Calculus M211

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Indiana University Bloomington

2013

Derivatives and the Shape of a Graph

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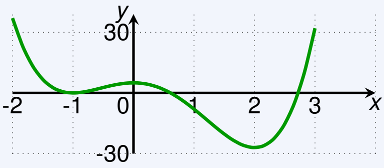

Recall Fermat's Theorem

If f has a local extremum at c, then c is a critical number.

Derivatives and the Shape of a Graph

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If f has a local extremum at c, then c is a critical number.

But not ever critical number is an extremum.

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Recall Fermat's Theorem

If f has a local extremum at c, then c is a critical number.

But not ever critical number is an extremum. We need a test!

First Derivative Test

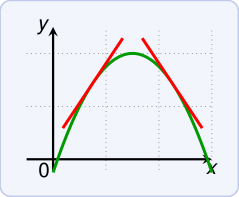
Suppose that c is a critical number of a continuous function f.

Derivatives and the Shape of a Graph

First Derivative Test

Suppose that c is a critical number of a continuous function f.

- If f' changes the sign from positive to negative, then f has a local maximum at c.



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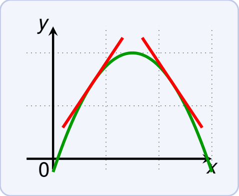

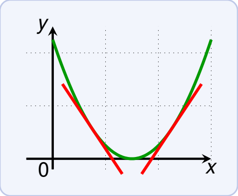

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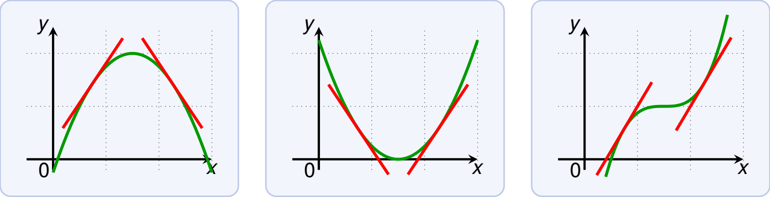
First Derivative Test

Suppose that c is a critical number of a continuous function f.

- If f' changes the sign from positive to negative, then f has a local maximum at c.
- If f' changes the sign from negative to positive, then f has a local minimum at c.
- If f' does not change sign at c, then f has no local extremum at c.



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The critical numbers are:

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The critical numbers are: -1, 0 and 2.

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We have already seen that:

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We have:

- $f(-1) = 0$ is a local minimum   (f' changes from $-$ to $+$)

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We have:

- $f(-1) = 0$ is a local minimum (f' changes from $-$ to $+$)
- $f(0) = 5$ is a local maximum (f' changes from $+$ to $-$)
- $f(2) = -27$ is

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What are the local extrema of $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$?

$$f'(x) = 12x(x-2)(x+1)$$

The critical numbers are: -1, 0 and 2.

We have already seen that:

Interval	$12x$	$x-2$	$x+1$	$f'(x)$	
x < -1	-	-	-	-	decreasing on $(-\infty, -1)$
-1 < x < 0	-	-	+	+	increasing on $(-1, 0)$
0 < x < 2	+	-	+	-	decreasing on $(0, 2)$
2 < x	+	+	+	+	increasing on $(2, \infty)$

We have:

- $f(-1) = 0$ is a local minimum (f' changes from $-$ to $+$)
- $f(0) = 5$ is a local maximum (f' changes from $+$ to $-$)
- $f(2) = -27$ is a local minimum (f' changes from $-$ to $+$)

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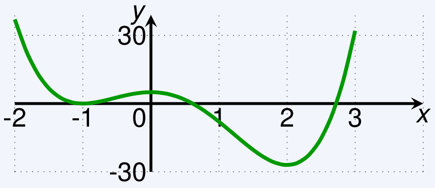

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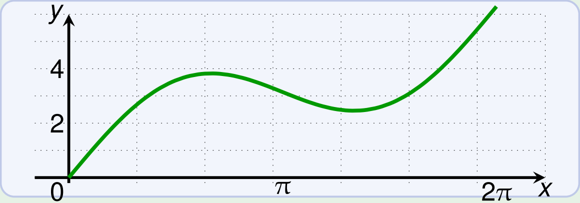

