Calculus M211

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L'Hospital's Rule

A limit of the form

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

where both

$$\lim_{x \to a} f(x) = 0 \qquad \text{and} \qquad \lim_{x \to a} g(x) = 0$$

is called **indeterminate form of type $\frac{0}{0}$.**

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Often cancellation of common factors helps:

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But not for examples like:

$$\lim_{x \to 0} \frac{\sin x}{x} \qquad \text{and} \qquad \lim_{x \to 1} \frac{\ln x}{x - 1}$$

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where both

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But not for examples like:

$$\lim_{x \to \infty} \frac{\ln x}{x - 1}$$

L'Hospital's Rule

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Suppose f and g are differentiable and $g'(x) \neq 0$ near a, and

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

is an indeterminate form of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$. Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if the limit on the right side exists or is $-\infty$ or ∞.

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Before applying L'Hospital's Rule it is important to verify that:

$$\lim_{x \to a} f(x) = 0 \quad \text{and} \quad \lim_{x \to a} g(x) = 0$$

or

$$\lim_{x \to a} f(x) = \pm\infty \quad \text{and} \quad \lim_{x \to a} g(x) = \pm\infty$$

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Find

$$\lim_{x \to 1} \frac{\ln x}{x - 1}$$

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We have

$$\lim_{x \to 1} \ln x = \ln 1 = 0 \qquad \text{and} \qquad \lim_{x \to 1} (x - 1) = 0$$

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$$\lim_{x \to \pi^-} \frac{\sin x}{1 - \cos x}$$

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However, this is wrong!

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If we were to apply l'Hospital's Rule:

$$\lim_{x \to \pi^-} \frac{\sin x}{1 - \cos x} = \lim_{x \to \pi^-} \frac{\cos x}{\sin x} = -\infty$$

However, this is wrong!

We have $\lim_{x \to \pi^-} (1 - \cos x) = 1 - (-1) = 2$.

$$\lim_{x \to \pi^-} \frac{\sin x}{1 - \cos x} = \frac{0}{1 - (-1)} = 0$$

L'Hospital's Rule

Find

$$\lim_{x\to\pi^-} \frac{\sin x}{1-\cos x}$$

If we were to apply l'Hospital's Rule:

$$\lim_{x\to\pi^-} \frac{\sin x}{1-\cos x} = \lim_{x\to\pi^-} \frac{\cos x}{\sin x} = -\infty$$

However, this is wrong!

We have $\lim_{x\to\pi^-}(1-\cos x) = 1-(-1) = 2$.

$$\lim_{x\to\pi^-} \frac{\sin x}{1-\cos x} = \frac{0}{1-(-1)} = 0$$

Before applying l'Hospital's Rule, always check that the limit is an indeterminate form $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

L'Hospital's Rule

L'Hospital's Rule is valid for one-sided limits and limits at infinity:

$$\lim_{x\to a^-} \frac{f(x)}{g(x)} \qquad \lim_{x\to a^+} \frac{f(x)}{g(x)} \qquad \lim_{x\to \infty} \frac{f(x)}{g(x)} \qquad \lim_{x\to -\infty} \frac{f(x)}{g(x)}$$

L'Hospital's Rule

A limit of the form

$$\lim_{x \to a} (f(x)g(x))$$

where

$$\lim_{x \to a} f(x) = 0 \qquad \text{and} \qquad \lim_{x \to a} g(x) = \pm\infty$$

is called **indeterminate form of type** $0 \cdot \infty$**.**

L'Hospital's Rule

A limit of the form

$$\lim_{x \to a} (f(x)g(x))$$

where

$$\lim_{x \to a} f(x) = 0 \qquad \text{and} \qquad \lim_{x \to a} g(x) = \pm\infty$$

is called **indeterminate form of type $0 \cdot \infty$.**

We then rewrite the limit as:

$$\lim_{x \to a} (f(x)g(x)) = \lim_{x \to a} \frac{f(x)}{1/g(x)}$$

an indeterminate form of type $\frac{0}{0}$

L'Hospital's Rule

A limit of the form

$$\lim_{x \to a} (f(x)g(x))$$

where

$$\lim_{x \to a} f(x) = 0 \qquad \text{and} \qquad \lim_{x \to a} g(x) = \pm\infty$$

