Calculus M211

Jörg Endrullis

Indiana University Bloomington

2013

Derivatives of Trigonometric Functions

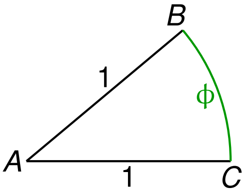
We investigate $\lim_{\phi \to 0} \frac{\sin \phi}{\phi}$

Derivatives of Trigonometric Functions

We investigate $\lim_{\phi \to 0} \frac{\sin \phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)

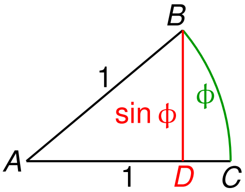
Derivatives of Trigonometric Functions

We investigate $\lim_{\phi \to 0} \frac{\sin \phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)



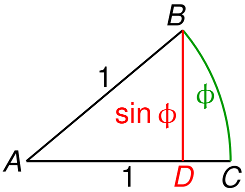
Derivatives of Trigonometric Functions

We investigate $\lim_{\phi \to 0} \frac{\sin \phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)



Derivatives of Trigonometric Functions

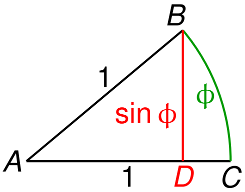
We investigate $\lim_{\phi \to 0} \frac{\sin \phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)



$\sin \phi = |BD| < \phi$

Derivatives of Trigonometric Functions

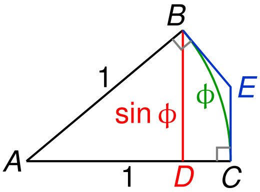
We investigate $\lim_{\phi \to 0} \frac{\sin \phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)



$$\sin \phi = |BD| < \phi \quad \Longrightarrow \quad \frac{\sin \phi}{\phi} < 1$$

Derivatives of Trigonometric Functions

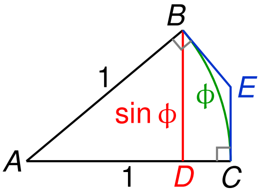
We investigate $\lim_{\phi \to 0} \frac{\sin \phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)



$$\sin \phi = |BD| < \phi \quad \Longrightarrow \quad \frac{\sin \phi}{\phi} < 1$$

Derivatives of Trigonometric Functions

We investigate $\lim_{\phi \to 0} \frac{\sin \phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)

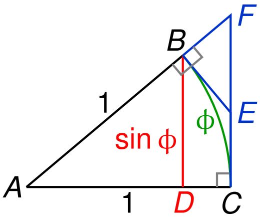


$$\sin \phi = |BD| < \phi \quad \Longrightarrow \quad \frac{\sin \phi}{\phi} < 1$$

$$\phi < |CE| + |EB|$$

Derivatives of Trigonometric Functions

We investigate $\lim_{\phi \to 0} \frac{\sin \phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)

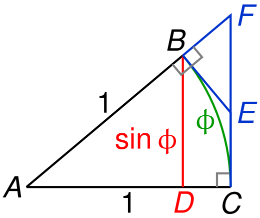


$$\sin \phi = |BD| < \phi \quad \Longrightarrow \quad \frac{\sin \phi}{\phi} < 1$$

$$\phi < |CE| + |EB|$$

Derivatives of Trigonometric Functions

We investigate $\lim_{\phi \to 0} \frac{\sin \phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)

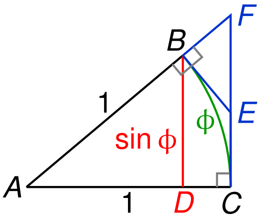


$$\sin \phi = |BD| < \phi \quad \Longrightarrow \quad \frac{\sin \phi}{\phi} < 1$$

$\phi < |CE| + |EB|$ & $|EB| < |EF|$

Derivatives of Trigonometric Functions

We investigate $\lim_{\phi \to 0} \frac{\sin\phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)

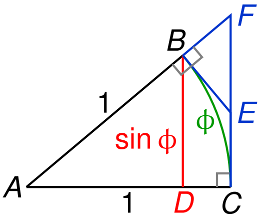


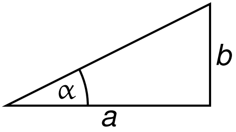
$$\sin\phi = |BD| < \phi \quad \Longrightarrow \quad \frac{\sin\phi}{\phi} < 1$$

$$\phi < |CE| + |EB| \;\;\&\;\; |EB| < |EF| \quad \Longrightarrow \quad \phi < |CE| + |EF| = |CF|$$

