Calculus M211

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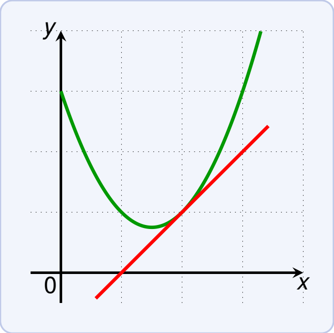
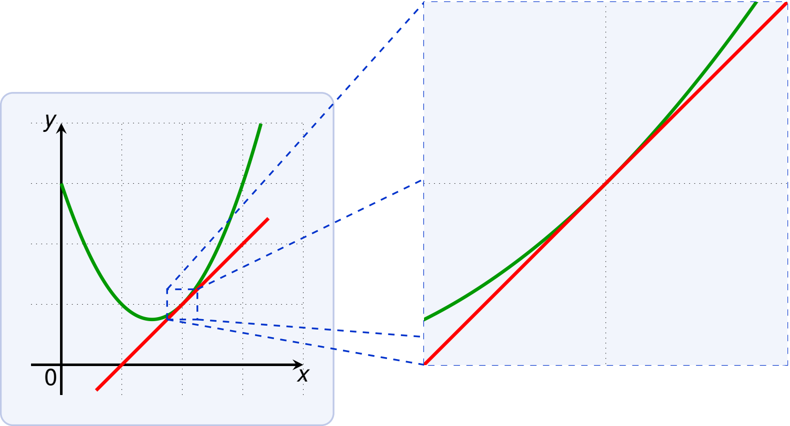
Indiana University Bloomington

2013

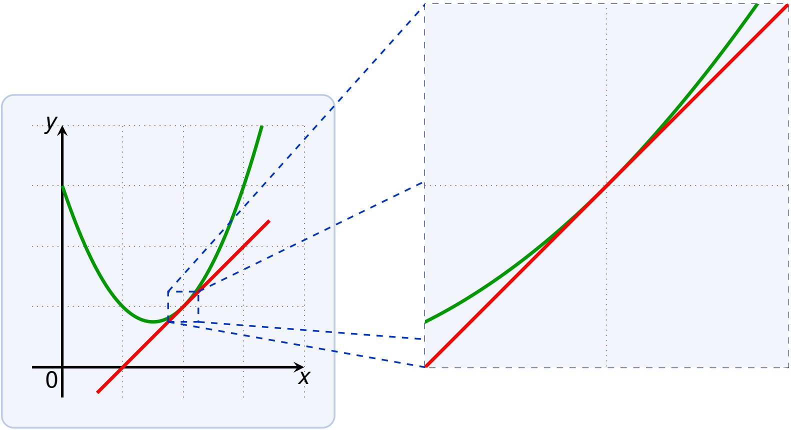
Linear Approximation and Differentials



Linear Approximation and Differentials



Linear Approximation and Differentials



A curve is very close to its tangent close to the point of tangency (touching).

We can use this for approximating values of the function. . .

Linear Approximation and Differentials

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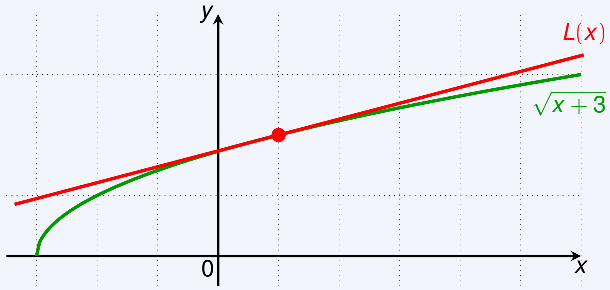

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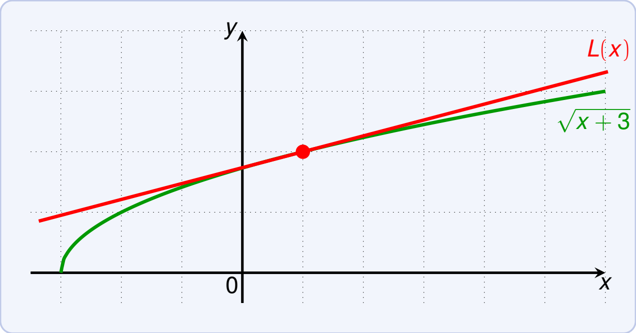

The linear approximation is close to the curve when x is near 1.