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Let I be an interval. If the graph of f is called

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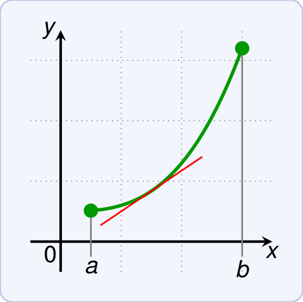
- **concave up** on I if it it lies above all its tangents on I



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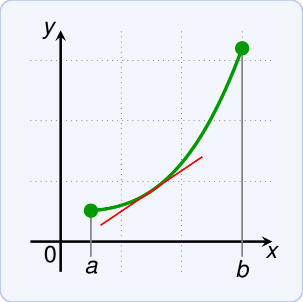

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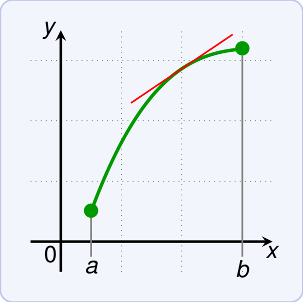

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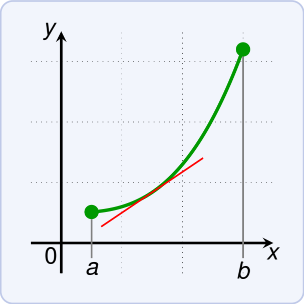

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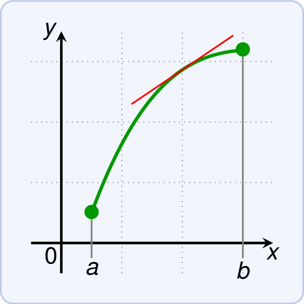

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Imagine the graph as a street & a car driving from left to right:

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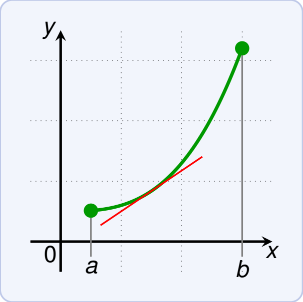

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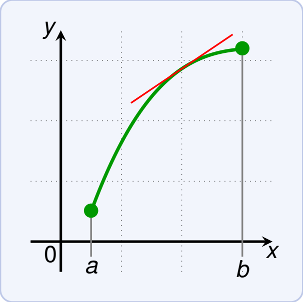

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Imagine the graph as a street & a car driving from left to right:

- then concave upward = turning left

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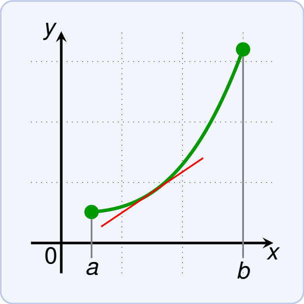

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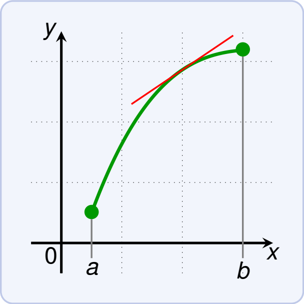

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Imagine the graph as a street & a car driving from left to right:

- then concave upward = turning left (increasing slope)

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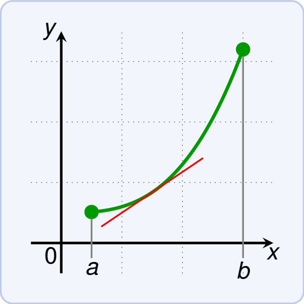

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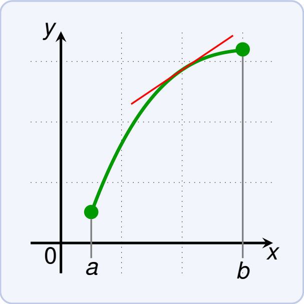

