

Calculus M211

Jörg Endrullis

Indiana University Bloomington

2013

Maximum and Minimum Values

An important application of derivatives are

optimization problems,

that is, finding the best way of doing something.

Maximum and Minimum Values

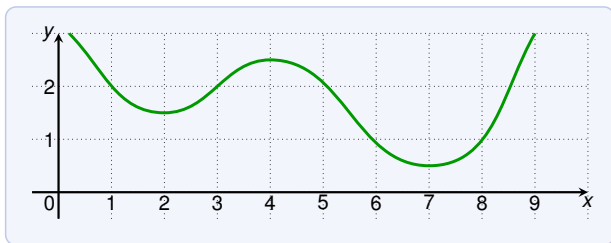
An important application of derivatives are

optimization problems,

that is, finding the best way of doing something.

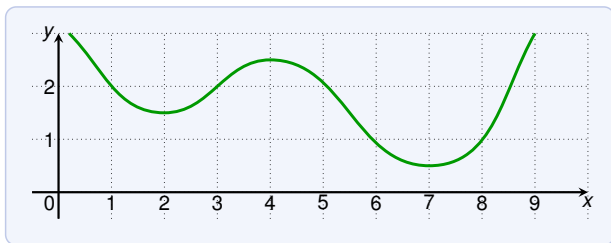
These problems can often be reduced to finding the minimum or maximum of a function.

Maximum and Minimum Values



Let c be in the domain D of f . Then $f(c)$ is the

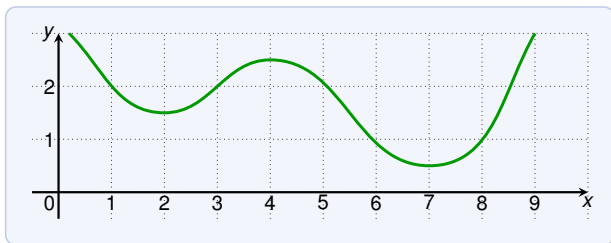
Maximum and Minimum Values



Let c be in the domain D of f . Then $f(c)$ is the

- ▶ **absolute maximum** value of f if $f(c) \geq f(x)$ for all x in D

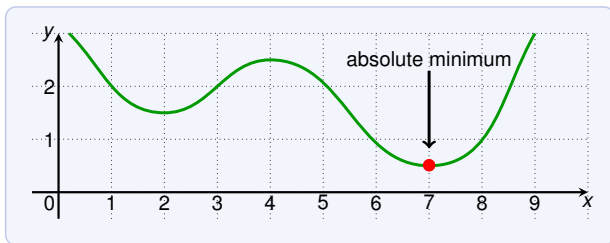
Maximum and Minimum Values



Let c be in the domain D of f . Then $f(c)$ is the

- ▶ **absolute maximum** value of f if $f(c) \geq f(x)$ for all x in D
- ▶ **absolute minimum** value of f if $f(c) \leq f(x)$ for all x in D

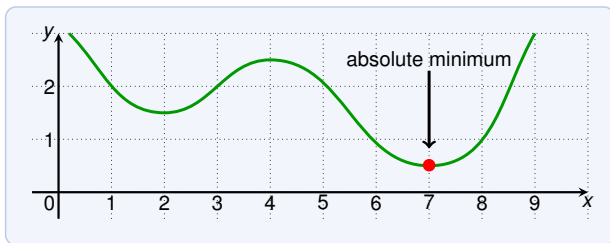
Maximum and Minimum Values



Let c be in the domain D of f . Then $f(c)$ is the

- ▶ **absolute maximum** value of f if $f(c) \geq f(x)$ for all x in D
- ▶ **absolute minimum** value of f if $f(c) \leq f(x)$ for all x in D

Maximum and Minimum Values



Let c be in the domain D of f . Then $f(c)$ is the

- ▶ **absolute maximum** value of f if $f(c) \geq f(x)$ for all x in D
- ▶ **absolute minimum** value of f if $f(c) \leq f(x)$ for all x in D

Often called **global maximum** or **global minimum**.

Maximum and Minimum Values



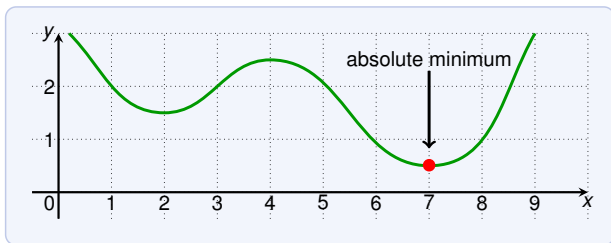
Let c be in the domain D of f . Then $f(c)$ is the

- ▶ **absolute maximum** value of f if $f(c) \geq f(x)$ for all x in D
- ▶ **absolute minimum** value of f if $f(c) \leq f(x)$ for all x in D

Often called **global maximum** or **global minimum**.

Minima and maxima are called **extreme values** of f .

Maximum and Minimum Values



Let c be in the domain D of f . Then $f(c)$ is the

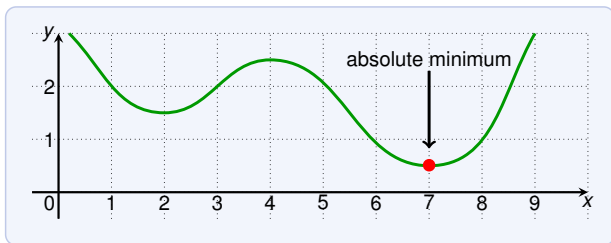
- ▶ **absolute maximum** value of f if $f(c) \geq f(x)$ for all x in D
- ▶ **absolute minimum** value of f if $f(c) \leq f(x)$ for all x in D

Often called **global maximum** or **global minimum**.

Minima and maxima are called **extreme values** of f .

The number $f(c)$ is a

Maximum and Minimum Values



Let c be in the domain D of f . Then $f(c)$ is the

- ▶ **absolute maximum** value of f if $f(c) \geq f(x)$ for all x in D
- ▶ **absolute minimum** value of f if $f(c) \leq f(x)$ for all x in D

Often called **global maximum** or **global minimum**.

Minima and maxima are called **extreme values** of f .

The number $f(c)$ is a

- ▶ **local maximum** value of f if $f(c) \geq f(x)$ when x is near c

Maximum and Minimum Values



Let c be in the domain D of f . Then $f(c)$ is the

- ▶ **absolute maximum** value of f if $f(c) \geq f(x)$ for all x in D
- ▶ **absolute minimum** value of f if $f(c) \leq f(x)$ for all x in D

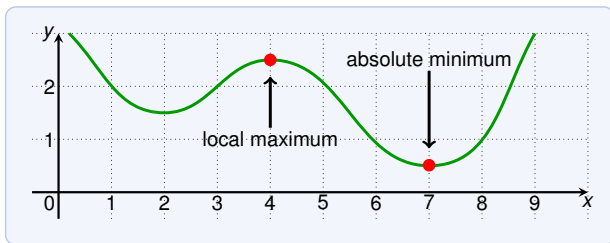
Often called **global maximum** or **global minimum**.

Minima and maxima are called **extreme values** of f .

The number $f(c)$ is a

- ▶ **local maximum** value of f if $f(c) \geq f(x)$ when x is near c
- ▶ **local minimum** value of f if $f(c) \leq f(x)$ when x is near c

Maximum and Minimum Values



Let c be in the domain D of f . Then $f(c)$ is the

- ▶ **absolute maximum** value of f if $f(c) \geq f(x)$ for all x in D
- ▶ **absolute minimum** value of f if $f(c) \leq f(x)$ for all x in D

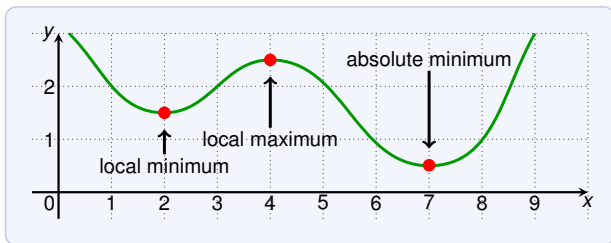
Often called **global maximum** or **global minimum**.

Minima and maxima are called **extreme values** of f .

The number $f(c)$ is a

- ▶ **local maximum** value of f if $f(c) \geq f(x)$ when x is near c
- ▶ **local minimum** value of f if $f(c) \leq f(x)$ when x is near c

Maximum and Minimum Values



Let c be in the domain D of f . Then $f(c)$ is the

- ▶ **absolute maximum** value of f if $f(c) \geq f(x)$ for all x in D
- ▶ **absolute minimum** value of f if $f(c) \leq f(x)$ for all x in D

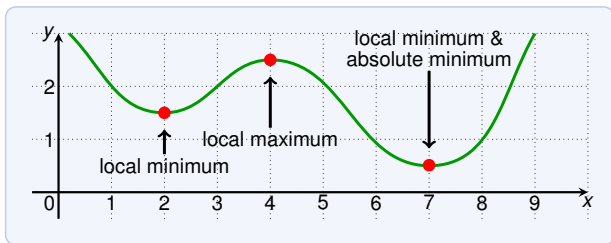
Often called **global maximum** or **global minimum**.

Minima and maxima are called **extreme values** of f .

The number $f(c)$ is a

- ▶ **local maximum** value of f if $f(c) \geq f(x)$ when x is near c
- ▶ **local minimum** value of f if $f(c) \leq f(x)$ when x is near c

Maximum and Minimum Values



Let c be in the domain D of f . Then $f(c)$ is the

- ▶ **absolute maximum** value of f if $f(c) \geq f(x)$ for all x in D
- ▶ **absolute minimum** value of f if $f(c) \leq f(x)$ for all x in D

Often called **global maximum** or **global minimum**.

Minima and maxima are called **extreme values** of f .

The number $f(c)$ is a

- ▶ **local maximum** value of f if $f(c) \geq f(x)$ when x is near c
- ▶ **local minimum** value of f if $f(c) \leq f(x)$ when x is near c

Maximum and Minimum Values

Where does

$$f(x) = x^2$$

have local / global minima or maxima?

Maximum and Minimum Values

Where does

$$f(x) = x^2$$

have local / global minima or maxima?

The value $f(0) = 0$ is absolute and local minimum since:

$$f(0) = 0 \leq x^2 = f(x) \quad \text{for all } x$$

Maximum and Minimum Values

Where does

$$f(x) = x^2$$

have local / global minima or maxima?

The value $f(0) = 0$ is absolute and local minimum since:

$$f(0) = 0 \leq x^2 = f(x) \quad \text{for all } x$$

The function has no local or global maxima.

Maximum and Minimum Values

Where does

$$f(x) = x^2$$

have local / global minima or maxima?

The value $f(0) = 0$ is absolute and local minimum since:

$$f(0) = 0 \leq x^2 = f(x) \quad \text{for all } x$$

The function has no local or global maxima.

Where does

$$f(x) = x^3$$

have (local or global) minima or maxima?

Maximum and Minimum Values

Where does

$$f(x) = x^2$$

have local / global minima or maxima?

The value $f(0) = 0$ is absolute and local minimum since:

$$f(0) = 0 \leq x^2 = f(x) \quad \text{for all } x$$

The function has no local or global maxima.

Where does

$$f(x) = x^3$$

have (local or global) minima or maxima?

The function has no local or global extrema.

Maximum and Minimum Values

Where does

$$f(x) = x^2$$

have local / global minima or maxima?

The value $f(0) = 0$ is absolute and local minimum since:

$$f(0) = 0 \leq x^2 = f(x) \quad \text{for all } x$$

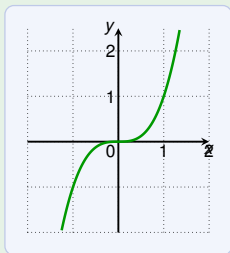
The function has no local or global maxima.

Where does

$$f(x) = x^3$$

have (local or global) minima or maxima?

The function has no local or global extrema.

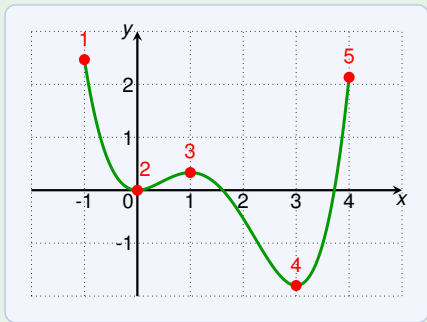


Maximum and Minimum Values

The graph of

$$f(x) = \frac{3x^4 - 16x^3 + 18x^2}{15}$$

for $-1 \leq x \leq 4$ is shown in this diagram:

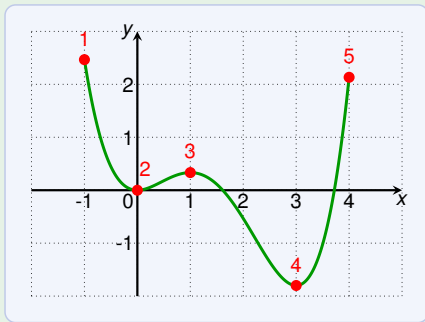


Maximum and Minimum Values

The graph of

$$f(x) = \frac{3x^4 - 16x^3 + 18x^2}{15}$$

for $-1 \leq x \leq 4$ is shown in this diagram:



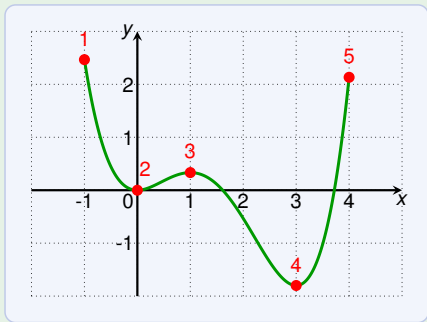
Which of the points are a local / global maxima or minima?