is called **indeterminate form of type** $0 \cdot \infty$**.**

We then rewrite the limit as:

$$\lim_{x \to a} (f(x)g(x)) = \lim_{x \to a} \frac{f(x)}{1/g(x)}$$

an indeterminate form of type $\frac{0}{0}$, or as

$$\lim_{x \to a} (f(x)g(x)) = \lim_{x \to a} \frac{g(x)}{1/f(x)}$$

an indeterminate form of type $\frac{\infty}{\infty}$.

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to 0^+} x \ln x$$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to 0^+} x \ln x$$

We have

$$\lim_{x \to 0^+} x = 0 \qquad \text{and} \qquad \lim_{x \to 0^+} \ln x =$$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to 0^+} x \ln x$$

We have

$$\lim_{x \to 0^+} x = 0 \qquad \text{and} \qquad \lim_{x \to 0^+} \ln x = -\infty$$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to 0^+} x \ln x$$

We have

$$\lim_{x \to 0^+} x = 0 \qquad \text{and} \qquad \lim_{x \to 0^+} \ln x = -\infty$$

Thus we can choose for rewriting to:

$$\lim_{x \to 0^+} \frac{x}{1/\ln x} \qquad \text{or} \qquad \lim_{x \to 0^+} \frac{\ln x}{1/x}$$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to 0^+} x \ln x$$

We have

$$\lim_{x \to 0^+} x = 0 \qquad \text{and} \qquad \lim_{x \to 0^+} \ln x = -\infty$$

Thus we can choose for rewriting to:

$$\lim_{x \to 0^+} \frac{x}{1/\ln x} \qquad \text{or} \qquad \lim_{x \to 0^+} \frac{\ln x}{1/x}$$

We choose the 2nd since the derivatives are easier:

$$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{1/x} =$$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to 0^+} x \ln x$$

We have

$$\lim_{x \to 0^+} x = 0 \qquad \text{and} \qquad \lim_{x \to 0^+} \ln x = -\infty$$

Thus we can choose for rewriting to:

$$\lim_{x \to 0^+} \frac{x}{1/\ln x} \qquad \text{or} \qquad \lim_{x \to 0^+} \frac{\ln x}{1/x}$$

We choose the 2nd since the derivatives are easier:

$$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{1/x} = \lim_{x \to 0^+} \frac{1/x}{-1/x^2}$$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to 0^+} x \ln x$$

We have

$$\lim_{x \to 0^+} x = 0 \qquad \text{and} \qquad \lim_{x \to 0^+} \ln x = -\infty$$

Thus we can choose for rewriting to:

$$\lim_{x \to 0^+} \frac{x}{1/\ln x} \qquad \text{or} \qquad \lim_{x \to 0^+} \frac{\ln x}{1/x}$$

We choose the 2nd since the derivatives are easier:

$$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{1/x} = \lim_{x \to 0^+} \frac{1/x}{-1/x^2} = \lim_{x \to 0^+} (-x)$$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to 0^+} x \ln x$$

We have

$$\lim_{x \to 0^+} x = 0 \qquad \text{and} \qquad \lim_{x \to 0^+} \ln x = -\infty$$

Thus we can choose for rewriting to:

$$\lim_{x \to 0^+} \frac{x}{1/\ln x} \qquad \text{or} \qquad \lim_{x \to 0^+} \frac{\ln x}{1/x}$$

We choose the 2nd since the derivatives are easier:

$$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{1/x} = \lim_{x \to 0^+} \frac{1/x}{-1/x^2} = \lim_{x \to 0^+} (-x) = 0$$

L'Hospital's Rule

A limit of the form

$$\lim_{x \to a} (f(x) - g(x))$$

where

$$\lim_{x \to a} f(x) = \infty \qquad \text{and} \qquad \lim_{x \to a} g(x) = \infty$$

is called **indeterminate form of type $\infty - \infty$.**

L'Hospital's Rule

A limit of the form

$$\lim_{x \to a} (f(x) - g(x))$$

where

$$\lim_{x \to a} f(x) = \infty \qquad \text{and} \qquad \lim_{x \to a} g(x) = \infty$$

is called **indeterminate form of type** $\infty - \infty$**.**

We then rewrite the limit as a **quotient.**

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x)$$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x)$$