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Imagine the graph as a street & a car driving from left to right:

- then concave upward = turning left (increasing slope)
- then concave downward = turning right

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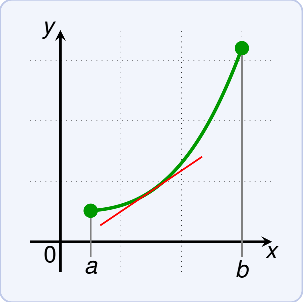

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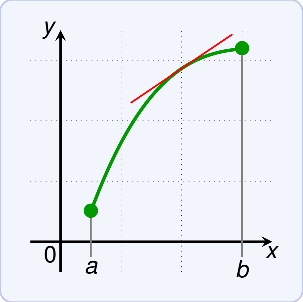

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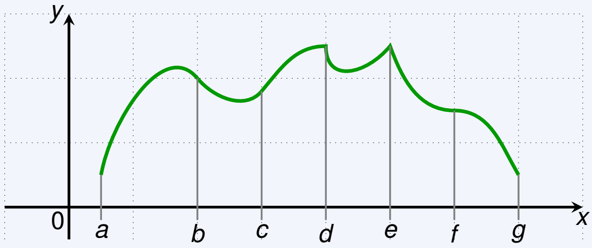

On which interval is the curve concave up / concave down?

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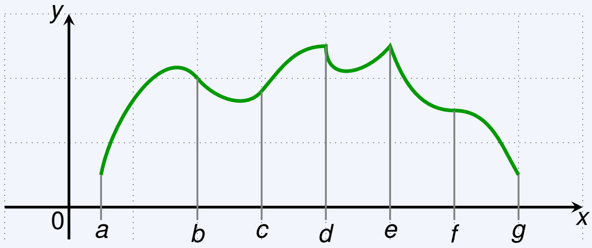

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- on (a,b)

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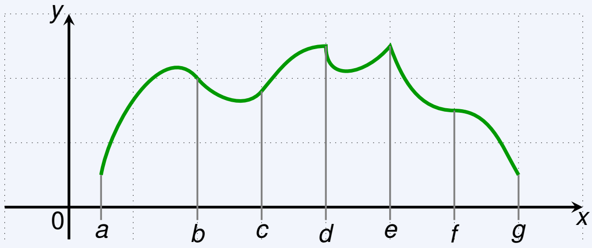

On which interval is the curve concave up / concave down?

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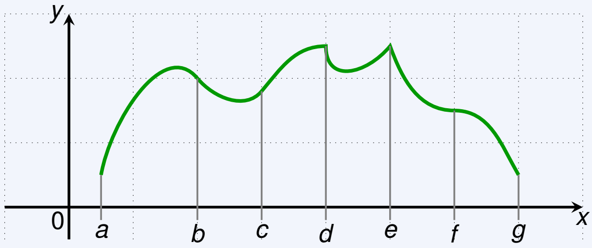

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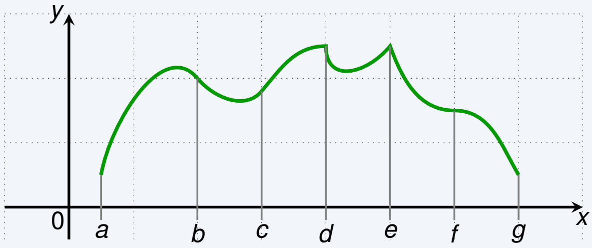

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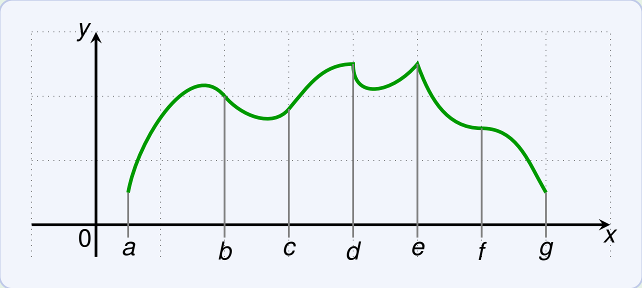