Maximum and Minimum Values

The graph of

$$f(x) = \frac{3x^4 - 16x^3 + 18x^2}{15}$$

for $-1 \leq x \leq 4$ is shown in this diagram:



Which of the points are a local / global maxima or minima?

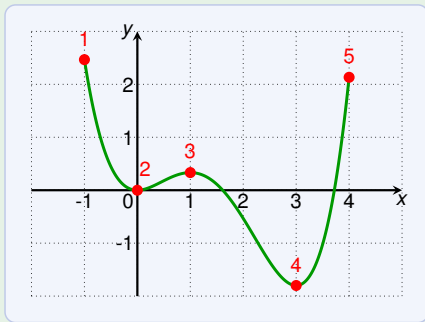
1.

Maximum and Minimum Values

The graph of

$$f(x) = \frac{3x^4 - 16x^3 + 18x^2}{15}$$

for $-1 \leq x \leq 4$ is shown in this diagram:



Which of the points are a local / global maxima or minima?

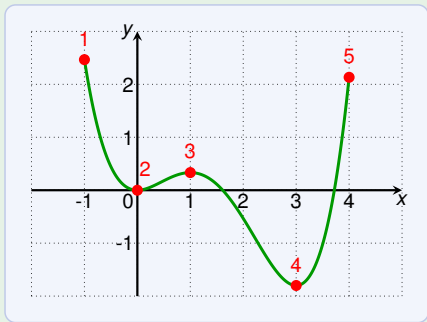
1. global (absolute) maximum;

Maximum and Minimum Values

The graph of

$$f(x) = \frac{3x^4 - 16x^3 + 18x^2}{15}$$

for $-1 \leq x \leq 4$ is shown in this diagram:



Which of the points are a local / global maxima or minima?

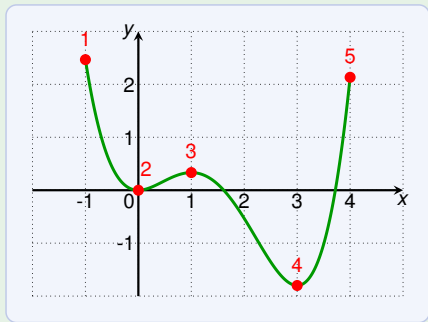
1. global (absolute) maximum;
not a local maximum since f is not defined near -1

Maximum and Minimum Values

The graph of

$$f(x) = \frac{3x^4 - 16x^3 + 18x^2}{15}$$

for $-1 \leq x \leq 4$ is shown in this diagram:



Which of the points are a local / global maxima or minima?

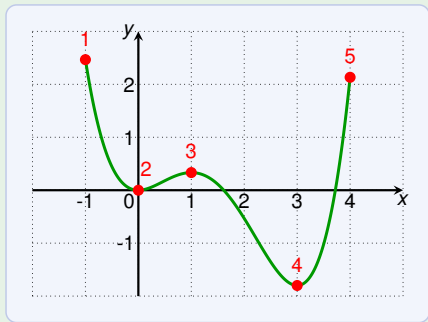
1. global (absolute) maximum;
not a local maximum since f is not defined near -1
- 2.

Maximum and Minimum Values

The graph of

$$f(x) = \frac{3x^4 - 16x^3 + 18x^2}{15}$$

for $-1 \leq x \leq 4$ is shown in this diagram:



Which of the points are a local / global maxima or minima?

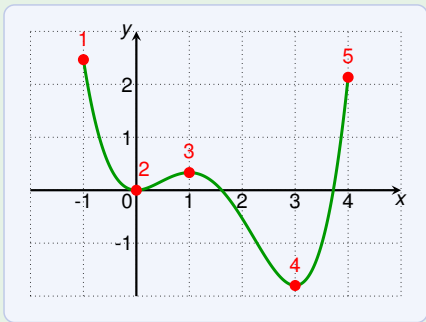
1. global (absolute) maximum;
not a local maximum since f is not defined near -1
2. local minimum

Maximum and Minimum Values

The graph of

$$f(x) = \frac{3x^4 - 16x^3 + 18x^2}{15}$$

for $-1 \leq x \leq 4$ is shown in this diagram:



Which of the points are a local / global maxima or minima?

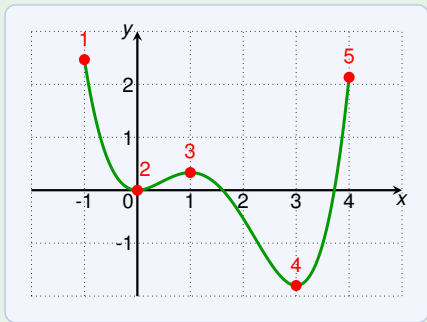
1. global (absolute) maximum;
not a local maximum since f is not defined near -1
2. local minimum
- 3.

Maximum and Minimum Values

The graph of

$$f(x) = \frac{3x^4 - 16x^3 + 18x^2}{15}$$

for $-1 \leq x \leq 4$ is shown in this diagram:



Which of the points are a local / global maxima or minima?

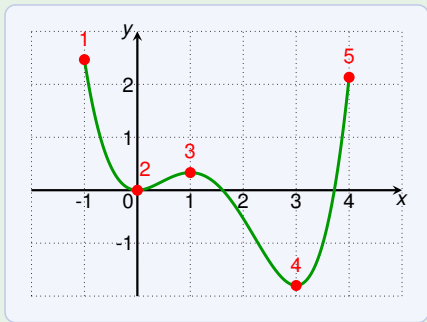
1. global (absolute) maximum;
not a local maximum since f is not defined near -1
2. local minimum
3. local maximum

Maximum and Minimum Values

The graph of

$$f(x) = \frac{3x^4 - 16x^3 + 18x^2}{15}$$

for $-1 \leq x \leq 4$ is shown in this diagram:



Which of the points are a local / global maxima or minima?

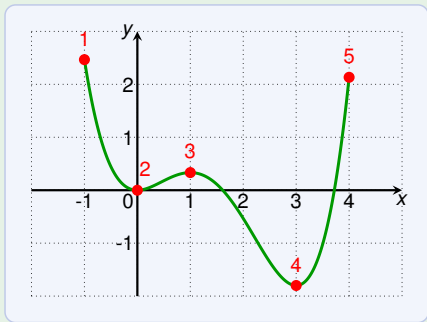
1. global (absolute) maximum;
not a local maximum since f is not defined near -1
2. local minimum
3. local maximum
- 4.

Maximum and Minimum Values

The graph of

$$f(x) = \frac{3x^4 - 16x^3 + 18x^2}{15}$$

for $-1 \leq x \leq 4$ is shown in this diagram:



Which of the points are a local / global maxima or minima?

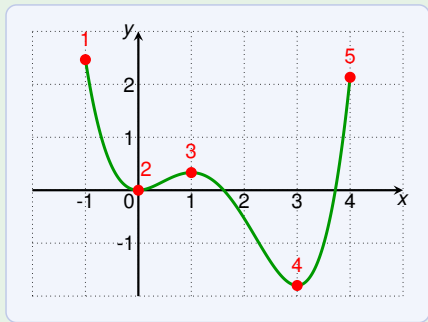
1. global (absolute) maximum;
not a local maximum since f is not defined near -1
2. local minimum
3. local maximum
4. global (absolute) and local minimum

Maximum and Minimum Values

The graph of

$$f(x) = \frac{3x^4 - 16x^3 + 18x^2}{15}$$

for $-1 \leq x \leq 4$ is shown in this diagram:



Which of the points are a local / global maxima or minima?

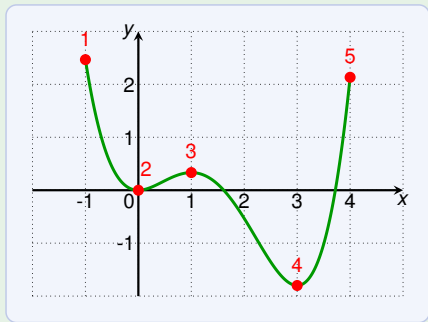
1. global (absolute) maximum;
not a local maximum since f is not defined near -1
2. local minimum
3. local maximum
4. global (absolute) and local minimum
- 5.

Maximum and Minimum Values

The graph of

$$f(x) = \frac{3x^4 - 16x^3 + 18x^2}{15}$$

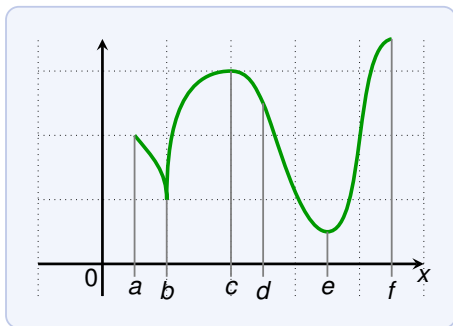
for $-1 \leq x \leq 4$ is shown in this diagram:



Which of the points are a local / global maxima or minima?

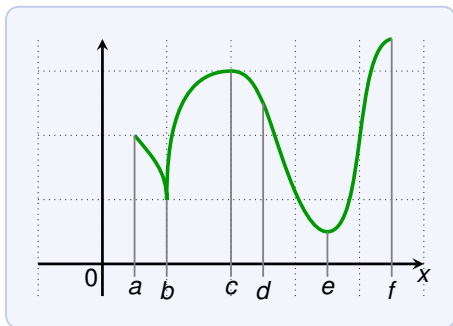
1. global (absolute) maximum;
not a local maximum since f is not defined near -1
2. local minimum
3. local maximum
4. global (absolute) and local minimum
5. nothing

Maximum and Minimum Values



Which of the points are global/local maxima/minima?

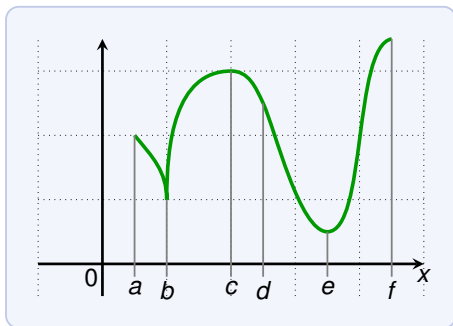
Maximum and Minimum Values



Which of the points are global/local maxima/minima?

a

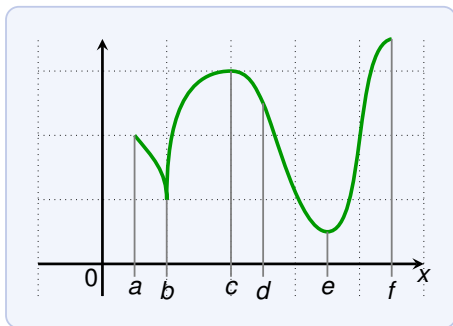
Maximum and Minimum Values



Which of the points are global/local maxima/minima?

a nothing

Maximum and Minimum Values

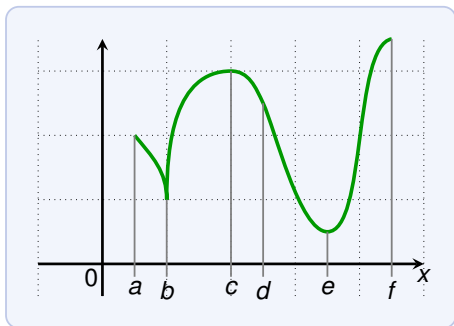


Which of the points are global/local maxima/minima?

a nothing

b

Maximum and Minimum Values

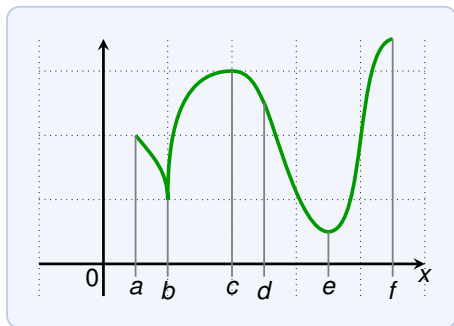


Which of the points are global/local maxima/minima?

a nothing

b local minimum

Maximum and Minimum Values



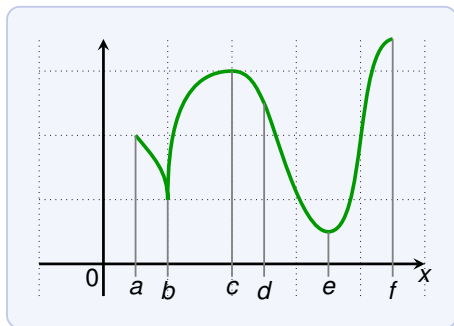
Which of the points are global/local maxima/minima?

a nothing

b local minimum

c

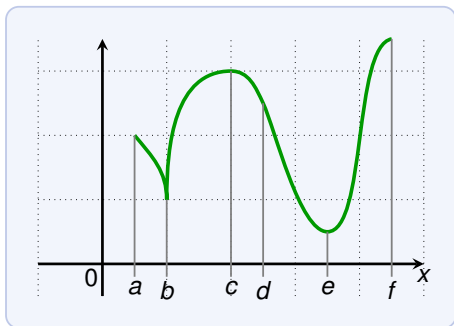
Maximum and Minimum Values



Which of the points are global/local maxima/minima?