Derivatives of Trigonometric Functions

We investigate $\lim_{\phi \to 0} \frac{\sin \phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)



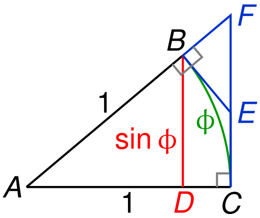


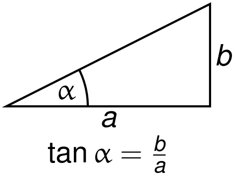
$$\sin \phi = |BD| < \phi \quad \Longrightarrow \quad \frac{\sin \phi}{\phi} < 1$$

$$\phi < |CE| + |EB| \;\;\&\;\; |EB| < |EF| \quad \Longrightarrow \quad \phi < |CE| + |EF| = |CF|$$

Derivatives of Trigonometric Functions

We investigate $\lim_{\phi \to 0} \frac{\sin \phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)



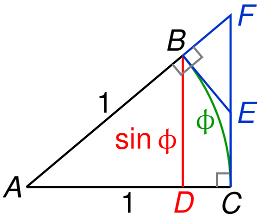


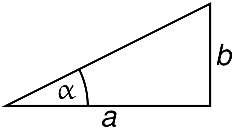
$$\sin \phi = |BD| < \phi \quad \implies \quad \frac{\sin \phi}{\phi} < 1$$

$$\phi < |CE| + |EB| \ \ \& \ \ |EB| < |EF| \quad \implies \quad \phi < |CE| + |EF| = |CF|$$

Derivatives of Trigonometric Functions

We investigate $\lim_{\phi \to 0} \frac{\sin\phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)





$\tan\alpha = \frac{b}{a}$

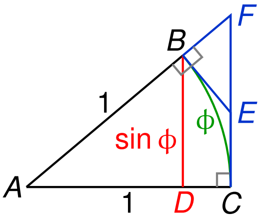
$b = a \cdot \tan\alpha$

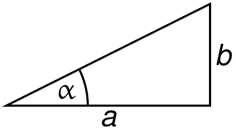
$$\sin\phi = |BD| < \phi \quad \Longrightarrow \quad \frac{\sin\phi}{\phi} < 1$$

$$\phi < |CE| + |EB| \;\;\&\;\; |EB| < |EF| \quad \Longrightarrow \quad \phi < |CE| + |EF| = |CF|$$

Derivatives of Trigonometric Functions

We investigate $\lim_{\phi \to 0} \frac{\sin \phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)





$\tan \alpha = \frac{b}{a}$

$b = a \cdot \tan \alpha$

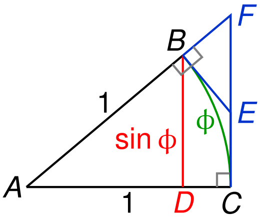
$$\sin \phi = |BD| < \phi \quad \Longrightarrow \quad \frac{\sin \phi}{\phi} < 1$$

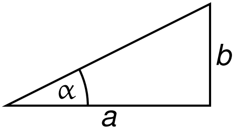
$$\phi < |CE| + |EB| \;\;\&\;\; |EB| < |EF| \quad \Longrightarrow \quad \phi < |CE| + |EF| = |CF|$$

$$\phi < |CF|$$

Derivatives of Trigonometric Functions

We investigate $\lim_{\phi \to 0} \frac{\sin\phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)





$$\tan\alpha = \frac{b}{a}$$
$$b = a \cdot \tan\alpha$$

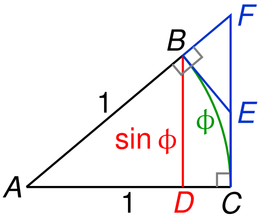
$$\sin\phi = |BD| < \phi \quad \Longrightarrow \quad \frac{\sin\phi}{\phi} < 1$$