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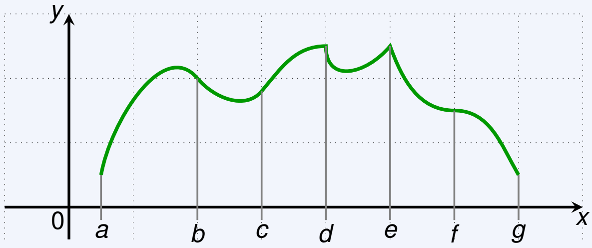

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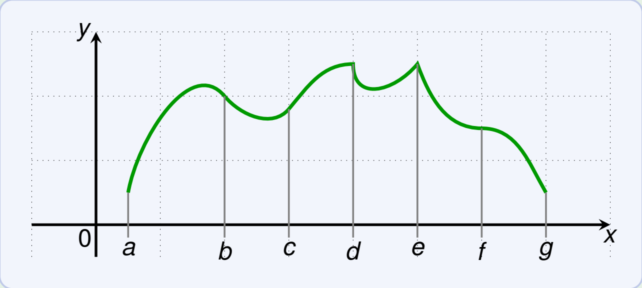
- on (c,d) concave downward

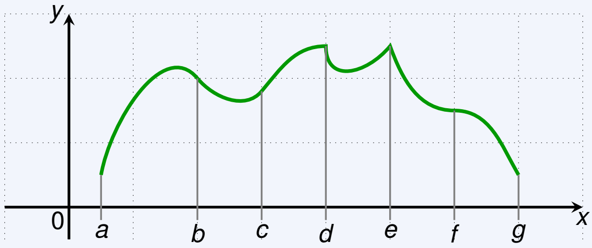
Derivatives and the Shape of a Graph



On which interval is the curve concave up / concave down?

- on (a,b) concave downward
- on (b,c) concave upward
- on (c,d) concave downward
- on (d,e)

Derivatives and the Shape of a Graph



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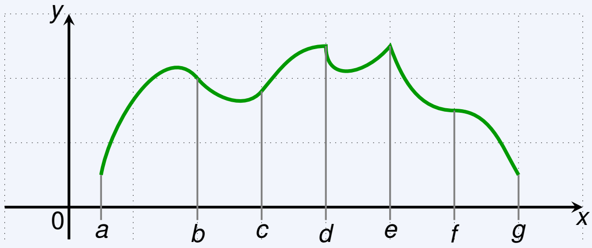
Derivatives and the Shape of a Graph



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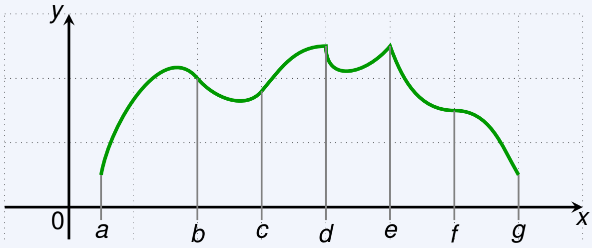
Derivatives and the Shape of a Graph



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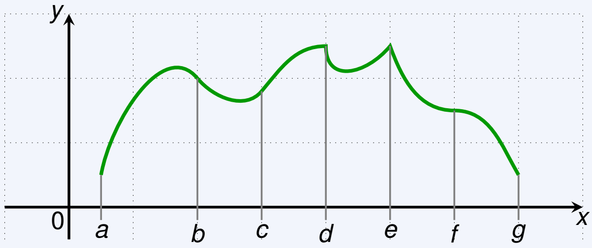
Derivatives and the Shape of a Graph



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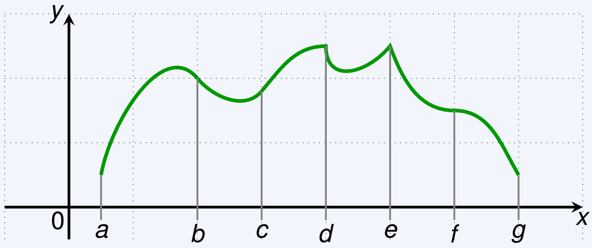
- on (e,f) concave upward
- on (f,g)

Derivatives and the Shape of a Graph



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- on (c,d) concave downward
- on (d,e) concave upward
- on (e,f) concave upward
- on (f,g) concave downward

Derivatives and the Shape of a Graph

Concavity Test

If $f''(x) > 0$ for all x in I, then f is concave upward on I.