- a* nothing
- b* local minimum
- c* local maximum

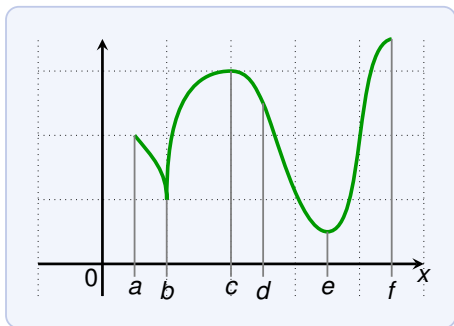
Maximum and Minimum Values



Which of the points are global/local maxima/minima?

- a nothing
- b local minimum
- c local maximum
- d

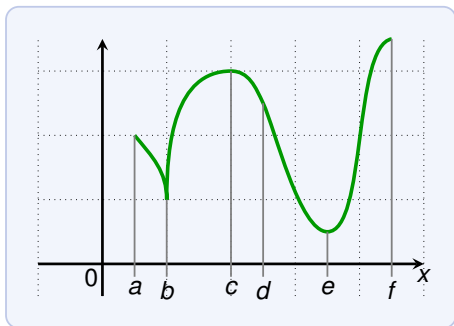
Maximum and Minimum Values



Which of the points are global/local maxima/minima?

- a* nothing
- b* local minimum
- c* local maximum
- d* nothing

Maximum and Minimum Values



Which of the points are global/local maxima/minima?

a nothing

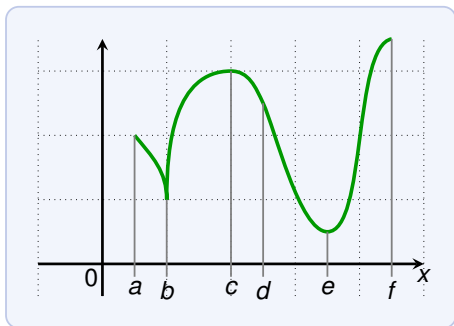
b local minimum

c local maximum

d nothing

e

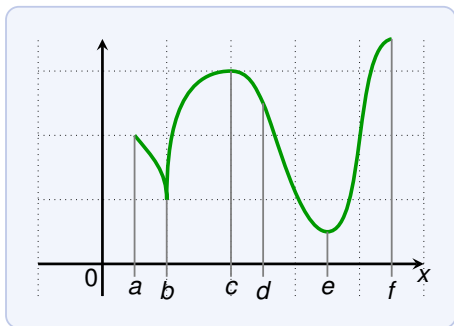
Maximum and Minimum Values



Which of the points are global/local maxima/minima?

- a* nothing
- b* local minimum
- c* local maximum
- d* nothing
- e* local and global (absolute) minimum

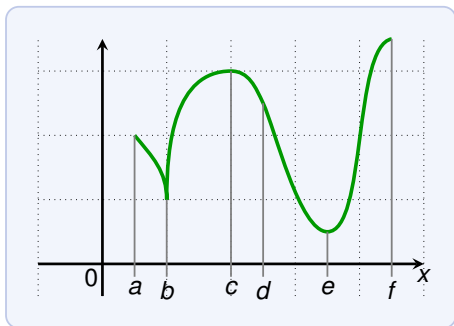
Maximum and Minimum Values



Which of the points are global/local maxima/minima?

- a* nothing
- b* local minimum
- c* local maximum
- d* nothing
- e* local and global (absolute) minimum
- f*

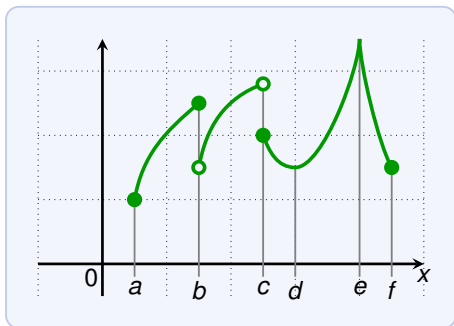
Maximum and Minimum Values



Which of the points are global/local maxima/minima?

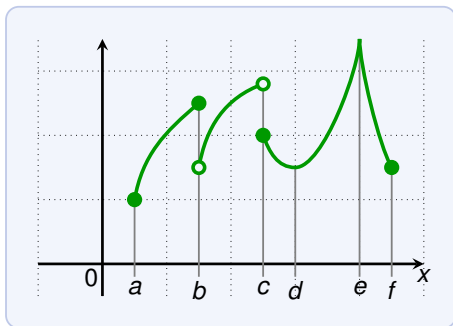
- a nothing
- b local minimum
- c local maximum
- d nothing
- e local and global (absolute) minimum
- f global (absolute) maximum, but not a local maximum

Maximum and Minimum Values



Which of the points are global/local maxima/minima?

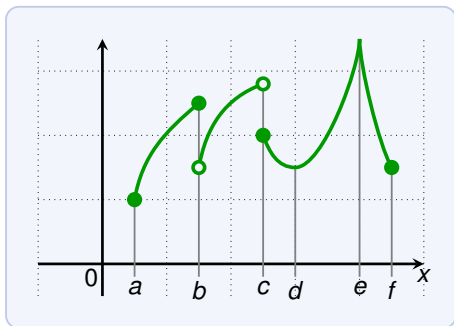
Maximum and Minimum Values



Which of the points are global/local maxima/minima?

a

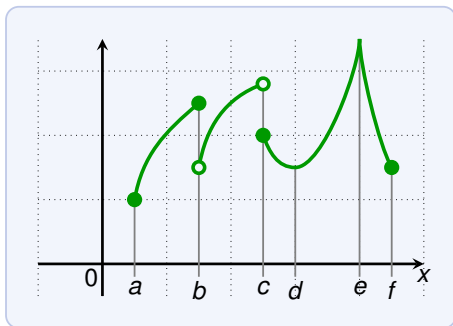
Maximum and Minimum Values



Which of the points are global/local maxima/minima?

a global (absolute) minimum, but not a local minimum

Maximum and Minimum Values

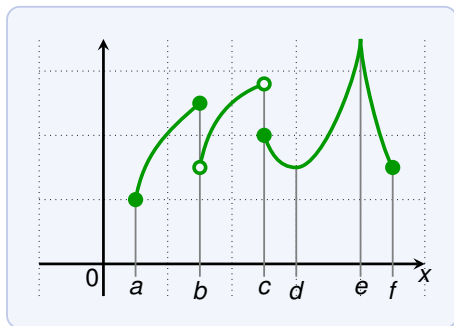


Which of the points are global/local maxima/minima?

a global (absolute) minimum, but not a local minimum

b

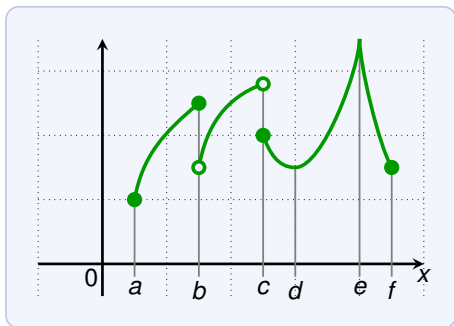
Maximum and Minimum Values



Which of the points are global/local maxima/minima?

- a global (absolute) minimum, but not a local minimum
- b local maximum

Maximum and Minimum Values



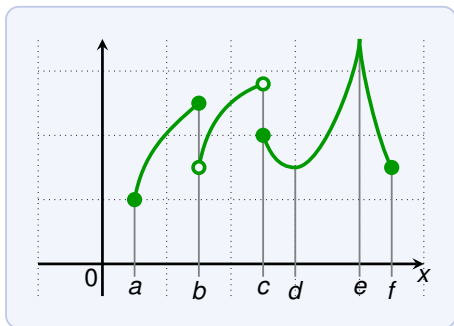
Which of the points are global/local maxima/minima?

a global (absolute) minimum, but not a local minimum

b local maximum

c

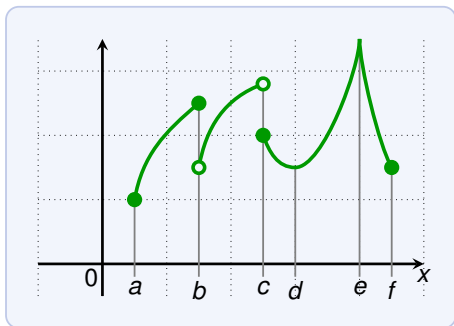
Maximum and Minimum Values



Which of the points are global/local maxima/minima?

- a* global (absolute) minimum, but not a local minimum
- b* local maximum
- c* nothing

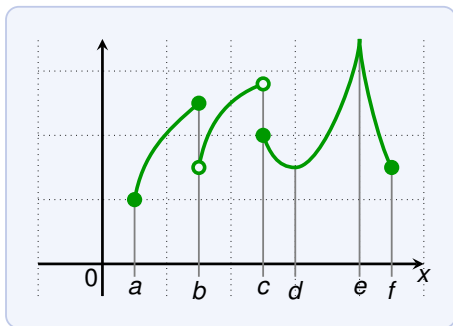
Maximum and Minimum Values



Which of the points are global/local maxima/minima?

- a* global (absolute) minimum, but not a local minimum
- b* local maximum
- c* nothing
- d*

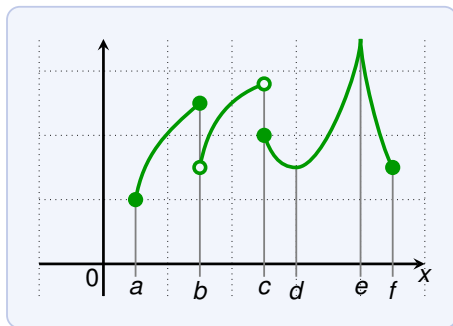
Maximum and Minimum Values



Which of the points are global/local maxima/minima?

- a* global (absolute) minimum, but not a local minimum
- b* local maximum
- c* nothing
- d* local minimum

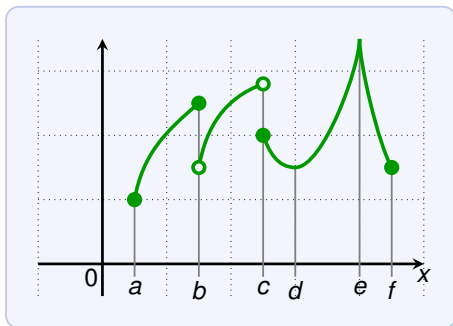
Maximum and Minimum Values



Which of the points are global/local maxima/minima?

- a global (absolute) minimum, but not a local minimum
- b local maximum
- c nothing
- d local minimum
- e

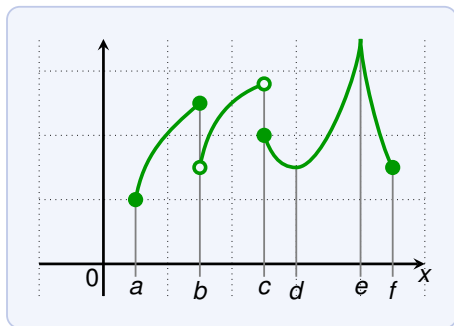
Maximum and Minimum Values



Which of the points are global/local maxima/minima?

- a global (absolute) minimum, but not a local minimum
- b local maximum
- c nothing
- d local minimum
- e local and global (absolute) maximum

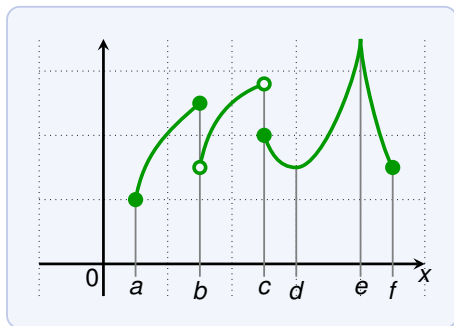
Maximum and Minimum Values



Which of the points are global/local maxima/minima?

- a global (absolute) minimum, but not a local minimum
- b local maximum
- c nothing
- d local minimum
- e local and global (absolute) maximum
- f

Maximum and Minimum Values



Which of the points are global/local maxima/minima?

- a* global (absolute) minimum, but not a local minimum
- b* local maximum
- c* nothing
- d* local minimum
- e* local and global (absolute) maximum
- f* nothing

Maximum and Minimum Values

Let f be a function, and $[a, b]$ a closed interval. Then $f(c)$ is an

Maximum and Minimum Values

Let f be a function, and $[a, b]$ a closed interval. Then $f(c)$ is an

- ▶ **absolute maximum** on $[a, b]$ if $f(c) \geq f(x)$ for all x in $[a, b]$

Maximum and Minimum Values

- Let f be a function, and $[a, b]$ a closed interval. Then $f(c)$ is an
- ▶ **absolute maximum** on $[a, b]$ if $f(c) \geq f(x)$ for all x in $[a, b]$
 - ▶ **absolute minimum** on $[a, b]$ if $f(c) \leq f(x)$ for all x in $[a, b]$

Maximum and Minimum Values

- Let f be a function, and $[a, b]$ a closed interval. Then $f(c)$ is an
- ▶ **absolute maximum** on $[a, b]$ if $f(c) \geq f(x)$ for all x in $[a, b]$
 - ▶ **absolute minimum** on $[a, b]$ if $f(c) \leq f(x)$ for all x in $[a, b]$

Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then

- ▶ f has an absolute maximum $f(c)$ for some c in $[a, b]$,
- ▶ f has an absolute minimum $f(d)$ for some d in $[a, b]$.