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Approximations for sin and cos are often applied in physics (e.g. optics).

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Use differential approximation, or the linearization method, to approximate $\sqrt[4]{15.5}$.

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$$L(x) = 2 + \frac{1}{32}(x - 16)$$

Linear Approximation and Differentials

Final Exam 2005

Use differential approximation, or the linearization method, to approximate $\sqrt[4]{15.5}$.

We have $f(x) = \sqrt[4]{x}$.

We need to choose where to compute the linearization: $a = 16$.

$$f(16) = 2$$

$$f'(x) = \frac{1}{4}x^{-\frac{3}{4}} \qquad f'(16) = \frac{1}{4}16^{-\frac{3}{4}} = \frac{1}{4}\sqrt[4]{16}^{-3} = \frac{1}{4}\cdot\frac{1}{8} = \frac{1}{32}$$

The linearization of f at 16 is:

$$L(x) = 2 + \frac{1}{32}(x - 16)$$

Then the approximation of $\sqrt[4]{15.5}$ is:

$$\sqrt[4]{15.5} \approx$$

Linear Approximation and Differentials

Final Exam 2005

Use differential approximation, or the linearization method, to approximate $\sqrt[4]{15.5}$.

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The linearization of f at 16 is:

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Then the approximation of $\sqrt[4]{15.5}$ is:

$$\sqrt[4]{15.5} \approx L(15.5)$$

Linear Approximation and Differentials

Final Exam 2005

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The linearization of f at 16 is:

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Then the approximation of $\sqrt[4]{15.5}$ is:

$$\sqrt[4]{15.5} \approx L(15.5) = 2 + \frac{1}{32}(15.5 - 16)$$

Linear Approximation and Differentials

Final Exam 2005

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The linearization of f at 16 is:

$$L(x) = 2 + \frac{1}{32}(x - 16)$$

Then the approximation of $\sqrt[4]{15.5}$ is:

$$\sqrt[4]{15.5} \approx L(15.5) = 2 + \frac{1}{32}(15.5 - 16) = 2 - \frac{1}{64}$$

Linear Approximation and Differentials

Final Exam 2005

Use differential approximation, or the linearization method, to approximate $\sqrt[4]{15.5}$.

We have $f(x) = \sqrt[4]{x}$.

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The linearization of f at 16 is:

$$L(x) = 2 + \frac{1}{32}(x - 16)$$

Then the approximation of $\sqrt[4]{15.5}$ is:

$$\sqrt[4]{15.5} \approx L(15.5) = 2 + \frac{1}{32}(15.5 - 16) = 2 - \frac{1}{64} = \frac{127}{64}$$

Linear Approximation and Differentials

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

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We have $f(x) = x^4$.

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

We have $f(x) = x^4$.

We need to choose where to compute the linearization: $a =$

Linear Approximation and Differentials

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

We have $f(x) = x^4$.

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Linear Approximation and Differentials

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

We have $f(x) = x^4$.

We need to choose where to compute the linearization: $a = 2$.

$$f(2) =$$

Linear Approximation and Differentials

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

We have $f(x) = x^4$.

We need to choose where to compute the linearization: $a = 2$.

$$f(2) = 16$$

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

We have $f(x) = x^4$.

We need to choose where to compute the linearization: $a = 2$.

$$f(2) = 16$$
$$f'(x) =$$

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

We have $f(x) = x^4$.

We need to choose where to compute the linearization: $a = 2$.

$$f(2) = 16$$
$$f'(x) = 4x^3$$

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

We have $f(x) = x^4$.

We need to choose where to compute the linearization: $a = 2$.

$$f(2) = 16$$

$$f'(x) = 4x^3 \qquad f'(2) =$$

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

We have $f(x) = x^4$.

We need to choose where to compute the linearization: $a = 2$.

$$f(2) = 16$$
$$f'(x) = 4x^3 \qquad f'(2) = 4 \cdot 2^3$$

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

We have $f(x) = x^4$.

