Calculus M211

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Indiana University Bloomington

2013

Related (Dependent) Rates

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- the volume increases with 100 cm^3/s

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$$V'(t) =$$

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How fast is the top sliding when the bottom is 6ft from the wall?

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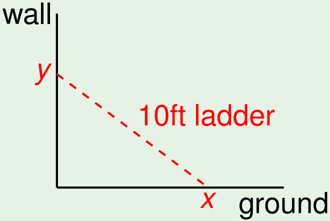

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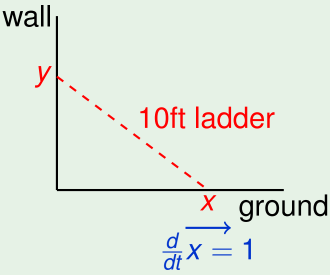

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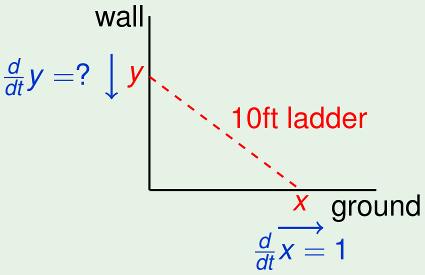

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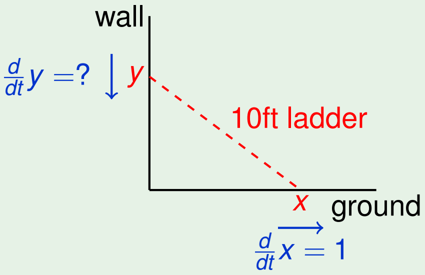

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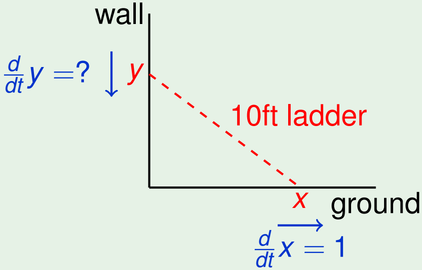

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$$x^2 + y^2 = 10^2$$

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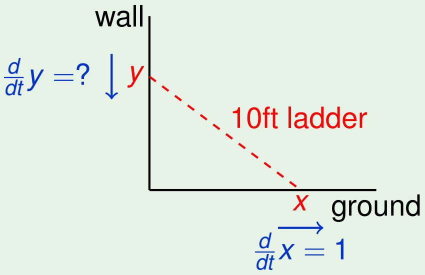

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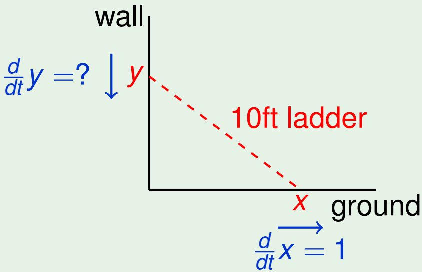

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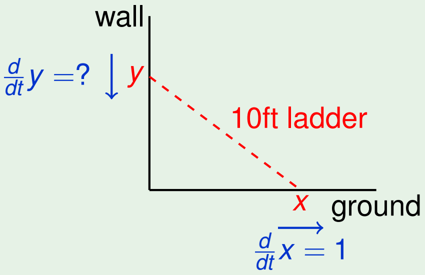

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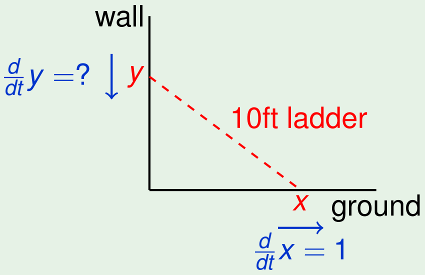

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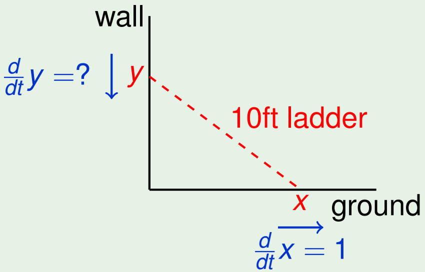