Maximum and Minimum Values

- Let f be a function, and $[a, b]$ a closed interval. Then $f(c)$ is an
- ▶ **absolute maximum** on $[a, b]$ if $f(c) \geq f(x)$ for all x in $[a, b]$
 - ▶ **absolute minimum** on $[a, b]$ if $f(c) \leq f(x)$ for all x in $[a, b]$

Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then

- ▶ f has an absolute maximum $f(c)$ for some c in $[a, b]$,
- ▶ f has an absolute minimum $f(d)$ for some d in $[a, b]$.



Maximum and Minimum Values

- Let f be a function, and $[a, b]$ a closed interval. Then $f(c)$ is an
- ▶ **absolute maximum** on $[a, b]$ if $f(c) \geq f(x)$ for all x in $[a, b]$
 - ▶ **absolute minimum** on $[a, b]$ if $f(c) \leq f(x)$ for all x in $[a, b]$

Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then

- ▶ f has an absolute maximum $f(c)$ for some c in $[a, b]$,
- ▶ f has an absolute minimum $f(d)$ for some d in $[a, b]$.



Continuous on $[1, 7]$.

Maximum and Minimum Values

- Let f be a function, and $[a, b]$ a closed interval. Then $f(c)$ is an
- ▶ **absolute maximum** on $[a, b]$ if $f(c) \geq f(x)$ for all x in $[a, b]$
 - ▶ **absolute minimum** on $[a, b]$ if $f(c) \leq f(x)$ for all x in $[a, b]$

Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then

- ▶ f has an absolute maximum $f(c)$ for some c in $[a, b]$,
- ▶ f has an absolute minimum $f(d)$ for some d in $[a, b]$.



Continuous on $[1, 7]$.

Absolute minimum:

Maximum and Minimum Values

- Let f be a function, and $[a, b]$ a closed interval. Then $f(c)$ is an
- ▶ **absolute maximum** on $[a, b]$ if $f(c) \geq f(x)$ for all x in $[a, b]$
 - ▶ **absolute minimum** on $[a, b]$ if $f(c) \leq f(x)$ for all x in $[a, b]$

Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then

- ▶ f has an absolute maximum $f(c)$ for some c in $[a, b]$,
- ▶ f has an absolute minimum $f(d)$ for some d in $[a, b]$.



Continuous on $[1, 7]$.

Absolute minimum:
 $f(4) = 1$

Maximum and Minimum Values

- Let f be a function, and $[a, b]$ a closed interval. Then $f(c)$ is an
- ▶ **absolute maximum** on $[a, b]$ if $f(c) \geq f(x)$ for all x in $[a, b]$
 - ▶ **absolute minimum** on $[a, b]$ if $f(c) \leq f(x)$ for all x in $[a, b]$

Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then

- ▶ f has an absolute maximum $f(c)$ for some c in $[a, b]$,
- ▶ f has an absolute minimum $f(d)$ for some d in $[a, b]$.



Continuous on $[1, 7]$.

Absolute minimum:
 $f(4) = 1$

Absolute maximum:

Maximum and Minimum Values

- Let f be a function, and $[a, b]$ a closed interval. Then $f(c)$ is an
- ▶ **absolute maximum** on $[a, b]$ if $f(c) \geq f(x)$ for all x in $[a, b]$
 - ▶ **absolute minimum** on $[a, b]$ if $f(c) \leq f(x)$ for all x in $[a, b]$

Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then

- ▶ f has an absolute maximum $f(c)$ for some c in $[a, b]$,
- ▶ f has an absolute minimum $f(d)$ for some d in $[a, b]$.



Continuous on $[1, 7]$.

Absolute minimum:
 $f(4) = 1$

Absolute maximum:
 $f(2) = 3$, and
 $f(6) = 3$

Maximum and Minimum Values

Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then

- ▶ f has an absolute maximum $f(c)$ for some c in $[a, b]$,
- ▶ f has an absolute minimum $f(d)$ for some d in $[a, b]$.

Maximum and Minimum Values

Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then

- ▶ f has an absolute maximum $f(c)$ for some c in $[a, b]$,
- ▶ f has an absolute minimum $f(d)$ for some d in $[a, b]$.

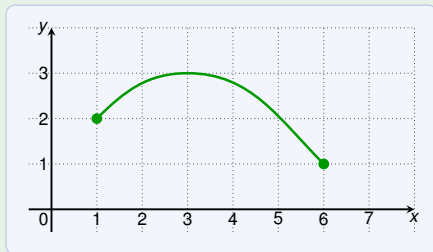


Maximum and Minimum Values

Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then

- ▶ f has an absolute maximum $f(c)$ for some c in $[a, b]$,
- ▶ f has an absolute minimum $f(d)$ for some d in $[a, b]$.



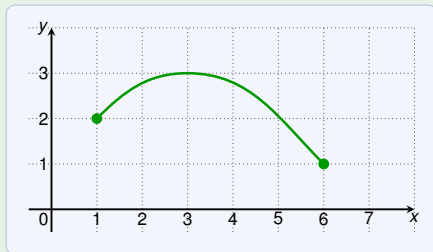
Continuous on $[1, 6]$.

Maximum and Minimum Values

Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then

- ▶ f has an absolute maximum $f(c)$ for some c in $[a, b]$,
- ▶ f has an absolute minimum $f(d)$ for some d in $[a, b]$.



Continuous on $[1, 6]$.

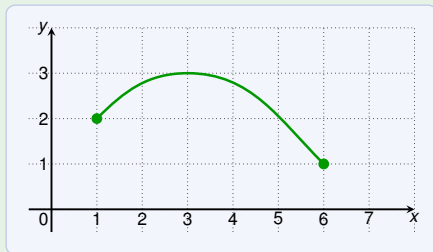
Absolute minimum:

Maximum and Minimum Values

Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then

- ▶ f has an absolute maximum $f(c)$ for some c in $[a, b]$,
- ▶ f has an absolute minimum $f(d)$ for some d in $[a, b]$.



Continuous on $[1, 6]$.

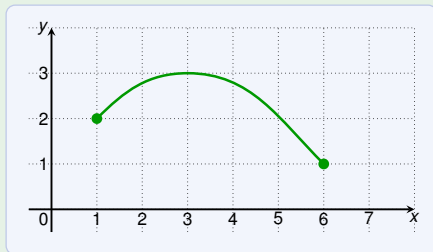
Absolute minimum:
 $f(6) = 1$

Maximum and Minimum Values

Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then

- ▶ f has an absolute maximum $f(c)$ for some c in $[a, b]$,
- ▶ f has an absolute minimum $f(d)$ for some d in $[a, b]$.



Continuous on $[1, 6]$.

Absolute minimum:
 $f(6) = 1$

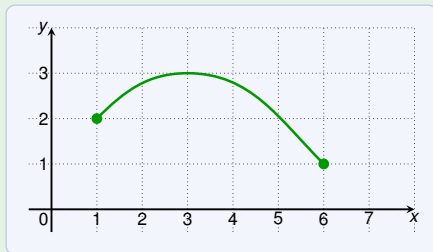
Absolute maximum:

Maximum and Minimum Values

Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then

- ▶ f has an absolute maximum $f(c)$ for some c in $[a, b]$,
- ▶ f has an absolute minimum $f(d)$ for some d in $[a, b]$.

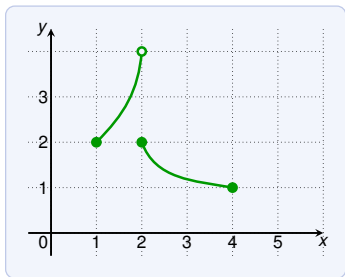


Continuous on $[1, 6]$.

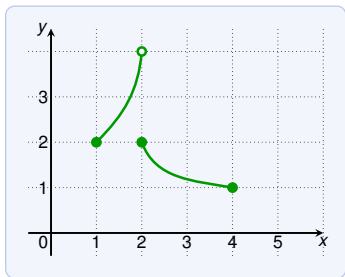
Absolute minimum:
 $f(6) = 1$

Absolute maximum:
 $f(3) = 3$

Maximum and Minimum Values

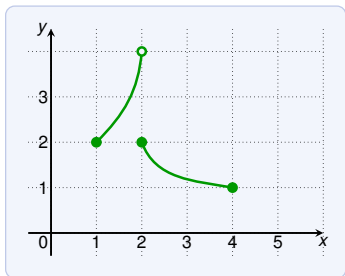


Maximum and Minimum Values



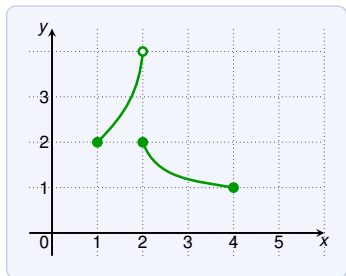
Absolute minimum:

Maximum and Minimum Values



Absolute minimum:
 $f(4) = 1$

Maximum and Minimum Values

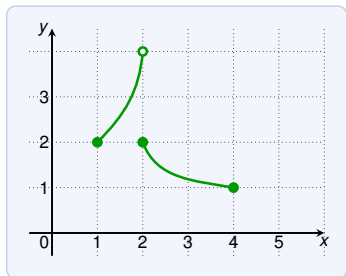


Absolute minimum:

$$f(4) = 1$$

Absolute maximum:

Maximum and Minimum Values



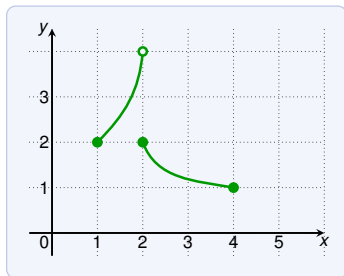
Absolute minimum:

$$f(4) = 1$$

Absolute maximum:

none

Maximum and Minimum Values



Absolute minimum:

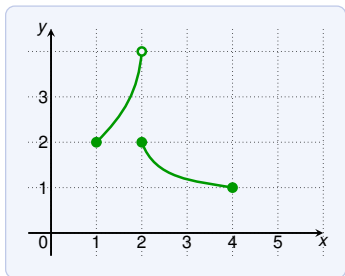
$$f(4) = 1$$

Absolute maximum:

none

Not continuous on $[1, 4]$!

Maximum and Minimum Values



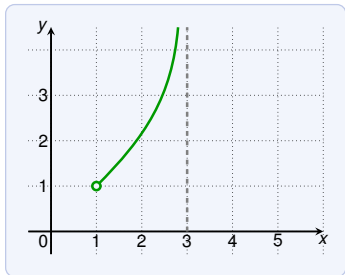
Absolute minimum:

$$f(4) = 1$$

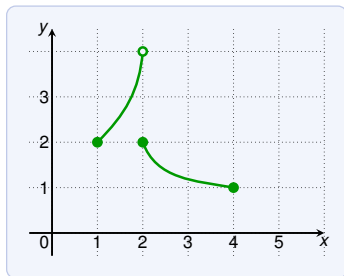
Absolute maximum:

none

Not continuous on $[1, 4]$!



Maximum and Minimum Values



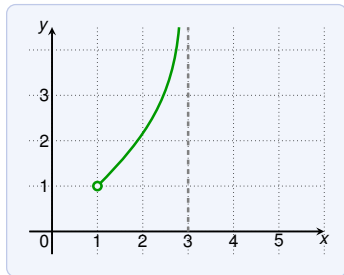
Absolute minimum:

$$f(4) = 1$$

Absolute maximum:

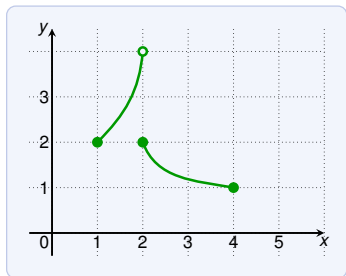
none

Not continuous on $[1, 4]$!



Absolute minimum:

Maximum and Minimum Values



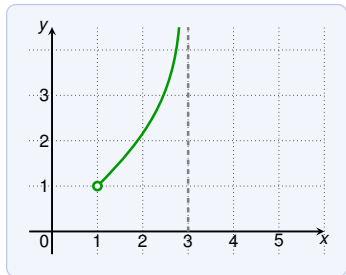
Absolute minimum:

$$f(4) = 1$$

Absolute maximum:

none

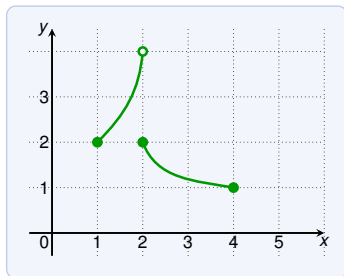
Not continuous on $[1, 4]$!



Absolute minimum:

none

Maximum and Minimum Values



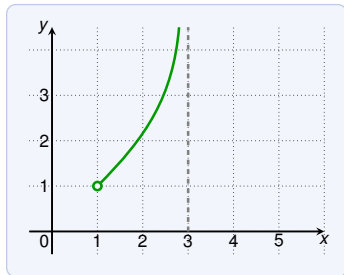
Absolute minimum:

$$f(4) = 1$$

Absolute maximum:

none

Not continuous on $[1, 4]$!