Then $\lim_{x \to (\pi/2)^-} \sec x =$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x)$$

Then $\lim_{x \to (\pi/2)^-} \sec x = \infty$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x)$$

Then $\lim_{x \to (\pi/2)^-} \sec x = \infty$ and $\lim_{x \to (\pi/2)^-} \tan x =$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x)$$

Then $\lim_{x \to (\pi/2)^-} \sec x = \infty$ and $\lim_{x \to (\pi/2)^-} \tan x = \infty$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x)$$

Then $\lim_{x \to (\pi/2)^-} \sec x = \infty$ and $\lim_{x \to (\pi/2)^-} \tan x = \infty$

We use a common denominator:

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x)$$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x\to(\pi/2)^-}(\sec x - \tan x)$$

Then $\lim_{x\to(\pi/2)^-}\sec x = \infty$ and $\lim_{x\to(\pi/2)^-}\tan x = \infty$

We use a common denominator:

$$\lim_{x\to(\pi/2)^-}(\sec x - \tan x) = \lim_{x\to(\pi/2)^-}\frac{1-\sin x}{\cos x}$$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x)$$

Then $\lim_{x \to (\pi/2)^-} \sec x = \infty$ and $\lim_{x \to (\pi/2)^-} \tan x = \infty$

We use a common denominator:

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x) = \lim_{x \to (\pi/2)^-} \frac{1 - \sin x}{\cos x}$$

Now $\lim_{x \to (\pi/2)^-} (1 - \sin x) =$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x)$$

Then $\lim_{x \to (\pi/2)^-} \sec x = \infty$ and $\lim_{x \to (\pi/2)^-} \tan x = \infty$

We use a common denominator:

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x) = \lim_{x \to (\pi/2)^-} \frac{1 - \sin x}{\cos x}$$

Now $\lim_{x \to (\pi/2)^-} (1 - \sin x) = 0$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x)$$

Then $\lim_{x \to (\pi/2)^-} \sec x = \infty$ and $\lim_{x \to (\pi/2)^-} \tan x = \infty$

We use a common denominator:

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x) = \lim_{x \to (\pi/2)^-} \frac{1 - \sin x}{\cos x}$$

Now $\lim_{x \to (\pi/2)^-} (1 - \sin x) = 0$ and $\lim_{x \to (\pi/2)^-} \cos x =$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x)$$

Then $\lim_{x \to (\pi/2)^-} \sec x = \infty$ and $\lim_{x \to (\pi/2)^-} \tan x = \infty$

We use a common denominator:

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x) = \lim_{x \to (\pi/2)^-} \frac{1 - \sin x}{\cos x}$$

Now $\lim_{x \to (\pi/2)^-} (1 - \sin x) = 0$ and $\lim_{x \to (\pi/2)^-} \cos x = 0$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x)$$

Then $\lim_{x \to (\pi/2)^-} \sec x = \infty$ and $\lim_{x \to (\pi/2)^-} \tan x = \infty$

We use a common denominator:

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x) = \lim_{x \to (\pi/2)^-} \frac{1 - \sin x}{\cos x}$$

Now $\lim_{x \to (\pi/2)^-} (1 - \sin x) = 0$ and $\lim_{x \to (\pi/2)^-} \cos x = 0$

Hence we can apply l'Hospital's Rule:

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x) = \lim_{x \to (\pi/2)^-} \frac{1 - \sin x}{\cos x}$$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x)$$

Then $\lim_{x \to (\pi/2)^-} \sec x = \infty$ and $\lim_{x \to (\pi/2)^-} \tan x = \infty$

We use a common denominator:

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x) = \lim_{x \to (\pi/2)^-} \frac{1 - \sin x}{\cos x}$$

