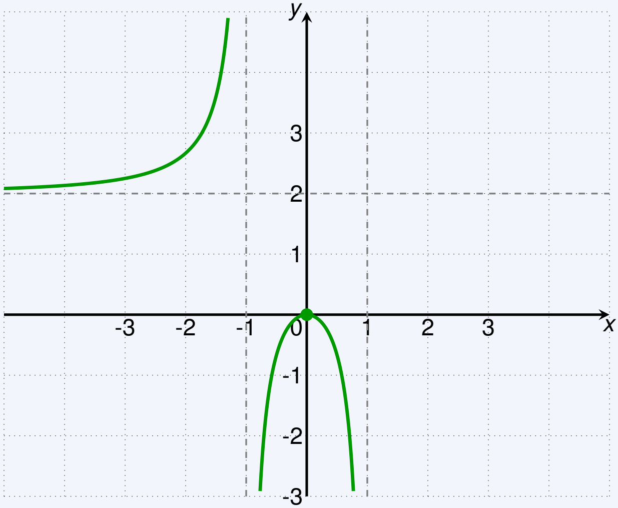

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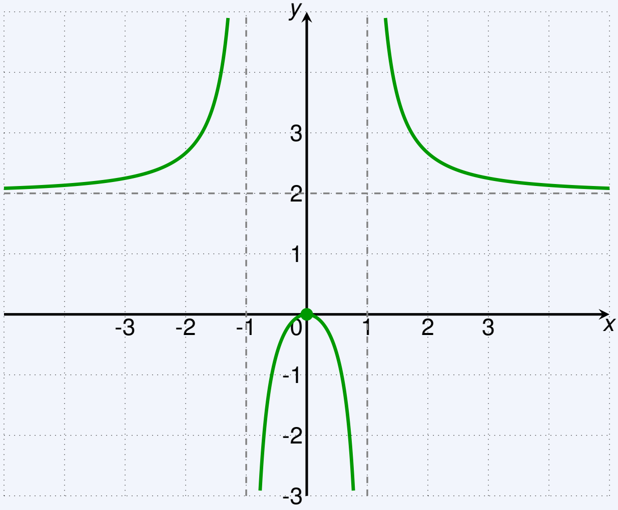


Slant Asymptotes

Asymptotes that are neither horizontal nor vertical:

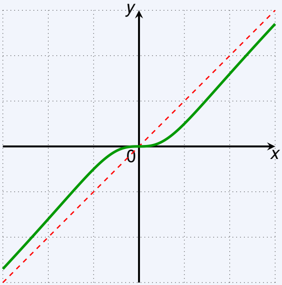
If

$$\lim_{x \to \infty} [f(x) - (mx + b)] = 0$$

or

$$\lim_{x \to -\infty} [f(x) - (mx + b)] = 0$$

the the line $y = mx + b$ is called **slant asymptote.**



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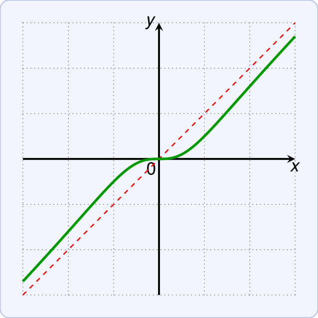

Note that the distance between curve and line approaches 0.

Slant Asymptotes

Sketch the graph of $f(x) = \frac{x^3}{2x^2+1}$.

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Vertical asymptotes:

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Vertical asymptotes: none.

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Vertical asymptotes: none. Horizontal asymptotes: none

Slant asymptotes: $\quad y = \frac{1}{2}x \quad$ since

$$\lim_{x\to\infty} \left(\frac{x^3}{2x^2+1} - \frac{x}{2} \right)$$

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Local minima, maxima:

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Local minima, maxima: none (since f' does not change sign)

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Interval	$f''(x)$	

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Sketch the graph of $f(x) = \frac{x^3}{2x^2+1}$.

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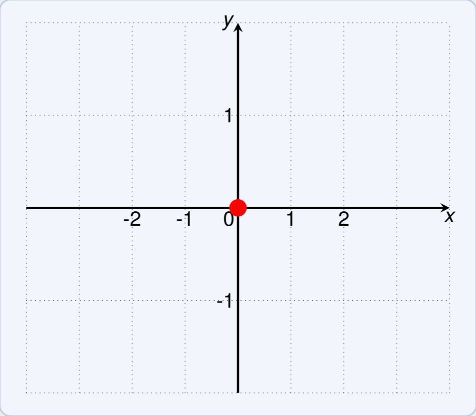

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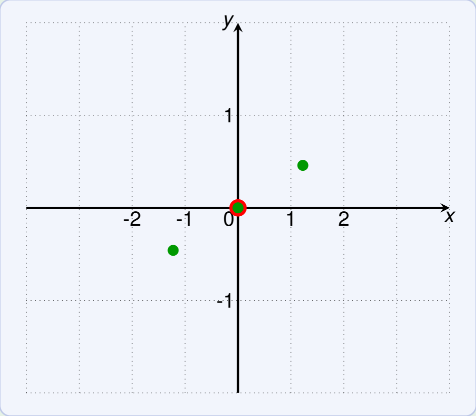

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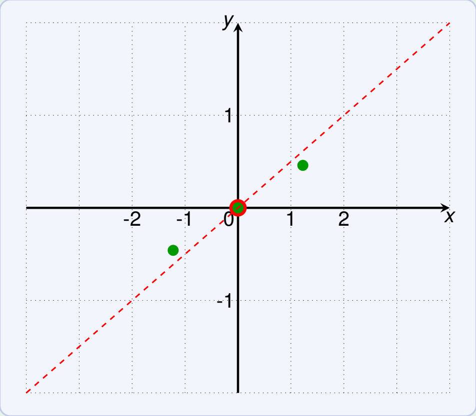

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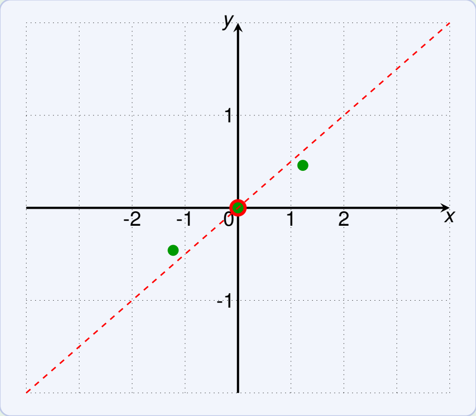

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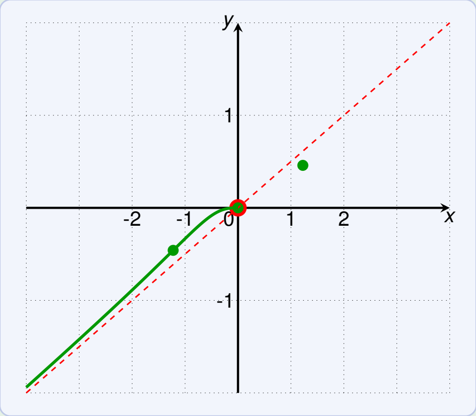

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