Derivatives and the Shape of a Graph

Concavity Test

If $f''(x) > 0$ for all x in I, then f is concave upward on I.

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A point P on a curve $f(x)$ is called **inflection point** if f is continuous at this point and the curve

- changes from concave upward to downward at P, or
- changes from concave downward to upward at P.

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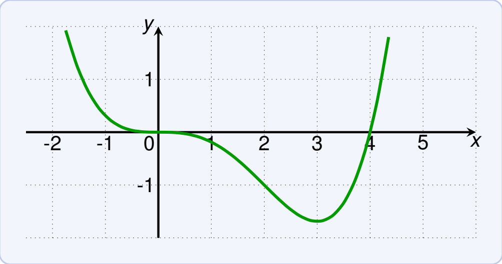

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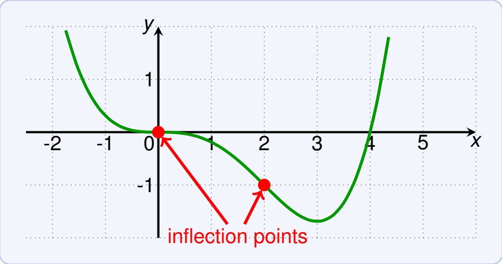

Where are inflection points of $f(x) = x^4 - 4x^3$?

Derivatives and the Shape of a Graph

Where are inflection points of $f(x) = x^4 - 4x^3$?

$$f'(x) = 4x^3 - 12x^2$$

Derivatives and the Shape of a Graph

Where are inflection points of $f(x) = x^4 - 4x^3$?

$$f'(x) = 4x^3 - 12x^2$$

$$f''(x) = 12x^2 - 24x$$

Derivatives and the Shape of a Graph

Where are inflection points of $f(x) = x^4 - 4x^3$?

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Derivatives and the Shape of a Graph

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Thus $f''(x) = 0$ for $x =$ and $x =$.

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Interval	$f''(x)$	

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Interval	$f''(x)$	
$x < 0$		

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Interval	$f''(x)$	
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Interval	$f''(x)$	
$x < 0$		
$0 < x < 2$		
$2 < x$		

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$2 < x$		

Derivatives and the Shape of a Graph

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Interval	$f''(x)$	
$x < 0$	$+$	concave upward on $(-\infty, 0)$
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$2 < x$		

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Thus the inflection points are:

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Thus the inflection points are:

- $(0, 0)$ since the curve changes from concave up to down

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Thus the inflection points are:

- $(0, 0)$ since the curve changes from concave up to down
- $(2, -16)$ since the curve changes from concave down to up

Derivatives and the Shape of a Graph

Second Derivative Test

Suppose f'' is continuous near c.

Derivatives and the Shape of a Graph

Second Derivative Test

Suppose f'' is continuous near c.

- If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

Derivatives and the Shape of a Graph

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Where does $f(x) = x^4 - 4x^3$ have local extrema?

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$$f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3)$$

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Thus $f'(x) = 0$ for $x = 0$ and $x = 3$. Second Derivative Test:

$$f''(0) = 0 \qquad\qquad f''(3) = 36$$

Derivatives and the Shape of a Graph

Second Derivative Test

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Thus $f'(x) = 0$ for $x = 0$ and $x = 3$. Second Derivative Test:

$$f''(0) = 0 \qquad\qquad f''(3) = 36 > 0$$

Derivatives and the Shape of a Graph

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$$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$$

$$f''(x) = 12x^2 - 24x = 12x(x-2)$$

Thus $f'(x) = 0$ for $x = 0$ and $x = 3$. Second Derivative Test:

$$f''(0) = 0 \qquad\qquad f''(3) = 36 > 0$$

Thus $f(3) = -27$ is a local minimum as $f'(3) = 0$ and $f''(3) > 0$.