Now $\lim_{x \to (\pi/2)^-} (1 - \sin x) = 0$ and $\lim_{x \to (\pi/2)^-} \cos x = 0$

Hence we can apply l'Hospital's Rule:

$$\begin{aligned} \lim_{x \to (\pi/2)^-} (\sec x - \tan x) &= \lim_{x \to (\pi/2)^-} \frac{1 - \sin x}{\cos x} \\ &= \lim_{x \to (\pi/2)^-} \frac{-\cos x}{-\sin x} \end{aligned}$$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x)$$

Then $\lim_{x \to (\pi/2)^-} \sec x = \infty$ and $\lim_{x \to (\pi/2)^-} \tan x = \infty$

We use a common denominator:

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x) = \lim_{x \to (\pi/2)^-} \frac{1 - \sin x}{\cos x}$$

Now $\lim_{x \to (\pi/2)^-} (1 - \sin x) = 0$ and $\lim_{x \to (\pi/2)^-} \cos x = 0$

Hence we can apply l'Hospital's Rule:

$$\begin{aligned}\lim_{x \to (\pi/2)^-} (\sec x - \tan x) &= \lim_{x \to (\pi/2)^-} \frac{1 - \sin x}{\cos x} \\ &= \lim_{x \to (\pi/2)^-} \frac{-\cos x}{-\sin x} = 0\end{aligned}$$

L'Hospital's Rule

A limit of the form

$$\lim_{x \to a} [f(x)]^{g(x)}$$

is an indeterminate form

- **of type** 0^0 if $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$
- **of type** ∞^0 if $\lim_{x \to a} f(x) = \infty$ and $\lim_{x \to a} g(x) = 0$
- **of type** 1^∞ if $\lim_{x \to a} f(x) = 1$ and $\lim_{x \to a} g(x) = \infty$

L'Hospital's Rule

A limit of the form

$$\lim_{x \to a} [f(x)]^{g(x)}$$

is an indeterminate form

- **of type** 0^0 if $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$
- **of type** ∞^0 if $\lim_{x \to a} f(x) = \infty$ and $\lim_{x \to a} g(x) = 0$
- **of type** 1^∞ if $\lim_{x \to a} f(x) = 1$ and $\lim_{x \to a} g(x) = \infty$

Each of these cases can be treated by writing the limit as:

$$\lim_{x \to a} [f(x)]^{g(x)} = \lim_{x \to a} e^{\ln\left([f(x)]^{g(x)}\right)}$$

L'Hospital's Rule

A limit of the form

$$\lim_{x \to a} [f(x)]^{g(x)}$$

is an indeterminate form

- **of type** 0^0 if $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$
- **of type** ∞^0 if $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = 0$
- **of type** 1^∞ if $\lim_{x\to a} f(x) = 1$ and $\lim_{x\to a} g(x) = \infty$

Each of these cases can be treated by writing the limit as:

$$\begin{aligned} \lim_{x \to a} [f(x)]^{g(x)} &= \lim_{x \to a} e^{\ln\left([f(x)]^{g(x)}\right)} \\ &= \lim_{x \to a} e^{g(x) \ln f(x)} \end{aligned}$$

L'Hospital's Rule

A limit of the form

$$\lim_{x \to a} [f(x)]^{g(x)}$$

is an indeterminate form

- **of type 0^0** if $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$
- **of type ∞^0** if $\lim_{x \to a} f(x) = \infty$ and $\lim_{x \to a} g(x) = 0$
- **of type 1^∞** if $\lim_{x \to a} f(x) = 1$ and $\lim_{x \to a} g(x) = \infty$

Each of these cases can be treated by writing the limit as:

$$\begin{aligned} \lim_{x \to a} [f(x)]^{g(x)} &= \lim_{x \to a} e^{\ln\left([f(x)]^{g(x)}\right)} \\ &= \lim_{x \to a} e^{g(x) \ln f(x)} = e^{\lim_{x \to a}(g(x) \ln f(x))} \end{aligned}$$