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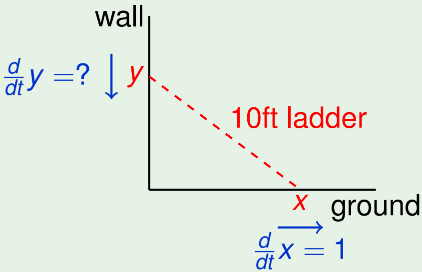

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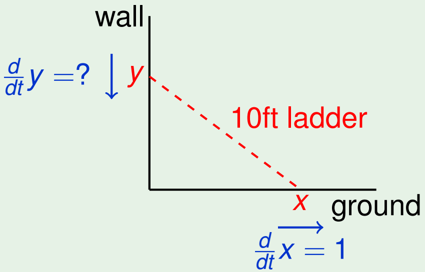

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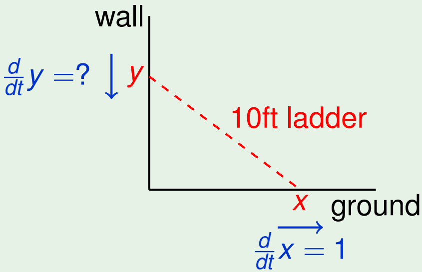

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The top slides with $\frac{3}{4}$ft/s when the bottom is 6ft from the wall.

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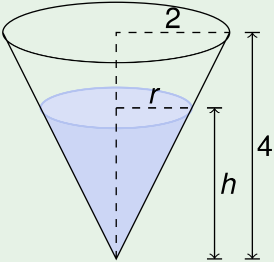
A water tank has the shape of an inverted circular cone:

- base radius $2m$ and the height is $4m$,
- water is pumped into the tank at a rate of $2\text{m}^3/\text{min}$.

At what rate is the water rising when the water is 3m deep?



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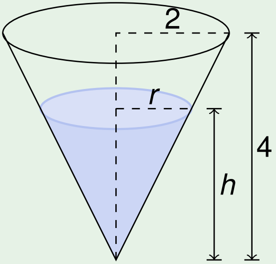

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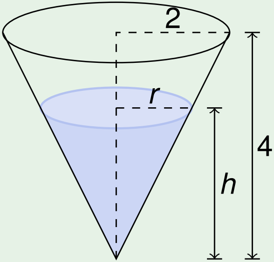

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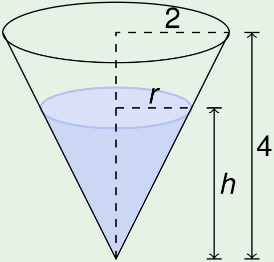

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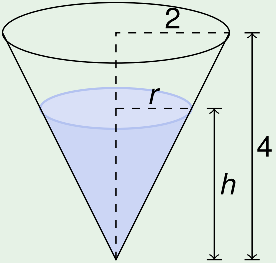

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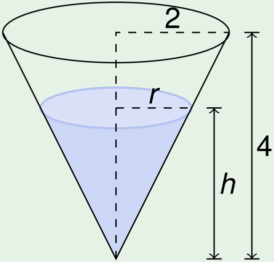

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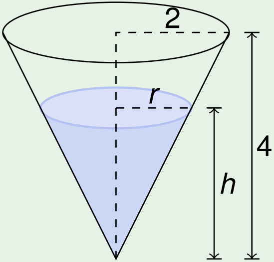

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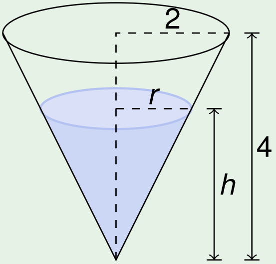

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We differentiate both sides with respect to t:

$$\frac{dV}{dt} = \frac{d}{dt}(\frac{1}{12}\pi h^3)$$

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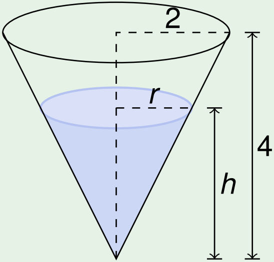

$$V = \frac{1}{3}\pi r^2 h$$

How is r related to h?