We need to choose where to compute the linearization: $a = 2$.

$$f(2) = 16$$

$$f'(x) = 4x^3 \qquad f'(2) = 4 \cdot 2^3 = 4 \cdot 8$$

Linear Approximation and Differentials

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

We have $f(x) = x^4$.

We need to choose where to compute the linearization: $a = 2$.

$$f(2) = 16$$

$$f'(x) = 4x^3 \qquad f'(2) = 4 \cdot 2^3 = 4 \cdot 8 = 32$$

Linear Approximation and Differentials

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

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We need to choose where to compute the linearization: $a = 2$.

$$f(2) = 16$$

$$f'(x) = 4x^3 \qquad f'(2) = 4 \cdot 2^3 = 4 \cdot 8 = 32$$

The linearization of f at 2 is:

$$L(x) =$$

Linear Approximation and Differentials

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

We have $f(x) = x^4$.

We need to choose where to compute the linearization: $a = 2$.

$$f(2) = 16$$

$$f'(x) = 4x^3 \qquad f'(2) = 4 \cdot 2^3 = 4 \cdot 8 = 32$$

The linearization of f at 2 is:

$$L(x) = 16 + 32(x - 2)$$

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

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We need to choose where to compute the linearization: $a = 2$.

$$f(2) = 16$$

$$f'(x) = 4x^3 \qquad f'(2) = 4 \cdot 2^3 = 4 \cdot 8 = 32$$

The linearization of f at 2 is:

$$L(x) = 16 + 32(x - 2)$$

Then the approximation of $(1.98)^4$ is:

$(1.98)^4 \approx$

Linear Approximation and Differentials

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

We have $f(x) = x^4$.

We need to choose where to compute the linearization: $a = 2$.

$$f(2) = 16$$

$$f'(x) = 4x^3 \qquad f'(2) = 4 \cdot 2^3 = 4 \cdot 8 = 32$$

The linearization of f at 2 is:

$$L(x) = 16 + 32(x - 2)$$

Then the approximation of $(1.98)^4$ is:

$(1.98)^4 \approx L(1.98)$

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

We have $f(x) = x^4$.

We need to choose where to compute the linearization: $a = 2$.

$$f(2) = 16$$

$$f'(x) = 4x^3 \qquad f'(2) = 4 \cdot 2^3 = 4 \cdot 8 = 32$$

The linearization of f at 2 is:

$$L(x) = 16 + 32(x - 2)$$

Then the approximation of $(1.98)^4$ is:

$$(1.98)^4 \approx L(1.98) = 16 + 32(1.98 - 2)$$

Linear Approximation and Differentials

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

We have $f(x) = x^4$.

We need to choose where to compute the linearization: $a = 2$.

$$f(2) = 16$$

$$f'(x) = 4x^3 \qquad\qquad f'(2) = 4 \cdot 2^3 = 4 \cdot 8 = 32$$

The linearization of f at 2 is:

$$L(x) = 16 + 32(x - 2)$$

Then the approximation of $(1.98)^4$ is:

$$(1.98)^4 \approx L(1.98) = 16 + 32(1.98 - 2) = 16 + 32(-0.02)$$

Linear Approximation and Differentials

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

We have $f(x) = x^4$.

We need to choose where to compute the linearization: $a = 2$.

$$f(2) = 16$$

$$f'(x) = 4x^3 \qquad f'(2) = 4 \cdot 2^3 = 4 \cdot 8 = 32$$

The linearization of f at 2 is:

$$L(x) = 16 + 32(x - 2)$$

Then the approximation of $(1.98)^4$ is:

$$(1.98)^4 \approx L(1.98) = 16 + 32(1.98 - 2) = 16 + 32(-0.02)$$
$$= 16 + 32(-\frac{1}{50})$$

Linear Approximation and Differentials

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

We have $f(x) = x^4$.