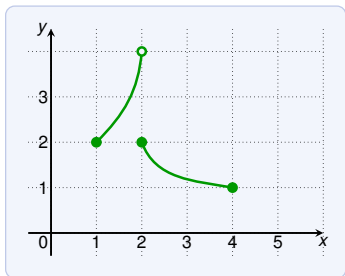


Absolute minimum:

none

Absolute maximum:

Maximum and Minimum Values



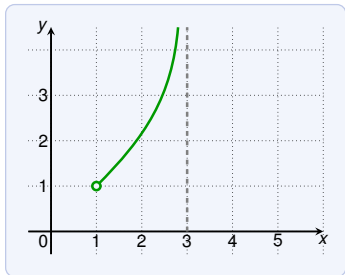
Absolute minimum:

$$f(4) = 1$$

Absolute maximum:

none

Not continuous on $[1, 4]$!



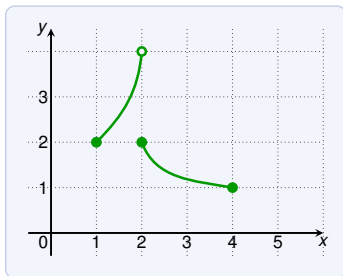
Absolute minimum:

none

Absolute maximum:

none

Maximum and Minimum Values



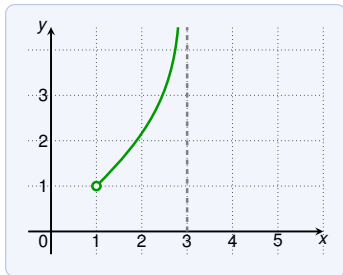
Absolute minimum:

$$f(4) = 1$$

Absolute maximum:

none

Not continuous on $[1, 4]$!



Absolute minimum:

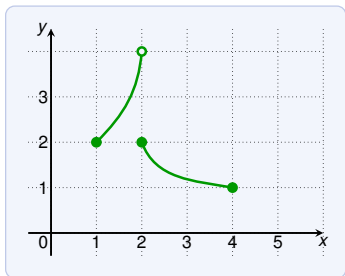
none

Absolute maximum:

none

Continuous on $(1, 3)$, but this is not a closed interval!

Maximum and Minimum Values



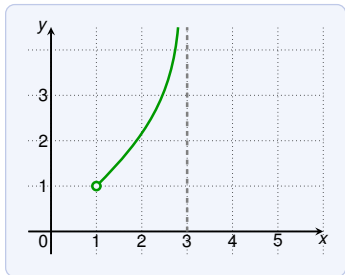
Absolute minimum:

$$f(4) = 1$$

Absolute maximum:

none

Not continuous on $[1, 4]$!



Absolute minimum:

none

Absolute maximum:

none

Continuous on $(1, 3)$, but this is not a closed interval!

The function needs to be **continuous** on a **closed** interval $[a, b]$.

Maximum and Minimum Values

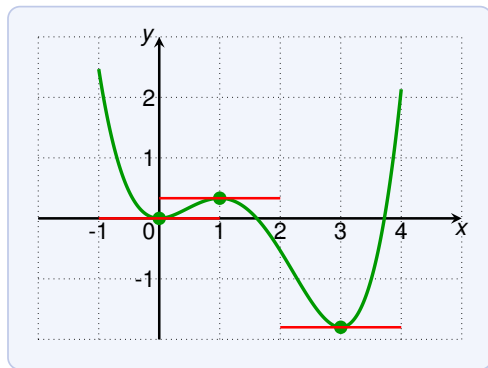
Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

Maximum and Minimum Values

Fermat's Theorem

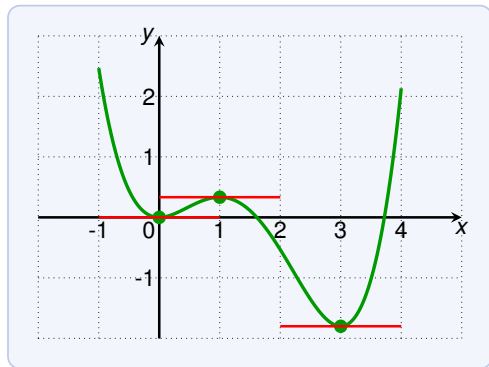
If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.



Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.



At every **local** maximum or minimum, the tangent is horizontal.
(if the derivative exists)

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

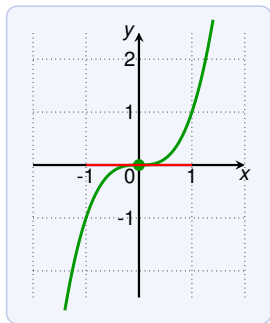
The reverse statement is not true! Having $f'(c) = 0$ does not guarantee that $f(c)$ is a minimum or maximum.

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

The reverse statement is not true! Having $f'(c) = 0$ does not guarantee that $f(c)$ is a minimum or maximum.



For example:

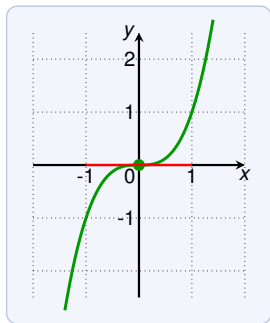
$$f(x) = x^3$$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

The reverse statement is not true! Having $f'(c) = 0$ does not guarantee that $f(c)$ is a minimum or maximum.



For example:

$$f(x) = x^3$$

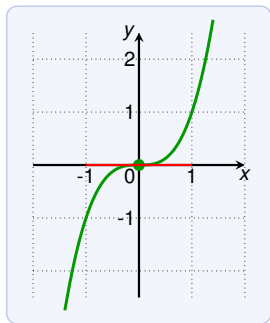
Then $f'(0) = 0$.

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

The reverse statement is not true! Having $f'(c) = 0$ does not guarantee that $f(c)$ is a minimum or maximum.



For example:

$$f(x) = x^3$$

Then $f'(0) = 0$.

But there is no minimum or maximum.

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists,
then $f'(c) = 0$.

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

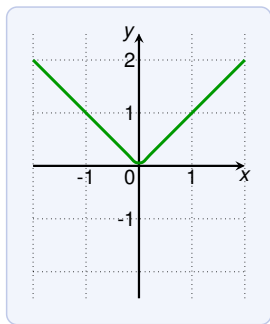
A local minimum/maximum does not guarantee that $f'(c)$ exists.

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

A local minimum/maximum does not guarantee that $f'(c)$ exists.



For example:

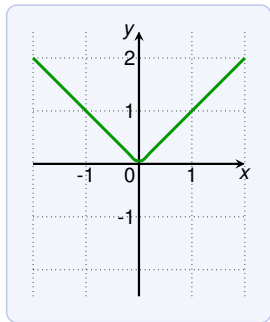
$$f(x) = |x|$$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

A local minimum/maximum does not guarantee that $f'(c)$ exists.



For example:

$$f(x) = |x|$$

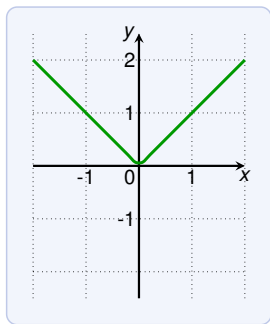
Then $f(0) = 0$ is a local minimum.

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

A local minimum/maximum does not guarantee that $f'(c)$ exists.



For example:

$$f(x) = |x|$$

Then $f(0) = 0$ is a local minimum.

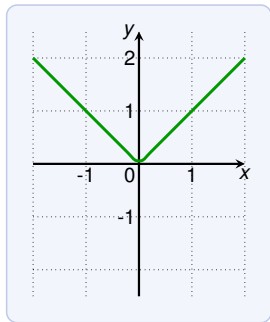
But $f'(0)$ does not exist.

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

A local minimum/maximum does not guarantee that $f'(c)$ exists.



For example:

$$f(x) = |x|$$

Then $f(0) = 0$ is a local minimum.

But $f'(0)$ does not exist.

Care needed for applying the theorem (check both conditions)!

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists,
then $f'(c) = 0$.

The theorem suggests where local extra can occur:

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

The theorem suggests where local extra can occur:

- ▶ where $f'(c) = 0$, or

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

The theorem suggests where local extrema can occur:

- ▶ where $f'(c) = 0$, or
- ▶ where $f'(c)$ does not exist.

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

The theorem suggests where local extrema can occur:

- ▶ where $f'(c) = 0$, or
- ▶ where $f'(c)$ does not exist.

A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$, or $f'(c)$ does not exist.

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

The theorem suggests where local extra can occur:

- ▶ where $f'(c) = 0$, or
- ▶ where $f'(c)$ does not exist.

A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$, or $f'(c)$ does not exist.

What are the critical numbers of $f(x) = x^{3/5}(5 - x)$?

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

The theorem suggests where local extra can occur:

- ▶ where $f'(c) = 0$, or
- ▶ where $f'(c)$ does not exist.

A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$, or $f'(c)$ does not exist.

What are the critical numbers of $f(x) = x^{3/5}(5 - x)$?

$$f(x) = x^{3/5}(5 - x)$$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

The theorem suggests where local extra can occur:

- ▶ where $f'(c) = 0$, or
- ▶ where $f'(c)$ does not exist.

A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$, or $f'(c)$ does not exist.

What are the critical numbers of $f(x) = x^{3/5}(5 - x)$?

$$f(x) = x^{3/5}(5 - x) = 5x^{3/5} - x^{8/5}$$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

The theorem suggests where local extra can occur:

- ▶ where $f'(c) = 0$, or
- ▶ where $f'(c)$ does not exist.

A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$, or $f'(c)$ does not exist.

What are the critical numbers of $f(x) = x^{3/5}(5 - x)$?

$$f(x) = x^{3/5}(5 - x) = 5x^{3/5} - x^{8/5}$$

$$f'(x) =$$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

The theorem suggests where local extra can occur:

- ▶ where $f'(c) = 0$, or
- ▶ where $f'(c)$ does not exist.

A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$, or $f'(c)$ does not exist.

What are the critical numbers of $f(x) = x^{3/5}(5 - x)$?

$$f(x) = x^{3/5}(5 - x) = 5x^{3/5} - x^{8/5}$$

$$f'(x) = \frac{3}{x^{2/5}} - \frac{8}{5}x^{3/5}$$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

The theorem suggests where local extra can occur:

- ▶ where $f'(c) = 0$, or
- ▶ where $f'(c)$ does not exist.

A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$, or $f'(c)$ does not exist.

What are the critical numbers of $f(x) = x^{3/5}(5 - x)$?

$$f(x) = x^{3/5}(5 - x) = 5x^{3/5} - x^{8/5}$$

$$f'(x) = \frac{3}{x^{2/5}} - \frac{8}{5}x^{3/5} = \frac{15}{5x^{2/5}} - \frac{8x}{5x^{2/5}}$$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

The theorem suggests where local extra can occur:

- ▶ where $f'(c) = 0$, or
- ▶ where $f'(c)$ does not exist.

A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$, or $f'(c)$ does not exist.

What are the critical numbers of $f(x) = x^{3/5}(5 - x)$?

$$f(x) = x^{3/5}(5 - x) = 5x^{3/5} - x^{8/5}$$

$$f'(x) = \frac{3}{x^{2/5}} - \frac{8}{5}x^{3/5} = \frac{15}{5x^{2/5}} - \frac{8x}{5x^{2/5}} = \frac{15 - 8x}{5x^{2/5}}$$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

The theorem suggests where local extra can occur:

- ▶ where $f'(c) = 0$, or
- ▶ where $f'(c)$ does not exist.

A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$, or $f'(c)$ does not exist.

What are the critical numbers of $f(x) = x^{3/5}(5 - x)$?

$$f(x) = x^{3/5}(5 - x) = 5x^{3/5} - x^{8/5}$$

$$f'(x) = \frac{3}{x^{2/5}} - \frac{8}{5}x^{3/5} = \frac{15}{5x^{2/5}} - \frac{8x}{5x^{2/5}} = \frac{15 - 8x}{5x^{2/5}}$$

The critical numbers are $\frac{15}{8}$ ($f'(c) = 0$) and 0 ($f'(c)$ does not exist)

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

What are the critical numbers of the function

$$f(x) = \sqrt{x} + |x - 2| \quad ?$$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

What are the critical numbers of the function

$$f(x) = \sqrt{x} + |x - 2| \quad ?$$

Due to $|x - 2|$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

What are the critical numbers of the function

$$f(x) = \sqrt{x} + |x - 2| \quad ?$$

Due to $|x - 2|$, the derivative is not defined at $x = 2$.

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

What are the critical numbers of the function

$$f(x) = \sqrt{x} + |x - 2| \quad ?$$

Due to $|x - 2|$, the derivative is not defined at $x = 2$.

For $x < 2$ we have $|x - 2| =$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

What are the critical numbers of the function

$$f(x) = \sqrt{x} + |x - 2| \quad ?$$

Due to $|x - 2|$, the derivative is not defined at $x = 2$.

For $x < 2$ we have $|x - 2| = -(x - 2)$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

What are the critical numbers of the function

$$f(x) = \sqrt{x} + |x - 2| \quad ?$$

Due to $|x - 2|$, the derivative is not defined at $x = 2$.