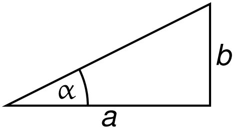
$$\phi < |CE| + |EB| \;\; \& \;\; |EB| < |EF| \quad \Longrightarrow \quad \phi < |CE| + |EF| = |CF|$$

$$\phi < |CF| = 1 \cdot \tan\phi$$

Derivatives of Trigonometric Functions

We investigate $\lim_{\phi \to 0} \frac{\sin \phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)





$$\tan \alpha = \frac{b}{a}$$
$$b = a \cdot \tan \alpha$$

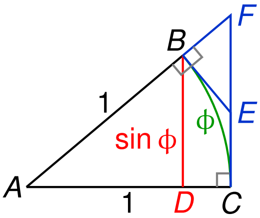
$$\sin \phi = |BD| < \phi \quad \Longrightarrow \quad \frac{\sin \phi}{\phi} < 1$$

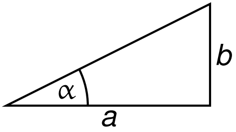
$$\phi < |CE| + |EB| \;\;\&\;\; |EB| < |EF| \quad \Longrightarrow \quad \phi < |CE| + |EF| = |CF|$$

$$\phi < |CF| = 1 \cdot \tan \phi = \frac{\sin \phi}{\cos \phi}$$

Derivatives of Trigonometric Functions

We investigate $\lim_{\phi \to 0} \frac{\sin \phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)





$\tan \alpha = \frac{b}{a}$

$b = a \cdot \tan \alpha$

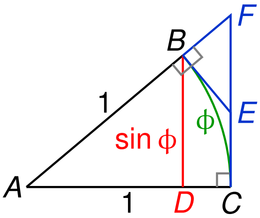
$$\sin \phi = |BD| < \phi \quad \Longrightarrow \quad \frac{\sin \phi}{\phi} < 1$$

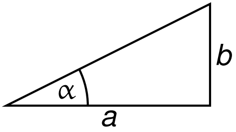
$$\phi < |CE| + |EB| \;\;\&\;\; |EB| < |EF| \quad \Longrightarrow \quad \phi < |CE| + |EF| = |CF|$$

$$\phi < |CF| = 1 \cdot \tan \phi = \frac{\sin \phi}{\cos \phi} \quad \Longrightarrow \quad \cos \phi < \frac{\sin \phi}{\phi} < 1$$

Derivatives of Trigonometric Functions

We investigate $\lim_{\phi \to 0} \frac{\sin\phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)





$\tan\alpha = \frac{b}{a}$

$b = a \cdot \tan\alpha$

$$\sin\phi = |BD| < \phi \quad \Longrightarrow \quad \frac{\sin\phi}{\phi} < 1$$

$$\phi < |CE| + |EB| \;\&\; |EB| < |EF| \quad \Longrightarrow \quad \phi < |CE| + |EF| = |CF|$$

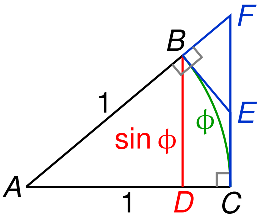
$$\phi < |CF| = 1 \cdot \tan\phi = \frac{\sin\phi}{\cos\phi} \quad \Longrightarrow \quad \cos\phi < \frac{\sin\phi}{\phi} < 1$$

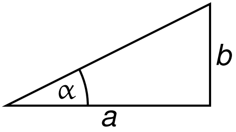
We use the Squeeze Theorem:

$$\lim_{\phi \to 0} \cos\phi = 1 = \lim_{\phi \to 0} 1$$

Derivatives of Trigonometric Functions

We investigate $\lim_{\phi\to 0} \frac{\sin\phi}{\phi}$ (for simplicity assume $0 < \phi < \pi/2$)





$\tan\alpha = \frac{b}{a}$

$b = a \cdot \tan\alpha$

$$\sin\phi = |BD| < \phi \quad\Longrightarrow\quad \frac{\sin\phi}{\phi} < 1$$

$$\phi < |CE| + |EB| \;\;\&\;\; |EB| < |EF| \quad\Longrightarrow\quad \phi < |CE| + |EF| = |CF|$$

$$\phi < |CF| = 1 \cdot \tan\phi = \frac{\sin\phi}{\cos\phi} \quad\Longrightarrow\quad \cos\phi < \frac{\sin\phi}{\phi} < 1$$

We use the Squeeze Theorem:

$$\lim_{\phi\to 0} \cos\phi = 1 = \lim_{\phi\to 0} 1 \quad\Longrightarrow\quad \lim_{\phi\to 0} \frac{\sin\phi}{\phi} = 1$$