Derivatives and the Shape of a Graph

Second Derivative Test

Suppose f'' is continuous near c.

- If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.
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$$f''(x) = 12x^2 - 24x = 12x(x-2)$$

Thus $f'(x) = 0$ for $x = 0$ and $x = 3$. Second Derivative Test:

$$f''(0) = 0 \qquad\qquad f''(3) = 36 > 0$$

Thus $f(3) = -27$ is a local minimum as $f'(3) = 0$ and $f''(3) > 0$.

The Second Derivative Test gives no information for $f''(0) = 0$.

Derivatives and the Shape of a Graph

Second Derivative Test

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However, the First Derivative Test . . .

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$$f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3)$$

$$f''(x) = 12x^2 - 24x = 12x(x - 2)$$

Thus $f'(x) = 0$ for $x = 0$ and $x = 3$. Second Derivative Test:

$$f''(0) = 0 \qquad\qquad f''(3) = 36 > 0$$

Thus $f(3) = -27$ is a local minimum as $f'(3) = 0$ and $f''(3) > 0$.

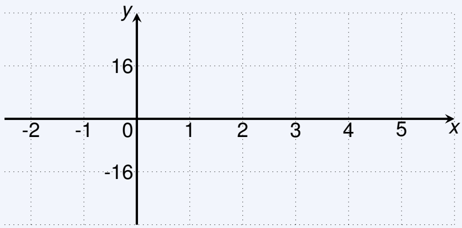
The Second Derivative Test gives no information for $f''(0) = 0$.

However, the First Derivative Test . . . yields that $f(0) = 0$ is no extremum since $f'(x) < 0$ for $x < 0$ and $0 < x < 3$.

Derivatives and the Shape of a Graph

Curve Sketching

$$f(x) = x^4 - 4x^3 = x^3(x-4) \qquad f'(x) = 4x^2(x-3)$$

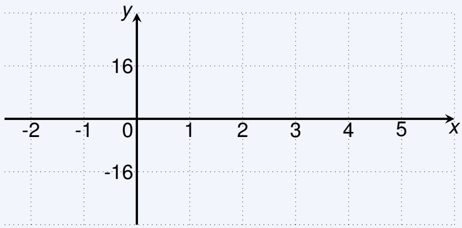


Derivatives and the Shape of a Graph

Curve Sketching

$$f(x) = x^4 - 4x^3 = x^3(x-4) \qquad f'(x) = 4x^2(x-3)$$

- $f(x) = 0 \iff$

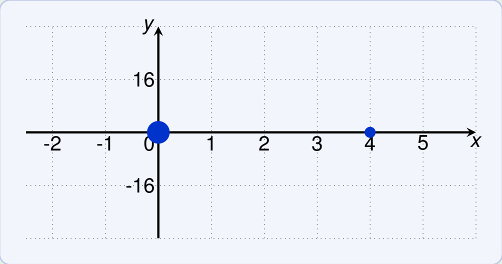


Derivatives and the Shape of a Graph

Curve Sketching

$$f(x) = x^4 - 4x^3 = x^3(x-4) \qquad f'(x) = 4x^2(x-3)$$

- $f(x) = 0 \iff x = 0$ or $x = 4$



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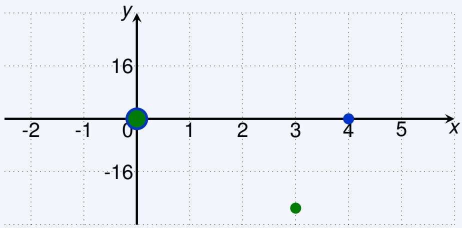
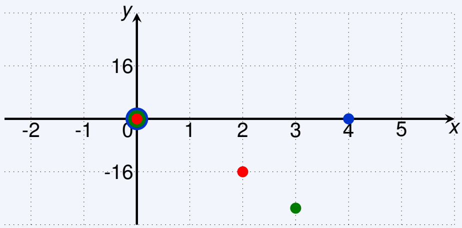