For $x < 2$ we have $|x - 2| = -(x - 2)$, thus:

$$f(x) = \sqrt{x} - (x - 2)$$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

What are the critical numbers of the function

$$f(x) = \sqrt{x} + |x - 2| \quad ?$$

Due to $|x - 2|$, the derivative is not defined at $x = 2$.

For $x < 2$ we have $|x - 2| = -(x - 2)$, thus:

$$f(x) = \sqrt{x} - (x - 2) \quad f'(x) =$$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

What are the critical numbers of the function

$$f(x) = \sqrt{x} + |x - 2| \quad ?$$

Due to $|x - 2|$, the derivative is not defined at $x = 2$.

For $x < 2$ we have $|x - 2| = -(x - 2)$, thus:

$$f(x) = \sqrt{x} - (x - 2) \qquad f'(x) = \frac{1}{2\sqrt{x}} - 1$$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

What are the critical numbers of the function

$$f(x) = \sqrt{x} + |x - 2| \quad ?$$

Due to $|x - 2|$, the derivative is not defined at $x = 2$.

For $x < 2$ we have $|x - 2| = -(x - 2)$, thus:

$$f(x) = \sqrt{x} - (x - 2) \qquad f'(x) = \frac{1}{2\sqrt{x}} - 1$$

Thus $f'(x) = 0 \iff$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

What are the critical numbers of the function

$$f(x) = \sqrt{x} + |x - 2| \quad ?$$

Due to $|x - 2|$, the derivative is not defined at $x = 2$.

For $x < 2$ we have $|x - 2| = -(x - 2)$, thus:

$$f(x) = \sqrt{x} - (x - 2) \qquad f'(x) = \frac{1}{2\sqrt{x}} - 1$$

Thus $f'(x) = 0 \iff x = 1/4$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

What are the critical numbers of the function

$$f(x) = \sqrt{x} + |x - 2| \quad ?$$

Due to $|x - 2|$, the derivative is not defined at $x = 2$.

For $x < 2$ we have $|x - 2| = -(x - 2)$, thus:

$$f(x) = \sqrt{x} - (x - 2) \qquad f'(x) = \frac{1}{2\sqrt{x}} - 1$$

Thus $f'(x) = 0 \iff x = 1/4$, and $f'(x)$ undefined for $x = 0$.

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

What are the critical numbers of the function

$$f(x) = \sqrt{x} + |x - 2| \quad ?$$

Due to $|x - 2|$, the derivative is not defined at $x = 2$.

For $x < 2$ we have $|x - 2| = -(x - 2)$, thus:

$$f(x) = \sqrt{x} - (x - 2) \qquad f'(x) = \frac{1}{2\sqrt{x}} - 1$$

Thus $f'(x) = 0 \iff x = 1/4$, and $f'(x)$ undefined for $x = 0$.

For $x > 2$ we have $|x - 2| = x - 2$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

What are the critical numbers of the function

$$f(x) = \sqrt{x} + |x - 2| \quad ?$$

Due to $|x - 2|$, the derivative is not defined at $x = 2$.

For $x < 2$ we have $|x - 2| = -(x - 2)$, thus:

$$f(x) = \sqrt{x} - (x - 2) \qquad f'(x) = \frac{1}{2\sqrt{x}} - 1$$

Thus $f'(x) = 0 \iff x = 1/4$, and $f'(x)$ undefined for $x = 0$.

For $x > 2$ we have $|x - 2| = x - 2$, thus:

$$f(x) = \sqrt{x} + (x - 2)$$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

What are the critical numbers of the function

$$f(x) = \sqrt{x} + |x - 2| \quad ?$$

Due to $|x - 2|$, the derivative is not defined at $x = 2$.

For $x < 2$ we have $|x - 2| = -(x - 2)$, thus:

$$f(x) = \sqrt{x} - (x - 2) \qquad f'(x) = \frac{1}{2\sqrt{x}} - 1$$

Thus $f'(x) = 0 \iff x = 1/4$, and $f'(x)$ undefined for $x = 0$.

For $x > 2$ we have $|x - 2| = x - 2$, thus:

$$f(x) = \sqrt{x} + (x - 2) \qquad f'(x) = \frac{1}{2\sqrt{x}} + 1$$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

What are the critical numbers of the function

$$f(x) = \sqrt{x} + |x - 2| \quad ?$$

Due to $|x - 2|$, the derivative is not defined at $x = 2$.

For $x < 2$ we have $|x - 2| = -(x - 2)$, thus:

$$f(x) = \sqrt{x} - (x - 2) \qquad f'(x) = \frac{1}{2\sqrt{x}} - 1$$

Thus $f'(x) = 0 \iff x = 1/4$, and $f'(x)$ undefined for $x = 0$.

For $x > 2$ we have $|x - 2| = x - 2$, thus:

$$f(x) = \sqrt{x} + (x - 2) \qquad f'(x) = \frac{1}{2\sqrt{x}} + 1 \geq 1$$

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

What are the critical numbers of the function

$$f(x) = \sqrt{x} + |x - 2| \quad ?$$

Due to $|x - 2|$, the derivative is not defined at $x = 2$.

For $x < 2$ we have $|x - 2| = -(x - 2)$, thus:

$$f(x) = \sqrt{x} - (x - 2) \qquad f'(x) = \frac{1}{2\sqrt{x}} - 1$$

Thus $f'(x) = 0 \iff x = 1/4$, and $f'(x)$ undefined for $x = 0$.

For $x > 2$ we have $|x - 2| = x - 2$, thus:

$$f(x) = \sqrt{x} + (x - 2) \qquad f'(x) = \frac{1}{2\sqrt{x}} + 1 \geq 1$$

Thus the critical numbers are 0 , $1/4$ and 2 .

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists,
then $f'(c) = 0$.

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

We can now rephrase the the theorem as follows:

If f has a local extremum at c , then c is a critical number of f .

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

We can now rephrase the the theorem as follows:

If f has a local extremum at c , then c is a critical number of f .

We can use this to look for global extrema on intervals:

Closed Interval Method

To find the **absolute** maximum and minimum values of a **continuous** function f on an **closed** interval $[a, b]$:

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

We can now rephrase the the theorem as follows:

If f has a local extremum at c , then c is a critical number of f .

We can use this to look for global extrema on intervals:

Closed Interval Method

To find the **absolute** maximum and minimum values of a **continuous** function f on an **closed** interval $[a, b]$:

1. Find the values of f at critical numbers of f in (a, b) .

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

We can now rephrase the the theorem as follows:

If f has a local extremum at c , then c is a critical number of f .

We can use this to look for global extrema on intervals:

Closed Interval Method

To find the **absolute** maximum and minimum values of a **continuous** function f on an **closed** interval $[a, b]$:

1. Find the values of f at critical numbers of f in (a, b) .
2. Find the values of f at the endpoints of the interval.

Maximum and Minimum Values

Fermat's Theorem

If f has a local maximum or minimum at c and $f'(c)$ exists, then $f'(c) = 0$.

We can now rephrase the the theorem as follows:

If f has a local extremum at c , then c is a critical number of f .

We can use this to look for global extrema on intervals:

Closed Interval Method

To find the **absolute** maximum and minimum values of a **continuous** function f on an **closed** interval $[a, b]$:

1. Find the values of f at critical numbers of f in (a, b) .
2. Find the values of f at the endpoints of the interval.
3. The largest value of (1) and (2) is the absolute maximum, the lowest the absolute minimum.

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Since f is cont. on $[-\frac{1}{2}, 4]$ we can use Closed Interval Method.

$$f'(x) =$$

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Since f is cont. on $[-\frac{1}{2}, 4]$ we can use Closed Interval Method.

$$f'(x) = 3x^2 - 6x$$

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Since f is cont. on $[-\frac{1}{2}, 4]$ we can use Closed Interval Method.

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Since f is cont. on $[-\frac{1}{2}, 4]$ we can use Closed Interval Method.

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

We have $f'(x) = 0$ if

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Since f is cont. on $[-\frac{1}{2}, 4]$ we can use Closed Interval Method.

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

We have $f'(x) = 0$ if $x = 0$ or $x = 2$.

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Since f is cont. on $[-\frac{1}{2}, 4]$ we can use Closed Interval Method.

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

We have $f'(x) = 0$ if $x = 0$ or $x = 2$. Both in $[-\frac{1}{2}, 4]$!

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Since f is cont. on $[-\frac{1}{2}, 4]$ we can use Closed Interval Method.

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

We have $f'(x) = 0$ if $x = 0$ or $x = 2$. Both in $[-\frac{1}{2}, 4]$!

No other critical values since $f'(x)$ exists for all x .

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Since f is cont. on $[-\frac{1}{2}, 4]$ we can use Closed Interval Method.

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

We have $f'(x) = 0$ if $x = 0$ or $x = 2$. Both in $[-\frac{1}{2}, 4]$!

No other critical values since $f'(x)$ exists for all x .

The values of f at the critical numbers are:

$$f(0) =$$

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Since f is cont. on $[-\frac{1}{2}, 4]$ we can use Closed Interval Method.

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

We have $f'(x) = 0$ if $x = 0$ or $x = 2$. Both in $[-\frac{1}{2}, 4]$!

No other critical values since $f'(x)$ exists for all x .

The values of f at the critical numbers are:

$$f(0) = 1$$

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Since f is cont. on $[-\frac{1}{2}, 4]$ we can use Closed Interval Method.

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

We have $f'(x) = 0$ if $x = 0$ or $x = 2$. Both in $[-\frac{1}{2}, 4]$!

No other critical values since $f'(x)$ exists for all x .

The values of f at the critical numbers are:

$$f(0) = 1$$

$$f(2) = -3$$

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Since f is cont. on $[-\frac{1}{2}, 4]$ we can use Closed Interval Method.

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

We have $f'(x) = 0$ if $x = 0$ or $x = 2$. Both in $[-\frac{1}{2}, 4]$!

No other critical values since $f'(x)$ exists for all x .

The values of f at the critical numbers are:

$$f(0) = 1$$

$$f(2) = -3$$

The values of f at the end points of the interval are:

$$f\left(-\frac{1}{2}\right) =$$

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Since f is cont. on $[-\frac{1}{2}, 4]$ we can use Closed Interval Method.

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

We have $f'(x) = 0$ if $x = 0$ or $x = 2$. Both in $[-\frac{1}{2}, 4]$!
No other critical values since $f'(x)$ exists for all x .

The values of f at the critical numbers are:

$$f(0) = 1$$

$$f(2) = -3$$

The values of f at the end points of the interval are:

$$f\left(-\frac{1}{2}\right) = -\frac{1}{8} - 3\frac{1}{4} + 1$$

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Since f is cont. on $[-\frac{1}{2}, 4]$ we can use Closed Interval Method.

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

We have $f'(x) = 0$ if $x = 0$ or $x = 2$. Both in $[-\frac{1}{2}, 4]$!
No other critical values since $f'(x)$ exists for all x .

The values of f at the critical numbers are:

$$f(0) = 1 \qquad f(2) = -3$$

The values of f at the end points of the interval are:

$$f\left(-\frac{1}{2}\right) = -\frac{1}{8} - 3\frac{1}{4} + 1 = \frac{1}{8}$$

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Since f is cont. on $[-\frac{1}{2}, 4]$ we can use Closed Interval Method.

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

We have $f'(x) = 0$ if $x = 0$ or $x = 2$. Both in $[-\frac{1}{2}, 4]$!

No other critical values since $f'(x)$ exists for all x .

The values of f at the critical numbers are:

$$f(0) = 1 \qquad f(2) = -3$$

The values of f at the end points of the interval are:

$$f\left(-\frac{1}{2}\right) = -\frac{1}{8} - 3\frac{1}{4} + 1 = \frac{1}{8} \qquad f(4) =$$

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Since f is cont. on $[-\frac{1}{2}, 4]$ we can use Closed Interval Method.

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

We have $f'(x) = 0$ if $x = 0$ or $x = 2$. Both in $[-\frac{1}{2}, 4]$!

No other critical values since $f'(x)$ exists for all x .

The values of f at the critical numbers are:

$$f(0) = 1 \qquad f(2) = -3$$

The values of f at the end points of the interval are:

$$f\left(-\frac{1}{2}\right) = -\frac{1}{8} - 3\frac{1}{4} + 1 = \frac{1}{8} \qquad f(4) = 4 \cdot 16 - 3 \cdot 16 + 1$$

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Since f is cont. on $[-\frac{1}{2}, 4]$ we can use Closed Interval Method.

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

We have $f'(x) = 0$ if $x = 0$ or $x = 2$. Both in $[-\frac{1}{2}, 4]$!

No other critical values since $f'(x)$ exists for all x .

The values of f at the critical numbers are:

$$f(0) = 1 \qquad f(2) = -3$$

The values of f at the end points of the interval are:

$$f\left(-\frac{1}{2}\right) = -\frac{1}{8} - 3\frac{1}{4} + 1 = \frac{1}{8} \qquad f(4) = 4 \cdot 16 - 3 \cdot 16 + 1 = 17$$

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Since f is cont. on $[-\frac{1}{2}, 4]$ we can use Closed Interval Method.

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

We have $f'(x) = 0$ if $x = 0$ or $x = 2$. Both in $[-\frac{1}{2}, 4]$!
No other critical values since $f'(x)$ exists for all x .