We need to choose where to compute the linearization: $a = 2$.

$$f(2) = 16$$

$$f'(x) = 4x^3 \qquad f'(2) = 4 \cdot 2^3 = 4 \cdot 8 = 32$$

The linearization of f at 2 is:

$$L(x) = 16 + 32(x - 2)$$

Then the approximation of $(1.98)^4$ is:

$$(1.98)^4 \approx L(1.98) = 16 + 32(1.98 - 2) = 16 + 32(-0.02)$$

$$= 16 + 32(-\frac{1}{50}) = 16 - \frac{16}{25}$$

Linear Approximation and Differentials

Final Exam 2004

Use the linearization method to approximate $(1.98)^4$.

We have $f(x) = x^4$.

We need to choose where to compute the linearization: $a = 2$.

$$f(2) = 16$$

$$f'(x) = 4x^3 \qquad f'(2) = 4 \cdot 2^3 = 4 \cdot 8 = 32$$

The linearization of f at 2 is:

$$L(x) = 16 + 32(x - 2)$$

Then the approximation of $(1.98)^4$ is:

$$(1.98)^4 \approx L(1.98) = 16 + 32(1.98 - 2) = 16 + 32(-0.02)$$

$$= 16 + 32(-\frac{1}{50}) = 16 - \frac{16}{25} = \frac{16 \cdot 24}{25}$$

Final Exam 2003 (Spring)

Use differentials or the linearization approximation method to approximate $\ln(0.9)$.

Linear Approximation and Differentials

Final Exam 2003 (Spring)

Use differentials or the linearization approximation method to approximate $\ln(0.9)$.

We have $f(x) = \ln x$.

Linear Approximation and Differentials

Final Exam 2003 (Spring)

Use differentials or the linearization approximation method to approximate $\ln(0.9)$.

We have $f(x) = \ln x$.

We need to choose where to compute the linearization: $a =$

Linear Approximation and Differentials

Final Exam 2003 (Spring)

Use differentials or the linearization approximation method to approximate $\ln(0.9)$.

We have $f(x) = \ln x$.

We need to choose where to compute the linearization: $a = 1$.

Final Exam 2003 (Spring)

Use differentials or the linearization approximation method to approximate $\ln(0.9)$.

We have $f(x) = \ln x$.

We need to choose where to compute the linearization: $a = 1$.

$$f(1) =$$

Final Exam 2003 (Spring)

Use differentials or the linearization approximation method to approximate $\ln(0.9)$.

We have $f(x) = \ln x$.

We need to choose where to compute the linearization: $a = 1$.

$$f(1) = 0$$

Linear Approximation and Differentials

Final Exam 2003 (Spring)

Use differentials or the linearization approximation method to approximate $\ln(0.9)$.

We have $f(x) = \ln x$.

We need to choose where to compute the linearization: $a = 1$.

$$f(1) = 0$$

$$f'(x) =$$

Linear Approximation and Differentials

Final Exam 2003 (Spring)

Use differentials or the linearization approximation method to approximate $\ln(0.9)$.

We have $f(x) = \ln x$.

We need to choose where to compute the linearization: $a = 1$.

$$f(1) = 0$$
$$f'(x) = \frac{1}{x}$$

Linear Approximation and Differentials

Final Exam 2003 (Spring)

Use differentials or the linearization approximation method to approximate $\ln(0.9)$.

We have $f(x) = \ln x$.

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$$f(1) = 0$$

$$f'(x) = \frac{1}{x} \qquad\qquad f'(1) = 1$$

Linear Approximation and Differentials

Final Exam 2003 (Spring)

Use differentials or the linearization approximation method to approximate $\ln(0.9)$.

We have $f(x) = \ln x$.

We need to choose where to compute the linearization: $a = 1$.

$$f(1) = 0$$

$$f'(x) = \frac{1}{x} \qquad\qquad f'(1) = 1$$

The linearization of f at 1 is:

$$L(x) =$$

Linear Approximation and Differentials

Final Exam 2003 (Spring)

Use differentials or the linearization approximation method to approximate $\ln(0.9)$.

We have $f(x) = \ln x$.