L'Hospital's Rule

A limit of the form

$$\lim_{x \to a} [f(x)]^{g(x)}$$

is an indeterminate form

- **of type** 0^0 if $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$
- **of type** ∞^0 if $\lim_{x \to a} f(x) = \infty$ and $\lim_{x \to a} g(x) = 0$
- **of type** 1^∞ if $\lim_{x \to a} f(x) = 1$ and $\lim_{x \to a} g(x) = \infty$

Each of these cases can be treated by writing the limit as:

$$\begin{aligned}\lim_{x \to a} [f(x)]^{g(x)} &= \lim_{x \to a} e^{\ln\left([f(x)]^{g(x)}\right)} \\ &= \lim_{x \to a} e^{g(x) \ln f(x)} = e^{\lim_{x \to a}(g(x) \ln f(x))}\end{aligned}$$

Other types are **not** indeterminate forms: 0^∞, 1^0 and ∞^1.

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to 0^+} x^x$$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to 0^+} x^x$$

Then $\lim_{x \to 0^+} x = 0.$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to 0^+} x^x$$

Then $\lim_{x \to 0^+} x = 0$.

We write the limit as:

$$\lim_{x \to 0^+} x^x$$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to 0^+} x^x$$

Then $\lim_{x \to 0^+} x = 0$.

We write the limit as:

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x}$$

Evaluate the limit

$$\lim_{x \to 0^+} x^x$$

Then $\lim_{x \to 0^+} x = 0$.

We write the limit as:

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x}$$
$$= e^{\lim_{x \to 0^+} (x \ln x)}$$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x \to 0^+} x^x$$

Then $\lim_{x \to 0^+} x = 0$.

We write the limit as:

$$\begin{aligned}\lim_{x \to 0^+} x^x &= \lim_{x \to 0^+} e^{\ln x^x} \\ &= e^{\lim_{x \to 0^+} (x \ln x)} \\ &= e^0\end{aligned}$$

L'Hospital's Rule

Evaluate the limit

$$\lim_{x\to 0^+} x^x$$

Then $\lim_{x\to 0^+} x = 0$.

We write the limit as:

$$\begin{aligned}\lim_{x\to 0^+} x^x &= \lim_{x\to 0^+} e^{\ln x^x} \\ &= e^{\lim_{x\to 0^+}(x \ln x)} \\ &= e^0 \\ &= 1\end{aligned}$$

L'Hospital's Rule

Evaluate the limit $\lim_{x \to 0^+} (1 + \sin 4x)^{\cot x}$

L'Hospital's Rule

Evaluate the limit $\lim_{x \to 0^+} (1 + \sin 4x)^{\cot x}$

Then $\lim_{x \to 0^+} (1 + \sin 4x) =$

L'Hospital's Rule

Evaluate the limit $\lim_{x \to 0^+} (1 + \sin 4x)^{\cot x}$

Then $\lim_{x \to 0^+} (1 + \sin 4x) = 1$

L'Hospital's Rule

Evaluate the limit $\lim_{x \to 0^+} (1 + \sin 4x)^{\cot x}$

Then $\lim_{x \to 0^+} (1 + \sin 4x) = 1$ and $\lim_{x \to 0^+} \cot x =$

L'Hospital's Rule

Evaluate the limit $\lim_{x \to 0^+} (1 + \sin 4x)^{\cot x}$

Then $\lim_{x \to 0^+} (1 + \sin 4x) = 1$ and $\lim_{x \to 0^+} \cot x = \infty$

L'Hospital's Rule

Evaluate the limit $\lim_{x \to 0^+} (1 + \sin 4x)^{\cot x}$

Then $\lim_{x \to 0^+} (1 + \sin 4x) = 1$ and $\lim_{x \to 0^+} \cot x = \infty$

We write the limit as:

$$\lim_{x \to 0^+} (1 + \sin 4x)^{cotx}$$

L'Hospital's Rule

Evaluate the limit $\lim_{x \to 0^+} (1 + \sin 4x)^{\cot x}$

Then $\lim_{x \to 0^+} (1 + \sin 4x) = 1$ and $\lim_{x \to 0^+} \cot x = \infty$

We write the limit as:

$$\lim_{x \to 0^+} (1 + \sin 4x)^{cotx} = \lim_{x \to 0^+} e^{\ln(1+\sin 4x)^{cotx}}$$