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$$\frac{dV}{dt} = \frac{d}{dt}(\frac{1}{12}\pi h^3) = \frac{1}{12}\pi 3h^2 \frac{dh}{dt}$$

Related (Dependent) Rates

A water tank has the shape of an inverted circular cone:

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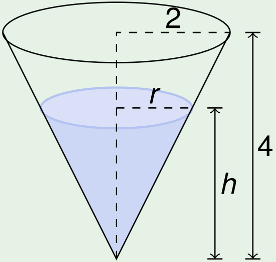

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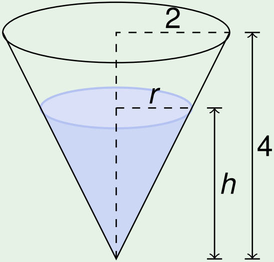

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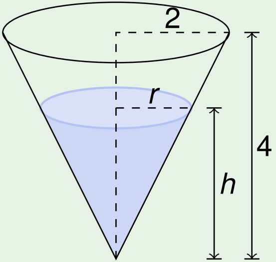

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Thus the water rises with $8/(\pi 9)$m/min when its is 3m deep.

Related (Dependent) Rates

Problem Solving Strategy

Important when solving textual problems:

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- Solve for the unknown rate, and substitute the given information into the resulting formula.

Related (Dependent) Rates

Two cars are headed for the same road intersection:

- car A is traveling west with 50mi/h
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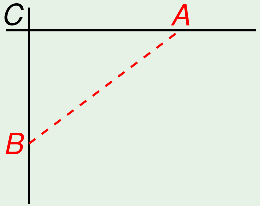
At what rate are the cars approaching
when A is 0.3mi and B is 0.4mi from the intersection?



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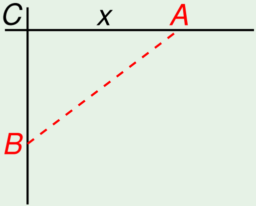

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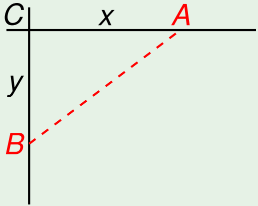

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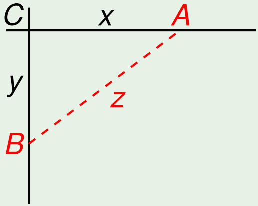

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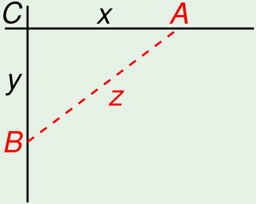

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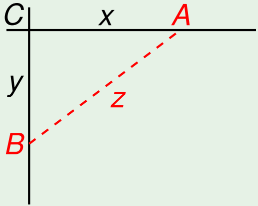

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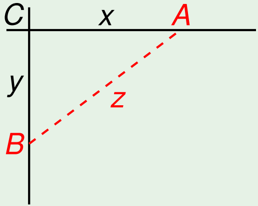

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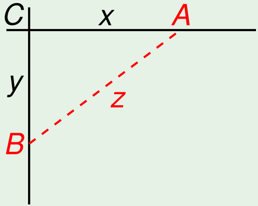

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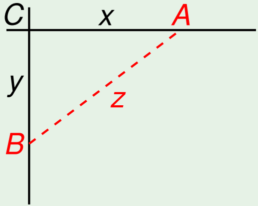

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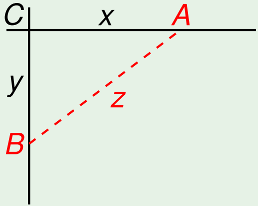

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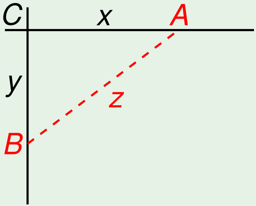

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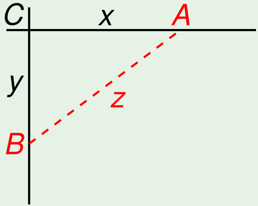

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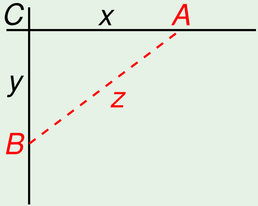

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When $x = 0.3$ & $y = 0.4$, we get $z = 0.5$. The answer is 78mi/h.