We need to choose where to compute the linearization: $a = 1$.

$$f(1) = 0$$

$$f'(x) = \frac{1}{x} \qquad\qquad f'(1) = 1$$

The linearization of f at 1 is:

$$L(x) = 0 + 1(x - 1)$$

Linear Approximation and Differentials

Final Exam 2003 (Spring)

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We need to choose where to compute the linearization: $a = 1$.

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$$f'(x) = \frac{1}{x} \qquad\qquad f'(1) = 1$$

The linearization of f at 1 is:

$$L(x) = 0 + 1(x - 1) = x - 1$$

Linear Approximation and Differentials

Final Exam 2003 (Spring)

Use differentials or the linearization approximation method to approximate $\ln(0.9)$.

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The linearization of f at 1 is:

$$L(x) = 0 + 1(x - 1) = x - 1$$

Then the approximation of $\ln(0.9)$ is:

$$\ln(0.9) \approx$$

Final Exam 2003 (Spring)

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The linearization of f at 1 is:

$$L(x) = 0 + 1(x - 1) = x - 1$$

Then the approximation of $\ln(0.9)$ is:

$$\ln(0.9) \approx L(0.9)$$

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$$f'(x) = \frac{1}{x} \qquad\qquad f'(1) = 1$$

The linearization of f at 1 is:

$$L(x) = 0 + 1(x - 1) = x - 1$$

Then the approximation of $\ln(0.9)$ is:

$$\ln(0.9) \approx L(0.9) = 0.9 - 1$$

Linear Approximation and Differentials

Final Exam 2003 (Spring)

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We need to choose where to compute the linearization: $a = 1$.

$$f(1) = 0$$

$$f'(x) = \frac{1}{x} \qquad\qquad f'(1) = 1$$

The linearization of f at 1 is:

$$L(x) = 0 + 1(x - 1) = x - 1$$

Then the approximation of $\ln(0.9)$ is:

$$\ln(0.9) \approx L(0.9) = 0.9 - 1 = -0.1$$

Final Exam 2003 (Fall)

Use differentials to approximate $\sqrt[3]{999}$.

Final Exam 2003 (Fall)

Use differentials to approximate $\sqrt[3]{999}$.

We have $f(x) = \sqrt[3]{x}$.

Linear Approximation and Differentials

Final Exam 2003 (Fall)

Use differentials to approximate $\sqrt[3]{999}$.

We have $f(x) = \sqrt[3]{x}$.

We choose where to compute the linearization: $a =$

Final Exam 2003 (Fall)

Use differentials to approximate $\sqrt[3]{999}$.

We have $f(x) = \sqrt[3]{x}$.

We choose where to compute the linearization: $a = 1000$.

Linear Approximation and Differentials

Final Exam 2003 (Fall)

Use differentials to approximate $\sqrt[3]{999}$.

We have $f(x) = \sqrt[3]{x}$.

We choose where to compute the linearization: $a = 1000$.

$f(1000) =$

Final Exam 2003 (Fall)

Use differentials to approximate $\sqrt[3]{999}$.

We have $f(x) = \sqrt[3]{x}$.

We choose where to compute the linearization: $a = 1000$.

$$f(1000) = 10$$

Linear Approximation and Differentials

Final Exam 2003 (Fall)

Use differentials to approximate $\sqrt[3]{999}$.

We have $f(x) = \sqrt[3]{x}$.

We choose where to compute the linearization: $a = 1000$.

$$f(1000) = 10$$

$$f'(x) =$$

Linear Approximation and Differentials

Final Exam 2003 (Fall)

Use differentials to approximate $\sqrt[3]{999}$.

We have $f(x) = \sqrt[3]{x}$.

We choose where to compute the linearization: $a = 1000$.

$$f(1000) = 10$$

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3(\sqrt[3]{x})^2}$$

Linear Approximation and Differentials

Final Exam 2003 (Fall)

Use differentials to approximate $\sqrt[3]{999}$.

We have $f(x) = \sqrt[3]{x}$.

We choose where to compute the linearization: $a = 1000$.

$$f(1000) = 10$$

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3(\sqrt[3]{x})^2} \qquad f'(1000) = \frac{1}{3 \cdot 10^2}$$

Linear Approximation and Differentials

Final Exam 2003 (Fall)

Use differentials to approximate $\sqrt[3]{999}$.