L'Hospital's Rule

Evaluate the limit $\lim_{x \to 0^+} (1 + \sin 4x)^{\cot x}$

Then $\lim_{x \to 0^+}(1 + \sin 4x) = 1$ and $\lim_{x \to 0^+} \cot x = \infty$

We write the limit as:

$$\begin{aligned} \lim_{x \to 0^+} (1 + \sin 4x)^{cotx} &= \lim_{x \to 0^+} e^{\ln(1+\sin 4x)^{cotx}} \\ &= e^{\lim_{x \to 0^+}(\cot x \cdot \ln(1+\sin 4x))} \end{aligned}$$

L'Hospital's Rule

Evaluate the limit $\lim_{x \to 0^+} (1 + \sin 4x)^{\cot x}$

Then $\lim_{x\to 0^+}(1+\sin 4x) = 1$ and $\lim_{x\to 0^+} \cot x = \infty$

We write the limit as:

$$\lim_{x \to 0^+} (1 + \sin 4x)^{cotx} = \lim_{x \to 0^+} e^{\ln(1+\sin 4x)^{cotx}}$$
$$= e^{\lim_{x\to 0^+}(\cot x \cdot \ln(1+\sin 4x))}$$

$$\lim_{x \to 0^+} (\cot x \cdot \ln(1 + \sin 4x))$$

L'Hospital's Rule

Evaluate the limit $\lim_{x\to 0^+}(1+\sin 4x)^{\cot x}$

Then $\lim_{x\to 0^+}(1+\sin 4x) = 1$ and $\lim_{x\to 0^+}\cot x = \infty$

We write the limit as:

$$\begin{aligned}\lim_{x\to 0^+}(1+\sin 4x)^{cotx} &= \lim_{x\to 0^+} e^{\ln(1+\sin 4x)^{cotx}} \\ &= e^{\lim_{x\to 0^+}(\cot x \cdot \ln(1+\sin 4x))}\end{aligned}$$

$$\lim_{x\to 0^+}(\cot x\cdot \ln(1+\sin 4x)) = \lim_{x\to 0^+}\frac{\ln(1+\sin 4x)}{\tan x}$$

L'Hospital's Rule

Evaluate the limit $\lim_{x \to 0^+} (1 + \sin 4x)^{\cot x}$

Then $\lim_{x\to 0^+}(1 + \sin 4x) = 1$ and $\lim_{x\to 0^+} \cot x = \infty$

We write the limit as:

$$\begin{aligned} \lim_{x \to 0^+} (1 + \sin 4x)^{cotx} &= \lim_{x \to 0^+} e^{\ln(1+\sin 4x)^{cotx}} \\ &= e^{\lim_{x\to 0^+}(\cot x \cdot \ln(1+\sin 4x))} \end{aligned}$$

$$\lim_{x \to 0^+} (\cot x \cdot \ln(1 + \sin 4x)) = \lim_{x \to 0^+} \frac{\ln(1 + \sin 4x)}{\tan x}$$

Now $\lim_{x\to 0^+} \ln(1 + \sin 4x) =$

L'Hospital's Rule

Evaluate the limit $\lim_{x \to 0^+} (1 + \sin 4x)^{\cot x}$

Then $\lim_{x \to 0^+} (1 + \sin 4x) = 1$ and $\lim_{x \to 0^+} \cot x = \infty$

We write the limit as:

$$\begin{aligned}\lim_{x \to 0^+} (1 + \sin 4x)^{cotx} &= \lim_{x \to 0^+} e^{\ln(1+\sin 4x)^{cotx}} \\ &= e^{\lim_{x \to 0^+} (\cot x \cdot \ln(1+\sin 4x))}\end{aligned}$$

$$\lim_{x \to 0^+} (\cot x \cdot \ln(1 + \sin 4x)) = \lim_{x \to 0^+} \frac{\ln(1 + \sin 4x)}{\tan x}$$