Related (Dependent) Rates

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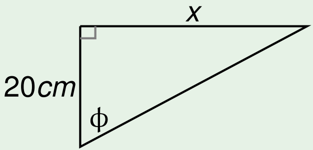

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How fast is the angle ϕ changing when $x = 15cm$?

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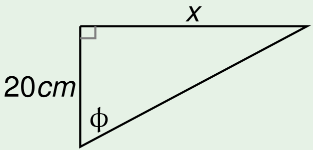

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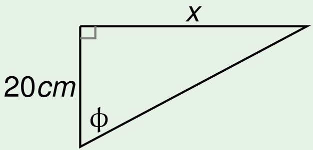

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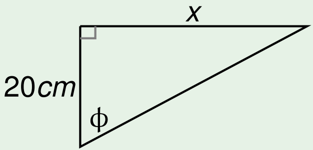

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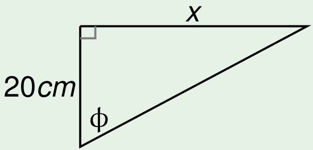

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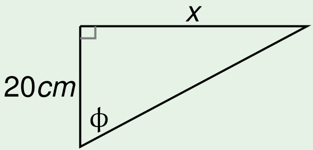

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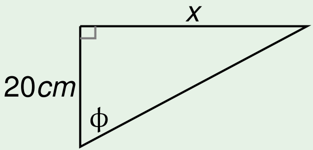

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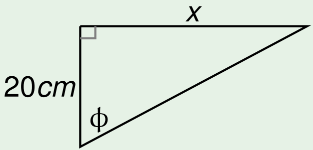

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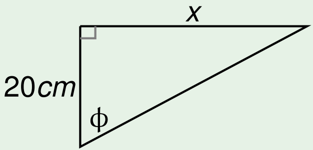

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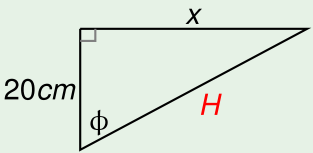

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How fast is the angle ϕ changing when $x = 15cm$?

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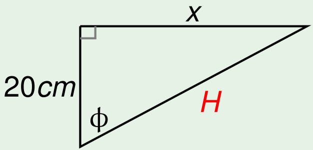
We have $\cos\phi = 20/H$

Related (Dependent) Rates

We have a right-angled triangle of the form



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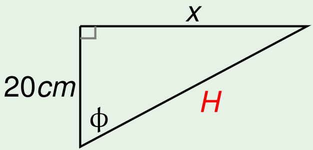

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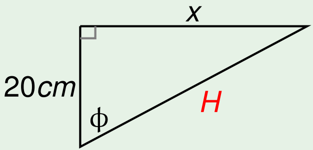

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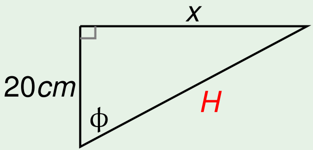

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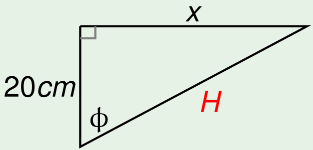

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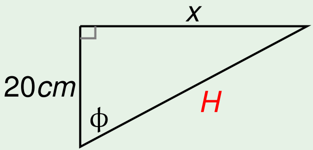

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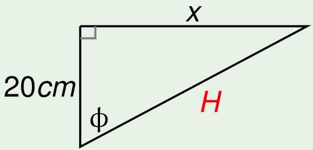

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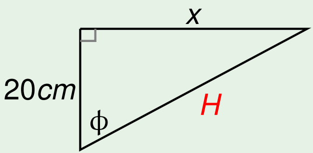

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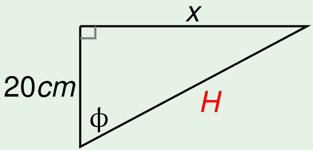

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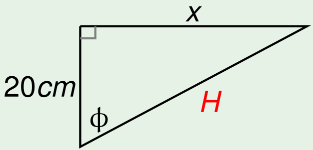

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