We have $f(x) = \sqrt[3]{x}$.

We choose where to compute the linearization: $a = 1000$.

$$f(1000) = 10$$

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3(\sqrt[3]{x})^2} \qquad f'(1000) = \frac{1}{3 \cdot 10^2} = \frac{1}{300}$$

Final Exam 2003 (Fall)

Use differentials to approximate $\sqrt[3]{999}$.

We have $f(x) = \sqrt[3]{x}$.

We choose where to compute the linearization: $a = 1000$.

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$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3(\sqrt[3]{x})^2} \qquad f'(1000) = \frac{1}{3 \cdot 10^2} = \frac{1}{300}$$

The linearization of f at 1000 is:

$$L(x) =$$

Final Exam 2003 (Fall)

Use differentials to approximate $\sqrt[3]{999}$.

We have $f(x) = \sqrt[3]{x}$.

We choose where to compute the linearization: $a = 1000$.

$$f(1000) = 10$$

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3(\sqrt[3]{x})^2} \qquad f'(1000) = \frac{1}{3 \cdot 10^2} = \frac{1}{300}$$

The linearization of f at 1000 is:

$$L(x) = 10 + \frac{1}{300}(x - 1000)$$

Linear Approximation and Differentials

Final Exam 2003 (Fall)

Use differentials to approximate $\sqrt[3]{999}$.

We have $f(x) = \sqrt[3]{x}$.

We choose where to compute the linearization: $a = 1000$.

$$f(1000) = 10$$

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3(\sqrt[3]{x})^2} \qquad f'(1000) = \frac{1}{3 \cdot 10^2} = \frac{1}{300}$$

The linearization of f at 1000 is:

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Then the approximation of $\sqrt[3]{999}$ is:

$$\sqrt[3]{999} \approx$$

Linear Approximation and Differentials

Final Exam 2003 (Fall)

Use differentials to approximate $\sqrt[3]{999}$.

We have $f(x) = \sqrt[3]{x}$.

We choose where to compute the linearization: $a = 1000$.

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$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3(\sqrt[3]{x})^2} \qquad f'(1000) = \frac{1}{3 \cdot 10^2} = \frac{1}{300}$$

The linearization of f at 1000 is:

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Then the approximation of $\sqrt[3]{999}$ is:

$$\sqrt[3]{999} \approx L(999)$$

Linear Approximation and Differentials

Final Exam 2003 (Fall)

Use differentials to approximate $\sqrt[3]{999}$.

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The linearization of f at 1000 is:

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Then the approximation of $\sqrt[3]{999}$ is:

$$\sqrt[3]{999} \approx L(999) = 10 + \frac{1}{300}(999 - 1000)$$

Linear Approximation and Differentials

Final Exam 2003 (Fall)

Use differentials to approximate $\sqrt[3]{999}$.

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The linearization of f at 1000 is:

$$L(x) = 10 + \frac{1}{300}(x - 1000)$$

Then the approximation of $\sqrt[3]{999}$ is:

$$\sqrt[3]{999} \approx L(999) = 10 + \frac{1}{300}(999 - 1000) = 10 - \frac{1}{300}$$

Final Exam 2003 (Fall)

Use differentials to approximate $\sqrt[3]{999}$.

We have $f(x) = \sqrt[3]{x}$.

We choose where to compute the linearization: $a = 1000$.

$$f(1000) = 10$$

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The linearization of f at 1000 is:

$$L(x) = 10 + \frac{1}{300}(x - 1000)$$

Then the approximation of $\sqrt[3]{999}$ is:

$$\sqrt[3]{999} \approx L(999) = 10 + \frac{1}{300}(999 - 1000) = 10 - \frac{1}{300} = \frac{2999}{300}$$

Linear Approximation and Differentials

The method of linear approximation with differentials:

$$f'(x) = \frac{dy}{dx}$$

Linear Approximation and Differentials

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We view dx and dy as variables, then:

$$dy = f'(x)\, dx$$

Linear Approximation and Differentials

The method of linear approximation with differentials:

$$f'(x) = \frac{dy}{dx}$$

We view dx and dy as variables, then:

$$dy = f'(x)\ dx$$

So dy depends on the value of x and dx.