The values of f at the critical numbers are:

$$f(0) = 1 \qquad f(2) = -3$$

The values of f at the end points of the interval are:

$$f\left(-\frac{1}{2}\right) = -\frac{1}{8} - 3\frac{1}{4} + 1 = \frac{1}{8} \qquad f(4) = 4 \cdot 16 - 3 \cdot 16 + 1 = 17$$

Absolute minimum is $f(2) = -3$,

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$

Since f is cont. on $[-\frac{1}{2}, 4]$ we can use Closed Interval Method.

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

We have $f'(x) = 0$ if $x = 0$ or $x = 2$. Both in $[-\frac{1}{2}, 4]$!

No other critical values since $f'(x)$ exists for all x .

The values of f at the critical numbers are:

$$f(0) = 1$$

$$f(2) = -3$$

The values of f at the end points of the interval are:

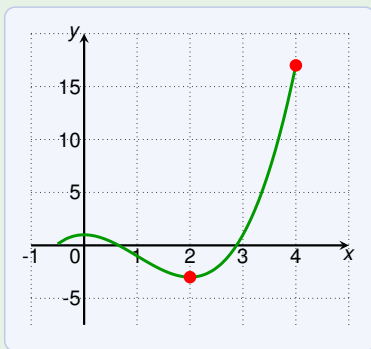
$$f\left(-\frac{1}{2}\right) = -\frac{1}{8} - 3\frac{1}{4} + 1 = \frac{1}{8} \quad f(4) = 4 \cdot 16 - 3 \cdot 16 + 1 = 17$$

Absolute minimum is $f(2) = -3$, absolute maximum $f(4) = 17$.

Maximum and Minimum Values

Find the absolute maximum and minimum values of

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$



Absolute minimum is $f(2) = -3$, absolute maximum $f(4) = 17$.

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t - 1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t - 1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) =$$

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t - 1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3(t - 1)^2 - 8t + 9$$

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t - 1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3(t - 1)^2 - 8t + 9 = 3t^2 - 14t + 12$$

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t - 1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3(t - 1)^2 - 8t + 9 = 3t^2 - 14t + 12$$

Since a is cont. on $[0, 5]$ we can use Closed Interval Method.

$$a'(t) =$$

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t - 1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3(t - 1)^2 - 8t + 9 = 3t^2 - 14t + 12$$

Since a is cont. on $[0, 5]$ we can use Closed Interval Method.

$$a'(t) = 6t - 14$$

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t - 1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3(t - 1)^2 - 8t + 9 = 3t^2 - 14t + 12$$

Since a is cont. on $[0, 5]$ we can use Closed Interval Method.

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff$$

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t - 1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3(t - 1)^2 - 8t + 9 = 3t^2 - 14t + 12$$

Since a is cont. on $[0, 5]$ we can use Closed Interval Method.

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t - 1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3(t - 1)^2 - 8t + 9 = 3t^2 - 14t + 12$$

Since a is cont. on $[0, 5]$ we can use Closed Interval Method.

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

The only critical number is $\frac{7}{3}$.

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t - 1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3(t - 1)^2 - 8t + 9 = 3t^2 - 14t + 12$$

Since a is cont. on $[0, 5]$ we can use Closed Interval Method.

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

The only critical number is $\frac{7}{3}$. Note that $\frac{7}{3}$ is in $[0, 5]$.

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t - 1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3(t - 1)^2 - 8t + 9 = 3t^2 - 14t + 12$$

Since a is cont. on $[0, 5]$ we can use Closed Interval Method.

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

The only critical number is $\frac{7}{3}$. Note that $\frac{7}{3}$ is in $[0, 5]$.

No other critical numbers since $a'(t)$ is defined everywhere.

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t - 1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3t^2 - 14t + 12$$

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t - 1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3t^2 - 14t + 12$$

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

The values at critical numbers and end points of the interval:

$$a\left(\frac{7}{3}\right) =$$

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t - 1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3t^2 - 14t + 12$$

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

The values at critical numbers and end points of the interval:

$$a\left(\frac{7}{3}\right) = 3\left(\frac{7}{3}\right)^2 - 14\frac{7}{3} + 12$$

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t-1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3t^2 - 14t + 12$$

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

The values at critical numbers and end points of the interval:

$$a\left(\frac{7}{3}\right) = 3\left(\frac{7}{3}\right)^2 - 14\frac{7}{3} + 12 = \frac{7 \cdot 7}{3} - \frac{14 \cdot 7}{3} + \frac{36}{3}$$

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t-1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3t^2 - 14t + 12$$

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

The values at critical numbers and end points of the interval:

$$a\left(\frac{7}{3}\right) = 3\left(\frac{7}{3}\right)^2 - 14\frac{7}{3} + 12 = \frac{7 \cdot 7}{3} - \frac{14 \cdot 7}{3} + \frac{36}{3} = -\frac{13}{3}$$

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t-1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3t^2 - 14t + 12$$

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

The values at critical numbers and end points of the interval:

$$a\left(\frac{7}{3}\right) = 3\left(\frac{7}{3}\right)^2 - 14\frac{7}{3} + 12 = \frac{7 \cdot 7}{3} - \frac{14 \cdot 7}{3} + \frac{36}{3} = -\frac{13}{3}$$

$$a(0) =$$

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t-1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3t^2 - 14t + 12$$

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

The values at critical numbers and end points of the interval:

$$a\left(\frac{7}{3}\right) = 3\left(\frac{7}{3}\right)^2 - 14\frac{7}{3} + 12 = \frac{7 \cdot 7}{3} - \frac{14 \cdot 7}{3} + \frac{36}{3} = -\frac{13}{3}$$

$$a(0) = 12$$

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t-1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3t^2 - 14t + 12$$

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

The values at critical numbers and end points of the interval:

$$a\left(\frac{7}{3}\right) = 3\left(\frac{7}{3}\right)^2 - 14\frac{7}{3} + 12 = \frac{7 \cdot 7}{3} - \frac{14 \cdot 7}{3} + \frac{36}{3} = -\frac{13}{3}$$

$$a(0) = 12$$

$$a(5) =$$

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t-1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3t^2 - 14t + 12$$

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

The values at critical numbers and end points of the interval:

$$a\left(\frac{7}{3}\right) = 3\left(\frac{7}{3}\right)^2 - 14\frac{7}{3} + 12 = \frac{7 \cdot 7}{3} - \frac{14 \cdot 7}{3} + \frac{36}{3} = -\frac{13}{3}$$

$$a(0) = 12$$

$$a(5) = 3 \cdot 5^2 - 14 \cdot 5 + 12$$

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t-1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3t^2 - 14t + 12$$

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

The values at critical numbers and end points of the interval:

$$a\left(\frac{7}{3}\right) = 3\left(\frac{7}{3}\right)^2 - 14\frac{7}{3} + 12 = \frac{7 \cdot 7}{3} - \frac{14 \cdot 7}{3} + \frac{36}{3} = -\frac{13}{3}$$

$$a(0) = 12$$

$$a(5) = 3 \cdot 5^2 - 14 \cdot 5 + 12 = 15 \cdot 5 - 14 \cdot 5 + 12$$

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t-1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3t^2 - 14t + 12$$

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

The values at critical numbers and end points of the interval:

$$a\left(\frac{7}{3}\right) = 3\left(\frac{7}{3}\right)^2 - 14\frac{7}{3} + 12 = \frac{7 \cdot 7}{3} - \frac{14 \cdot 7}{3} + \frac{36}{3} = -\frac{13}{3}$$

$$a(0) = 12$$

$$a(5) = 3 \cdot 5^2 - 14 \cdot 5 + 12 = 15 \cdot 5 - 14 \cdot 5 + 12 = 17$$

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t-1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3t^2 - 14t + 12$$

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

The values at critical numbers and end points of the interval:

$$a\left(\frac{7}{3}\right) = 3\left(\frac{7}{3}\right)^2 - 14\frac{7}{3} + 12 = \frac{7 \cdot 7}{3} - \frac{14 \cdot 7}{3} + \frac{36}{3} = -\frac{13}{3}$$

$$a(0) = 12$$

$$a(5) = 3 \cdot 5^2 - 14 \cdot 5 + 12 = 15 \cdot 5 - 14 \cdot 5 + 12 = 17$$

The absolute minimum acceleration is

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t-1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3t^2 - 14t + 12$$

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

The values at critical numbers and end points of the interval:

$$a\left(\frac{7}{3}\right) = 3\left(\frac{7}{3}\right)^2 - 14\frac{7}{3} + 12 = \frac{7 \cdot 7}{3} - \frac{14 \cdot 7}{3} + \frac{36}{3} = -\frac{13}{3}$$

$$a(0) = 12$$

$$a(5) = 3 \cdot 5^2 - 14 \cdot 5 + 12 = 15 \cdot 5 - 14 \cdot 5 + 12 = 17$$

The absolute minimum acceleration is $a\left(\frac{7}{3}\right) = -\frac{13}{3}$.

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t-1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3t^2 - 14t + 12$$

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

The values at critical numbers and end points of the interval:

$$a\left(\frac{7}{3}\right) = 3\left(\frac{7}{3}\right)^2 - 14\frac{7}{3} + 12 = \frac{7 \cdot 7}{3} - \frac{14 \cdot 7}{3} + \frac{36}{3} = -\frac{13}{3}$$

$$a(0) = 12$$

$$a(5) = 3 \cdot 5^2 - 14 \cdot 5 + 12 = 15 \cdot 5 - 14 \cdot 5 + 12 = 17$$

The absolute minimum acceleration is $a\left(\frac{7}{3}\right) = -\frac{13}{3}$.

The absolute maximum acceleration is

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t-1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

The acceleration is:

$$a(t) = v'(t) = 3t^2 - 14t + 12$$

$$a'(t) = 6t - 14 \quad a'(t) = 0 \iff t = \frac{7}{3}$$

The values at critical numbers and end points of the interval:

$$a\left(\frac{7}{3}\right) = 3\left(\frac{7}{3}\right)^2 - 14\frac{7}{3} + 12 = \frac{7 \cdot 7}{3} - \frac{14 \cdot 7}{3} + \frac{36}{3} = -\frac{13}{3}$$

$$a(0) = 12$$

$$a(5) = 3 \cdot 5^2 - 14 \cdot 5 + 12 = 15 \cdot 5 - 14 \cdot 5 + 12 = 17$$

The absolute minimum acceleration is $a\left(\frac{7}{3}\right) = -\frac{13}{3}$.

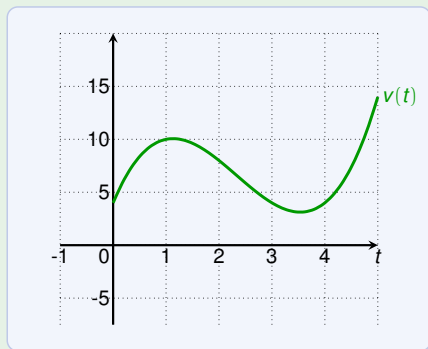
The absolute maximum acceleration is $a(5) = 17$.

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t - 1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.



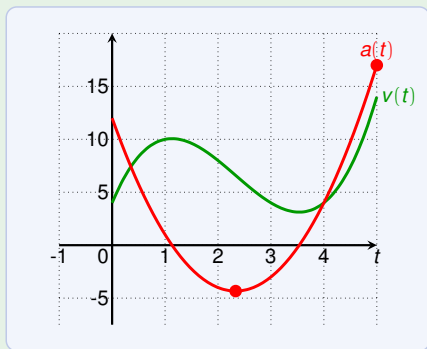
The absolute minimum acceleration is $a\left(\frac{7}{3}\right) = -\frac{13}{3}$.
The absolute maximum acceleration is $a(5) = 17$.

Maximum and Minimum Values

Assume that an object is moving with speed

$$v(t) = (t - 1)^3 - 4t^2 + 9t + 5 \quad 0 \leq t \leq 5$$

Find the absolute minimum and maximum acceleration.

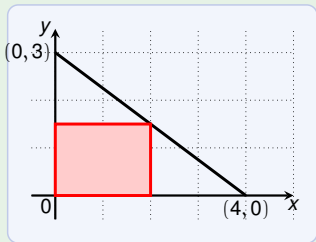


The absolute minimum acceleration is $a\left(\frac{7}{3}\right) = -\frac{13}{3}$.

The absolute maximum acceleration is $a(5) = 17$.

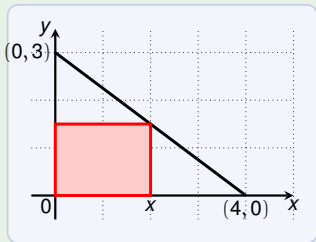
Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



Exam Task from 2003

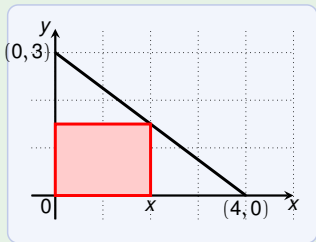
Find the area of the largest rectangle that can be inscribed as shown in the triangle.



The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) =$

Exam Task from 2003

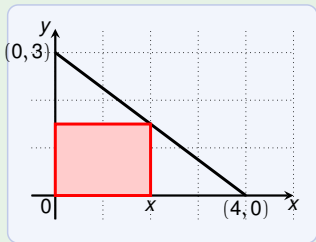
Find the area of the largest rectangle that can be inscribed as shown in the triangle.