Now $\lim_{x \to 0^+} \ln(1 + \sin 4x) = 0$

L'Hospital's Rule

Evaluate the limit $\lim_{x\to 0^+}(1+\sin 4x)^{\cot x}$

Then $\lim_{x\to 0^+}(1+\sin 4x)=1$ and $\lim_{x\to 0^+}\cot x=\infty$

We write the limit as:

$$\begin{aligned}\lim_{x\to 0^+}(1+\sin 4x)^{cotx} &= \lim_{x\to 0^+} e^{\ln(1+\sin 4x)^{cotx}}\\ &= e^{\lim_{x\to 0^+}(\cot x\cdot\ln(1+\sin 4x))}\end{aligned}$$

$$\lim_{x\to 0^+}(\cot x\cdot\ln(1+\sin 4x)) = \lim_{x\to 0^+}\frac{\ln(1+\sin 4x)}{\tan x}$$

Now $\lim_{x\to 0^+}\ln(1+\sin 4x)=0$ and $\lim_{x\to 0^+}\tan x=$

L'Hospital's Rule

Evaluate the limit $\lim_{x \to 0^+} (1 + \sin 4x)^{\cot x}$

Then $\lim_{x \to 0^+} (1 + \sin 4x) = 1$ and $\lim_{x \to 0^+} \cot x = \infty$

We write the limit as:

$$\lim_{x \to 0^+} (1 + \sin 4x)^{cotx} = \lim_{x \to 0^+} e^{\ln(1+\sin 4x)^{cotx}}$$
$$= e^{\lim_{x \to 0^+} (\cot x \cdot \ln(1+\sin 4x))}$$

$$\lim_{x \to 0^+} (\cot x \cdot \ln(1 + \sin 4x)) = \lim_{x \to 0^+} \frac{\ln(1 + \sin 4x)}{\tan x}$$

Now $\lim_{x \to 0^+} \ln(1 + \sin 4x) = 0$ and $\lim_{x \to 0^+} \tan x = 0$

L'Hospital's Rule

Evaluate the limit $\lim_{x\to 0^+}(1+\sin 4x)^{\cot x}$

Then $\lim_{x\to 0^+}(1+\sin 4x) = 1$ and $\lim_{x\to 0^+}\cot x = \infty$

We write the limit as:

$$\begin{aligned}\lim_{x\to 0^+}(1+\sin 4x)^{cotx} &= \lim_{x\to 0^+} e^{\ln(1+\sin 4x)^{cotx}} \\ &= e^{\lim_{x\to 0^+}(\cot x\cdot\ln(1+\sin 4x))}\end{aligned}$$

$$\lim_{x\to 0^+}(\cot x\cdot\ln(1+\sin 4x)) = \lim_{x\to 0^+}\frac{\ln(1+\sin 4x)}{\tan x}$$

Now $\lim_{x\to 0^+}\ln(1+\sin 4x) = 0$ and $\lim_{x\to 0^+}\tan x = 0$

Hence we can apply l'Hospital's Rule:

$$\lim_{x\to 0^+}\frac{\ln(1+\sin 4x)}{\tan x}$$

L'Hospital's Rule

Evaluate the limit $\lim_{x \to 0^+} (1 + \sin 4x)^{\cot x}$

Then $\lim_{x \to 0^+} (1 + \sin 4x) = 1$ and $\lim_{x \to 0^+} \cot x = \infty$

We write the limit as:

$$\begin{aligned} \lim_{x \to 0^+} (1 + \sin 4x)^{cotx} &= \lim_{x \to 0^+} e^{\ln(1+\sin 4x)^{cotx}} \\ &= e^{\lim_{x \to 0^+} (\cot x \cdot \ln(1+\sin 4x))} \end{aligned}$$

$$\lim_{x \to 0^+} (\cot x \cdot \ln(1 + \sin 4x)) = \lim_{x \to 0^+} \frac{\ln(1 + \sin 4x)}{\tan x}$$

Now $\lim_{x \to 0^+} \ln(1 + \sin 4x) = 0$ and $\lim_{x \to 0^+} \tan x = 0$

Hence we can apply l'Hospital's Rule:

$$\lim_{x \to 0^+} \frac{\ln(1 + \sin 4x)}{\tan x} = \lim_{x \to 0^+} \frac{\frac{4 \cos 4x}{1+\sin 4x}}{(\sec x)^2}$$

