Calculus M211

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2013

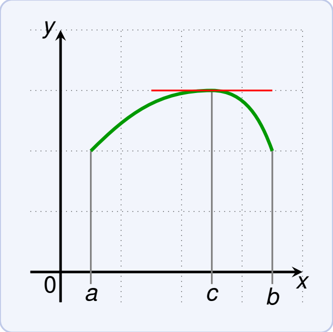
Mean Value Theorem

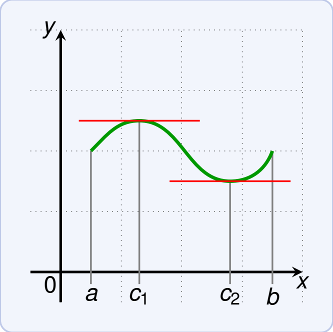
Rolle's Theorem

Let f be a function satisfying the all of the following:

- f is continuous on $[a, b]$
- f is differentiable on (a, b)
- $f(a) = f(b)$

Then there is a number c in (a, b) such that $f'(c) = 0$.





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- If f is constant, then $f'(c) = 0$ for all c in (a, b).

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- If f is constant, then $f'(c) = 0$ for all c in (a, b).
- If f is not constant, then there is x in (a, b) such that

$$f(x) > f(a) \qquad \text{or} \qquad f(x) < f(a)$$

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  Assume $f(x) > f(a)$.

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  Then c must be in (a, b) and hence is a local maximum.

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  Then c must be in (a, b) and hence is a local maximum. Hence $f'(c) = 0$ by Fermat's Theorem. □

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Let $s(t)$ be the position of an object after time t.
The object is in the same place at time $t = 2s$ and $t = 10s$.

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What does Rolle's Theorem tell us about the object?

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Let $s(t)$ be the position of an object after time t.
The object is in the same place at time $t = 2s$ and $t = 10s$.

What does Rolle's Theorem tell us about the object?

It tells that there is a time c between $2s$ and $10s$ such that the

$$s'(t) = 0$$

that is, the velocity of the object at time c is 0.

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Show that the function f is one-to-one (never takes the same value twice):

$$f(x) = x^3 + x - 1$$

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By Rolle's Theorem there exists c in (x_1, x_2) with $f'(c) = 0$.

This is a contradiction since $f'(x) = 3x^2 + 1 \geq 1$ for all x.
There no $x_1 < x_2$ such that $f(x_1) = f(x_2)$. Thus f is one-to-one.

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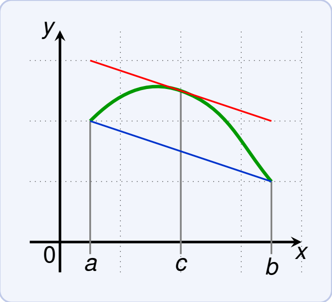

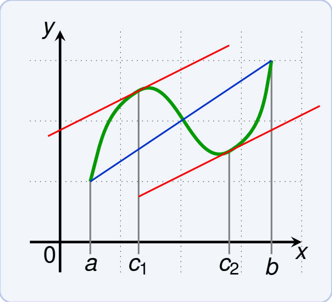

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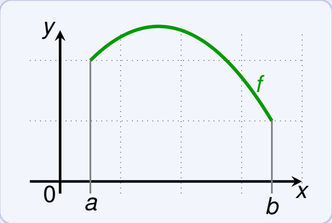

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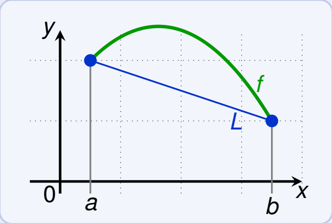

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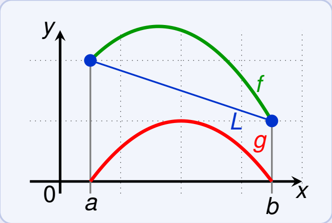

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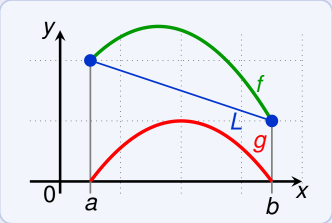

Let $L = mx + n$ be the line through $(a, f(a))$ and $(b, f(b))$.

Define $g = f - L$. Then $g(a) = 0$ and $g(b) = 0$.

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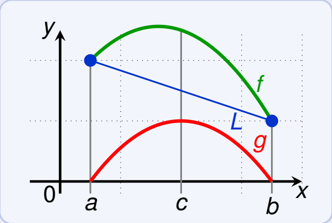

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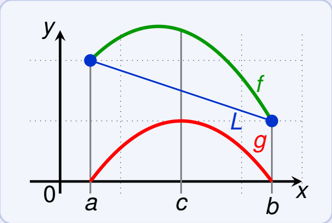

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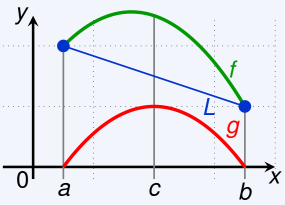

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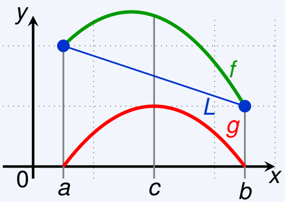

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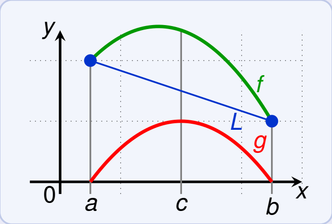

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Since $f = g + L$ we get $f'(c) = g'(c) + m = m = \frac{f(b)-f(a)}{b-a}$.

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Consider the function

$$f(x) = x^3 - x$$

on the interval $[a, b]$ with $a = 0$ and $b = 2$.

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By the Mean Value Theorem, there is a c in $(0, 2)$ such that

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By the Mean Value Theorem, there is a c in $(0, 2)$ such that

$$f'(c) = \frac{f(2) - f(0)}{2 - 0} = \frac{6}{2} = 3$$

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$$f'(c) = \frac{f(2) - f(0)}{2 - 0} = \frac{6}{2} = 3$$

Indeed, we can find such a c, namely: $f'\left(\frac{2}{\sqrt{3}}\right) = 3$.

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Let $s(t)$ be the position of an object after time t.

Then the average velocity between time $t = a$ and $t = b$ is:

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What does the Mean Value Theorem tell us?

It states that there is a time c between a and b such that

$$f'(c) = \frac{s(b) - s(a)}{b - a} \quad , \quad \text{that is}$$

the instantaneous velocity at c is equal to the average velocity.

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We can interpret the Mean Value Theorem as follows:

> There is a number c in the interval (a, b) such that the instantaneous rate of change at c is equal to the average rate of change over the interval $[a, b]$.

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Let f be a function satisfying the all of the following:

- f is continuous on $[a, b]$
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Then there is a number c in (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

Suppose that $f(0) = -3$ and $f'(x) \leq 5$ for all x.

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How large can $f(2)$ possibly be?

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Thus the largest possible value for $f(2)$ is 7.

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Let $h = f - g$. Then $h' = f' - g' = 0$ on (a, b). Thus h is constant on (a, b). □