The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) = -\frac{3}{4}x + 3$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



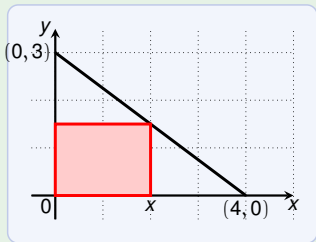
The line through $(0, 3)$ & $(4, 0)$ has the equation: $l(x) = -\frac{3}{4}x + 3$

The area A of the rectangle depends on the width x :

$$A(x) =$$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



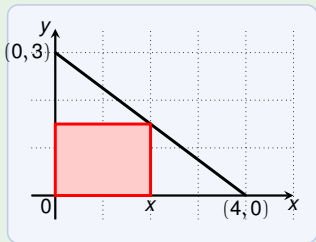
The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) = -\frac{3}{4}x + 3$

The area A of the rectangle depends on the width x :

$$A(x) = x \cdot \ell(x)$$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



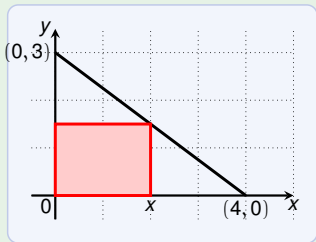
The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) = -\frac{3}{4}x + 3$

The area A of the rectangle depends on the width x :

$$A(x) = x \cdot \ell(x) = x \cdot \left(-\frac{3}{4}x + 3\right)$$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



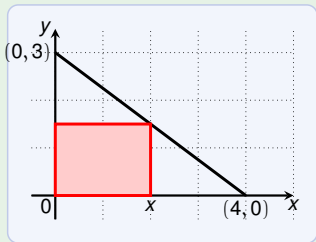
The line through $(0, 3)$ & $(4, 0)$ has the equation: $l(x) = -\frac{3}{4}x + 3$

The area A of the rectangle depends on the width x :

$$A(x) = x \cdot l(x) = x \cdot \left(-\frac{3}{4}x + 3\right) = -\frac{3}{4}x^2 + 3x$$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



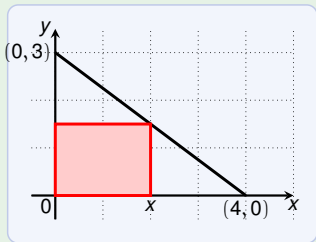
The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) = -\frac{3}{4}x + 3$

The area A of the rectangle depends on the width x :

$$A(x) = x \cdot \ell(x) = x \cdot \left(-\frac{3}{4}x + 3\right) = -\frac{3}{4}x^2 + 3x \quad \text{for } x \text{ in } [0, 4]$$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



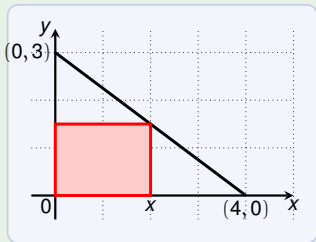
The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) = -\frac{3}{4}x + 3$

The area A of the rectangle depends on the width x :

$$A(x) = x \cdot \ell(x) = x \cdot \left(-\frac{3}{4}x + 3\right) = -\frac{3}{4}x^2 + 3x \quad \text{for } x \text{ in } [0, 4]$$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) = -\frac{3}{4}x + 3$

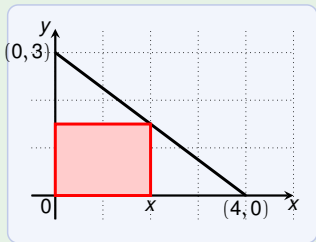
The area A of the rectangle depends on the width x :

$$A(x) = x \cdot \ell(x) = x \cdot \left(-\frac{3}{4}x + 3\right) = -\frac{3}{4}x^2 + 3x \quad \text{for } x \text{ in } [0, 4]$$

$$A'(x)$$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) = -\frac{3}{4}x + 3$

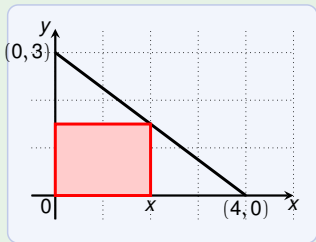
The area A of the rectangle depends on the width x :

$$A(x) = x \cdot \ell(x) = x \cdot \left(-\frac{3}{4}x + 3\right) = -\frac{3}{4}x^2 + 3x \quad \text{for } x \text{ in } [0, 4]$$

$$A'(x) =$$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) = -\frac{3}{4}x + 3$

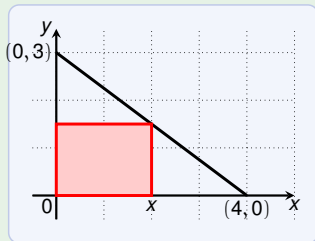
The area A of the rectangle depends on the width x :

$$A(x) = x \cdot \ell(x) = x \cdot \left(-\frac{3}{4}x + 3\right) = -\frac{3}{4}x^2 + 3x \quad \text{for } x \text{ in } [0, 4]$$

$$A'(x) = -\frac{3}{2}x + 3$$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) = -\frac{3}{4}x + 3$

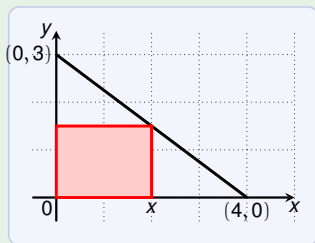
The area A of the rectangle depends on the width x :

$$A(x) = x \cdot \ell(x) = x \cdot \left(-\frac{3}{4}x + 3\right) = -\frac{3}{4}x^2 + 3x \quad \text{for } x \text{ in } [0, 4]$$

$$A'(x) = -\frac{3}{2}x + 3 \quad A'(x) = 0 \iff$$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) = -\frac{3}{4}x + 3$

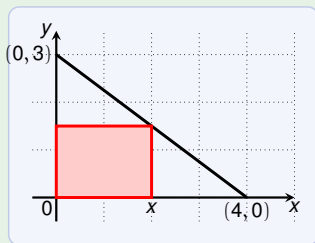
The area A of the rectangle depends on the width x :

$$A(x) = x \cdot \ell(x) = x \cdot \left(-\frac{3}{4}x + 3\right) = -\frac{3}{4}x^2 + 3x \quad \text{for } x \text{ in } [0, 4]$$

$$A'(x) = -\frac{3}{2}x + 3 \quad A'(x) = 0 \iff \frac{3}{2}x = 3$$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) = -\frac{3}{4}x + 3$

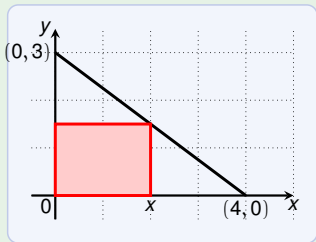
The area A of the rectangle depends on the width x :

$$A(x) = x \cdot \ell(x) = x \cdot \left(-\frac{3}{4}x + 3\right) = -\frac{3}{4}x^2 + 3x \quad \text{for } x \text{ in } [0, 4]$$

$$A'(x) = -\frac{3}{2}x + 3 \quad A'(x) = 0 \iff \frac{3}{2}x = 3 \iff x = 2$$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) = -\frac{3}{4}x + 3$

The area A of the rectangle depends on the width x :

$$A(x) = x \cdot \ell(x) = x \cdot \left(-\frac{3}{4}x + 3\right) = -\frac{3}{4}x^2 + 3x \quad \text{for } x \text{ in } [0, 4]$$

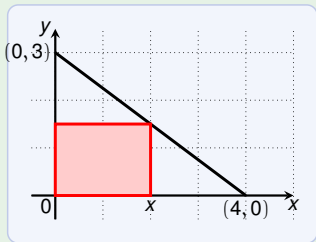
$$A'(x) = -\frac{3}{2}x + 3 \quad A'(x) = 0 \iff \frac{3}{2}x = 3 \iff x = 2$$

Thus the only critical number is 2. The value of $A(x)$ at 0, 2, 4:

$$A(0) =$$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) = -\frac{3}{4}x + 3$

The area A of the rectangle depends on the width x :

$$A(x) = x \cdot \ell(x) = x \cdot \left(-\frac{3}{4}x + 3\right) = -\frac{3}{4}x^2 + 3x \quad \text{for } x \text{ in } [0, 4]$$

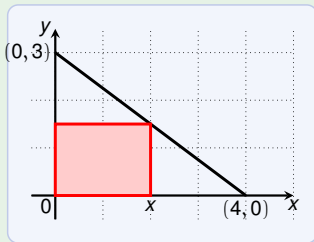
$$A'(x) = -\frac{3}{2}x + 3 \quad A'(x) = 0 \iff \frac{3}{2}x = 3 \iff x = 2$$

Thus the only critical number is 2. The value of $A(x)$ at 0, 2, 4:

$$A(0) = 0$$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) = -\frac{3}{4}x + 3$

The area A of the rectangle depends on the width x :

$$A(x) = x \cdot \ell(x) = x \cdot \left(-\frac{3}{4}x + 3\right) = -\frac{3}{4}x^2 + 3x \quad \text{for } x \text{ in } [0, 4]$$

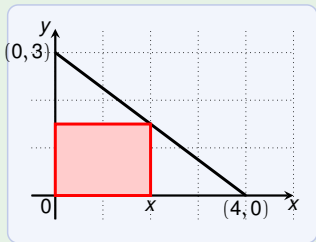
$$A'(x) = -\frac{3}{2}x + 3 \quad A'(x) = 0 \iff \frac{3}{2}x = 3 \iff x = 2$$

Thus the only critical number is 2. The value of $A(x)$ at 0, 2, 4:

$$A(0) = 0 \quad A(2) =$$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) = -\frac{3}{4}x + 3$

The area A of the rectangle depends on the width x :

$$A(x) = x \cdot \ell(x) = x \cdot \left(-\frac{3}{4}x + 3\right) = -\frac{3}{4}x^2 + 3x \quad \text{for } x \text{ in } [0, 4]$$

$$A'(x) = -\frac{3}{2}x + 3 \quad A'(x) = 0 \iff \frac{3}{2}x = 3 \iff x = 2$$

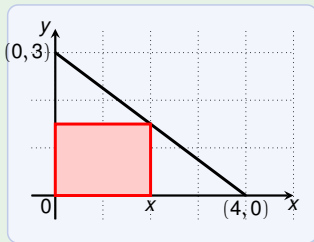
Thus the only critical number is 2. The value of $A(x)$ at 0, 2, 4:

$$A(0) = 0$$

$$A(2) = 3$$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) = -\frac{3}{4}x + 3$

The area A of the rectangle depends on the width x :

$$A(x) = x \cdot \ell(x) = x \cdot \left(-\frac{3}{4}x + 3\right) = -\frac{3}{4}x^2 + 3x \quad \text{for } x \text{ in } [0, 4]$$

$$A'(x) = -\frac{3}{2}x + 3 \quad A'(x) = 0 \iff \frac{3}{2}x = 3 \iff x = 2$$

Thus the only critical number is 2. The value of $A(x)$ at 0, 2, 4:

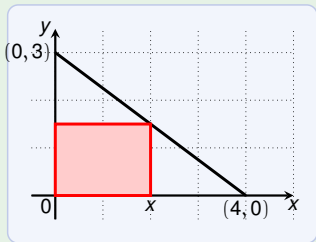
$$A(0) = 0$$

$$A(2) = 3$$

$$A(4) =$$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) = -\frac{3}{4}x + 3$

The area A of the rectangle depends on the width x :

$$A(x) = x \cdot \ell(x) = x \cdot \left(-\frac{3}{4}x + 3\right) = -\frac{3}{4}x^2 + 3x \quad \text{for } x \text{ in } [0, 4]$$

$$A'(x) = -\frac{3}{2}x + 3 \quad A'(x) = 0 \iff \frac{3}{2}x = 3 \iff x = 2$$

Thus the only critical number is 2. The value of $A(x)$ at 0, 2, 4:

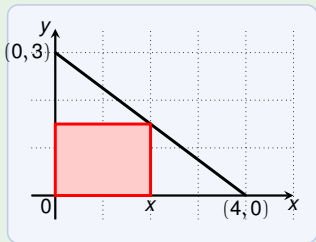
$$A(0) = 0$$

$$A(2) = 3$$

$$A(4) = 0$$

Exam Task from 2003

Find the area of the largest rectangle that can be inscribed as shown in the triangle.



The line through $(0, 3)$ & $(4, 0)$ has the equation: $\ell(x) = -\frac{3}{4}x + 3$

The area A of the rectangle depends on the width x :

$$A(x) = x \cdot \ell(x) = x \cdot \left(-\frac{3}{4}x + 3\right) = -\frac{3}{4}x^2 + 3x \quad \text{for } x \text{ in } [0, 4]$$

$$A'(x) = -\frac{3}{2}x + 3 \quad A'(x) = 0 \iff \frac{3}{2}x = 3 \iff x = 2$$

Thus the only critical number is 2. The value of $A(x)$ at 0, 2, 4:

$$A(0) = 0$$

$$A(2) = 3$$

$$A(4) = 0$$

The area of the largest rectangle is **3**.