Derivatives and the Shape of a Graph

Curve Sketching

$$f(x) = x^4 - 4x^3 = x^3(x-4) \qquad f'(x) = 4x^2(x-3)$$

- $f(x) = 0 \iff x = 0$ or $x = 4$
- local minimum at $(3, -27)$ and $f'(0) = 0$
- inflection points $(0, 0)$ and $(2, -16)$

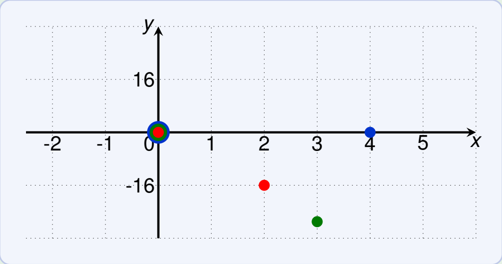


Derivatives and the Shape of a Graph

Curve Sketching

$$f(x) = x^4 - 4x^3 = x^3(x-4) \qquad f'(x) = 4x^2(x-3)$$

- $f(x) = 0 \iff x = 0$ or $x = 4$
- local minimum at $(3, -27)$ and $f'(0) = 0$
- inflection points $(0, 0)$ and $(2, -16)$
- decreasing on



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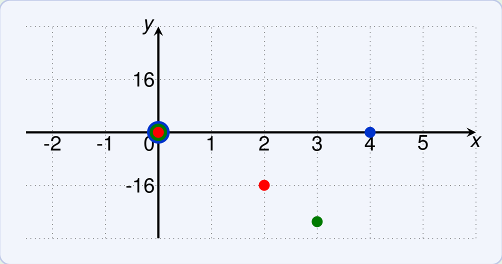

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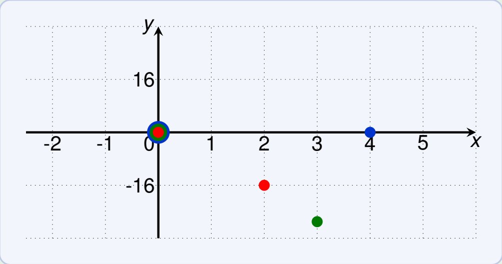

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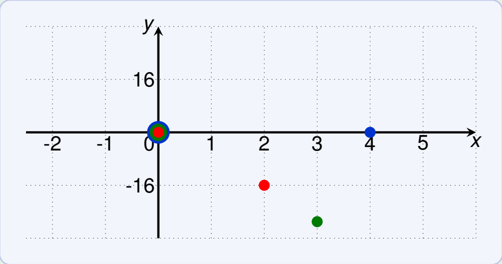

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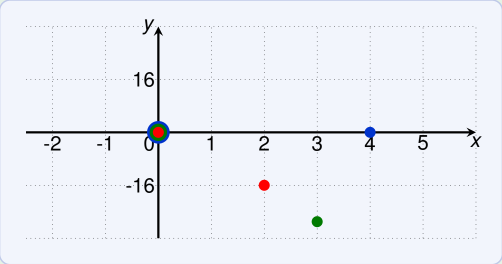

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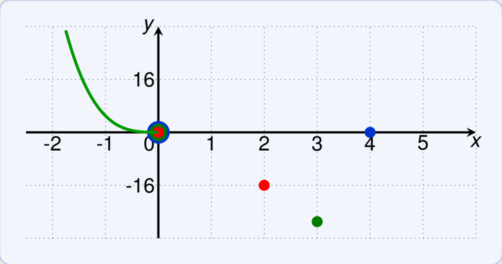