Linear Approximation and Differentials

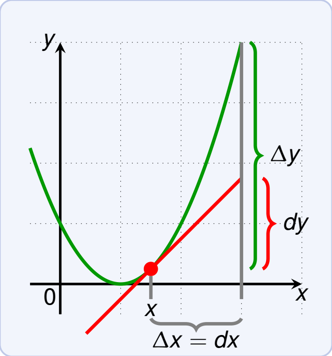
The method of linear approximation with differentials:

$$f'(x) = \frac{dy}{dx}$$

We view *dx* and *dy* as variables, then:

$$dy = f'(x)\, dx$$

So *dy* depends on the value of *x* and *dx*.



Linear Approximation and Differentials

The method of linear approximation with differentials:

$$f'(x) = \frac{dy}{dx}$$

We view *dx* and *dy* as variables, then:

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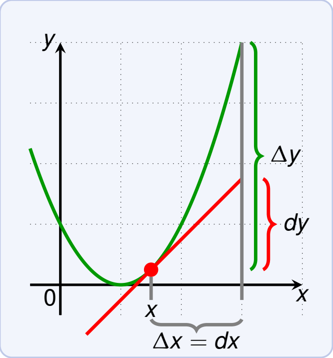
So *dy* depends on the value of *x* and *dx*.



- x = point of linearization

Linear Approximation and Differentials

The method of linear approximation with differentials:

$$f'(x) = \frac{dy}{dx}$$

We view *dx* and *dy* as variables, then:

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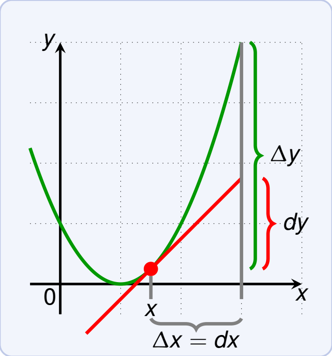
So *dy* depends on the value of *x* and *dx*.



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Linear Approximation and Differentials

The method of linear approximation with differentials:

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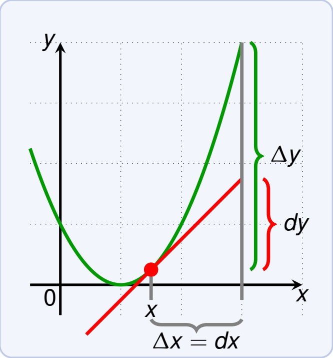

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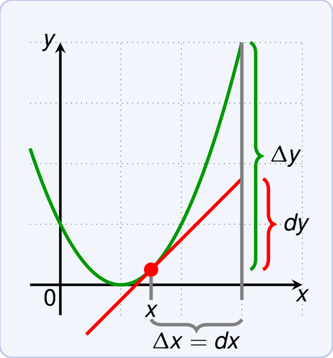

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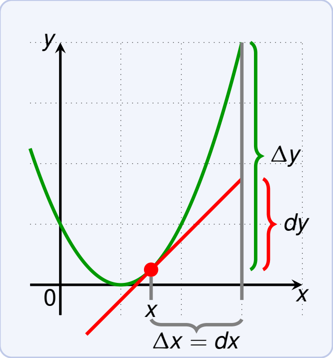

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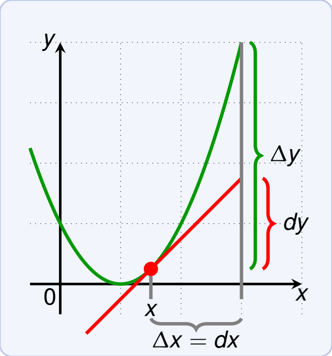

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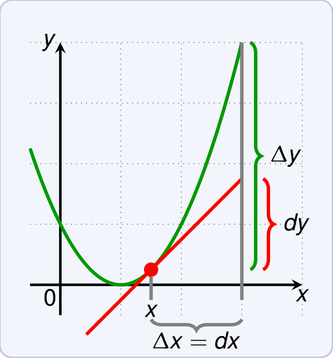

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