L'Hospital's Rule

Evaluate the limit $\lim_{x\to 0^+} (1+\sin 4x)^{\cot x}$

Then $\lim_{x\to 0^+}(1+\sin 4x) = 1$ and $\lim_{x\to 0^+} \cot x = \infty$

We write the limit as:

$$\begin{aligned}\lim_{x\to 0^+} (1+\sin 4x)^{cotx} &= \lim_{x\to 0^+} e^{\ln(1+\sin 4x)^{cotx}} \\ &= e^{\lim_{x\to 0^+}(\cot x\cdot\ln(1+\sin 4x))}\end{aligned}$$

$$\lim_{x\to 0^+} (\cot x \cdot \ln(1+\sin 4x)) = \lim_{x\to 0^+} \frac{\ln(1+\sin 4x)}{\tan x}$$

Now $\lim_{x\to 0^+} \ln(1+\sin 4x) = 0$ and $\lim_{x\to 0^+} \tan x = 0$

Hence we can apply l'Hospital's Rule:

$$\lim_{x\to 0^+} \frac{\ln(1+\sin 4x)}{\tan x} = \lim_{x\to 0^+} \frac{\frac{4\cos 4x}{1+\sin 4x}}{(\sec x)^2} = \frac{\left(\frac{4}{1}\right)}{1}$$

L'Hospital's Rule

Evaluate the limit $$\lim_{x\to 0^+}(1+\sin 4x)^{\cot x}$$

Then $\lim_{x\to 0^+}(1+\sin 4x) = 1$ and $\lim_{x\to 0^+}\cot x = \infty$

We write the limit as:

$$\lim_{x\to 0^+}(1+\sin 4x)^{cotx} = \lim_{x\to 0^+} e^{\ln(1+\sin 4x)^{cotx}} = e^{\lim_{x\to 0^+}(\cot x\cdot \ln(1+\sin 4x))}$$

$$\lim_{x\to 0^+}(\cot x \cdot \ln(1+\sin 4x)) = \lim_{x\to 0^+}\frac{\ln(1+\sin 4x)}{\tan x}$$

Now $\lim_{x\to 0^+}\ln(1+\sin 4x) = 0$ and $\lim_{x\to 0^+}\tan x = 0$

Hence we can apply l'Hospital's Rule:

$$\lim_{x\to 0^+}\frac{\ln(1+\sin 4x)}{\tan x} = \lim_{x\to 0^+}\frac{\frac{4\cos 4x}{1+\sin 4x}}{(\sec x)^2} = \frac{\left(\frac{4}{1}\right)}{1} = 4$$

L'Hospital's Rule

Evaluate the limit $\lim_{x\to 0^+} (1+\sin 4x)^{\cot x}$

Then $\lim_{x\to 0^+}(1+\sin 4x) = 1$ and $\lim_{x\to 0^+} \cot x = \infty$

We write the limit as:

$$\lim_{x\to 0^+} (1+\sin 4x)^{cotx} = \lim_{x\to 0^+} e^{\ln(1+\sin 4x)^{cotx}}$$
$$= e^{\lim_{x\to 0^+}(\cot x \cdot \ln(1+\sin 4x))}$$

$$\lim_{x\to 0^+} (\cot x \cdot \ln(1+\sin 4x)) = \lim_{x\to 0^+} \frac{\ln(1+\sin 4x)}{\tan x}$$

Now $\lim_{x\to 0^+} \ln(1+\sin 4x) = 0$ and $\lim_{x\to 0^+} \tan x = 0$

Hence we can apply l'Hospital's Rule:

$$\lim_{x\to 0^+} \frac{\ln(1+\sin 4x)}{\tan x} = \lim_{x\to 0^+} \frac{\frac{4\cos 4x}{1+\sin 4x}}{(\sec x)^2} = \frac{\left(\frac{4}{1}\right)}{1} = 4$$

Thus $\lim_{x\to 0^+}(1+\sin 4x)^{\cot x} = e^4$