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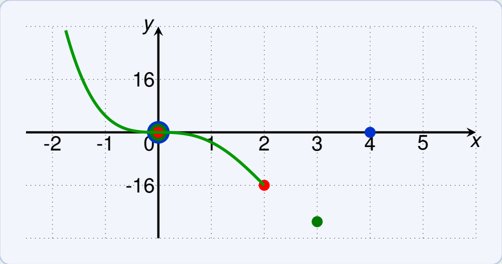

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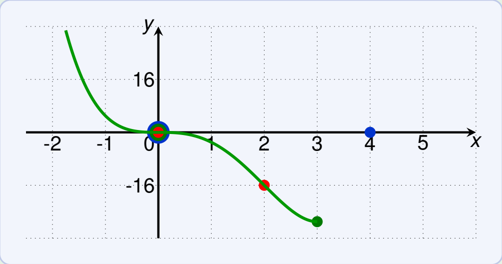

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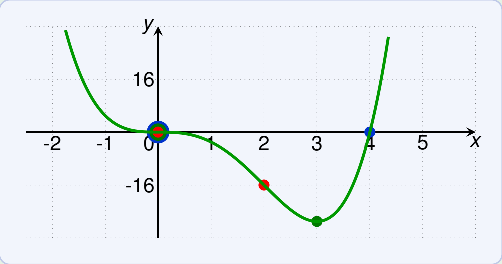

Summary: Finding Local Extrema

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Find critical numbers c: $f'(c) = 0$ or $f'(c)$ does not exist.

Derivatives and the Shape of a Graph

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First **Derivative Test** (f needs to be continuous at c):

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First **Derivative Test** (f needs to be continuous at c):

- If f' changes from $+$ to $-$ at $c \implies$ local maximum

Derivatives and the Shape of a Graph

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- If f' changes from $+$ to $-$ at $c \implies$ local maximum
- If f' changes from $-$ to $+$ at $c \implies$ local minimum

Summary: Finding Local Extrema

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First **Derivative Test** (f needs to be continuous at c):

- If f' changes from $+$ to $-$ at $c \implies$ local maximum
- If f' changes from $-$ to $+$ at $c \implies$ local minimum
- If f' does not change sign at $c \implies$ no local extremum

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The **Second Derivative Test**:

Derivatives and the Shape of a Graph

Summary: Finding Local Extrema

Find critical numbers c: $f'(c) = 0$ or $f'(c)$ does not exist.

First **Derivative Test** (f needs to be continuous at c):

- If f' changes from $+$ to $-$ at $c \implies$ local maximum
- If f' changes from $-$ to $+$ at $c \implies$ local minimum
- If f' does not change sign at $c \implies$ no local extremum

The **Second Derivative Test**:

1. $f'(c) = 0$ and $f''(c) > 0 \implies$ local minimum

Derivatives and the Shape of a Graph

Summary: Finding Local Extrema

Find critical numbers c: $f'(c) = 0$ or $f'(c)$ does not exist.

First **Derivative Test** (f needs to be continuous at c):

- If f' changes from $+$ to $-$ at $c \implies$ local maximum
- If f' changes from $-$ to $+$ at $c \implies$ local minimum
- If f' does not change sign at $c \implies$ no local extremum

The **Second Derivative Test**:

1. $f'(c) = 0$ and $f''(c) > 0 \implies$ local minimum
2. $f'(c) = 0$ and $f''(c) < 0 \implies$ local maximum

Derivatives and the Shape of a Graph

Summary: Finding Local Extrema

Find critical numbers c: $f'(c) = 0$ or $f'(c)$ does not exist.

First **Derivative Test** (f needs to be continuous at c):

- If f' changes from $+$ to $-$ at $c \implies$ local maximum
- If f' changes from $-$ to $+$ at $c \implies$ local minimum
- If f' does not change sign at $c \implies$ no local extremum

The **Second Derivative Test**:

1. $f'(c) = 0$ and $f''(c) > 0 \implies$ local minimum
2. $f'(c) = 0$ and $f''(c) < 0 \implies$ local maximum
3. $f'(c)$ or $f''(c)$ does not exist or $f''(c) = 0$
   $\implies$ use the First Derivative Test