Derivatives of Trigonometric Functions

We have the following identities for sin and cos:

$$\begin{aligned}\sin(x+y) &= \sin x \cdot \cos y + \cos x \cdot \sin y \\ \cos(x+y) &= \cos x \cdot \cos y - \sin x \cdot \sin y\end{aligned}$$

Derivatives of Trigonometric Functions

We will prove that

$$\frac{d}{dx}\sin(x) \;=\; \cos(x)$$

Derivatives of Trigonometric Functions

We will prove that

$$\frac{d}{dx}\sin(x) \;=\; \cos(x)$$

We have

$$\frac{d}{dx}\sin(x) =$$

Derivatives of Trigonometric Functions

We will prove that

$$\frac{d}{dx}\sin(x) = \cos(x)$$

We have

$$\frac{d}{dx}\sin(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

Derivatives of Trigonometric Functions

We will prove that

$$\frac{d}{dx}\sin(x) \;=\; \cos(x)$$

We have

$$\begin{aligned}\frac{d}{dx}\sin(x) &= \lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h}\\ &= \lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin(x)}{h}\end{aligned}$$

Derivatives of Trigonometric Functions

We will prove that

$$\frac{d}{dx}\sin(x) \;=\; \cos(x)$$

We have

$$\begin{aligned}
\frac{d}{dx}\sin(x) &= \lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h} \\
&= \lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin(x)}{h} \\
&= \lim_{h\to 0}\left[\frac{\sin x\cos h-\sin(x)}{h}+\frac{\cos x\sin h}{h}\right]
\end{aligned}$$

Derivatives of Trigonometric Functions

We will prove that

$$\frac{d}{dx}\sin(x) = \cos(x)$$

We have

$$\begin{aligned}
\frac{d}{dx}\sin(x) &= \lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h} \\
&= \lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin(x)}{h} \\
&= \lim_{h\to 0}\left[\frac{\sin x\cos h-\sin(x)}{h}+\frac{\cos x\sin h}{h}\right] \\
&= \lim_{h\to 0}\left[\sin x\frac{\cos h-1}{h}+\cos x\frac{\sin h}{h}\right]
\end{aligned}$$

Derivatives of Trigonometric Functions

We will prove that

$$\frac{d}{dx}\sin(x) \;=\; \cos(x)$$

We have

$$\begin{aligned}
\frac{d}{dx}\sin(x) &= \lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h}\\
&= \lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin(x)}{h}\\
&= \lim_{h\to 0}\left[\frac{\sin x\cos h-\sin(x)}{h}+\frac{\cos x\sin h}{h}\right]\\
&= \lim_{h\to 0}\left[\sin x\frac{\cos h-1}{h}+\cos x\frac{\sin h}{h}\right]\\
&= \sin x\cdot\lim_{h\to 0}\frac{\cos h-1}{h}+\cos x\cdot\underbrace{\lim_{h\to 0}\frac{\sin h}{h}}_{1}
\end{aligned}$$

Derivatives of Trigonometric Functions

We will prove that

$$\frac{d}{dx}\sin(x) = \cos(x)$$

We have

$$\begin{aligned}
\frac{d}{dx}\sin(x) &= \lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h} \\
&= \lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin(x)}{h} \\
&= \lim_{h\to 0}\left[\frac{\sin x\cos h-\sin(x)}{h}+\frac{\cos x\sin h}{h}\right] \\
&= \lim_{h\to 0}\left[\sin x\frac{\cos h-1}{h}+\cos x\frac{\sin h}{h}\right] \\
&= \sin x\cdot\lim_{h\to 0}\frac{\cos h-1}{h}+\cos x\cdot\underbrace{\lim_{h\to 0}\frac{\sin h}{h}}_{1}
\end{aligned}$$

Derivatives of Trigonometric Functions

We will prove that

$$\frac{d}{dx}\sin(x) \;=\; \cos(x)$$

We have

$$\frac{d}{dx}\sin(x) = \sin x \cdot \lim_{h\to 0}\frac{\cos h - 1}{h} + \cos x$$

Derivatives of Trigonometric Functions

We will prove that

$$\frac{d}{dx}\sin(x) \;=\; \cos(x)$$

We have

$$\begin{aligned}\frac{d}{dx}\sin(x) &= \sin x \cdot \lim_{h\to 0}\frac{\cos h - 1}{h} + \cos x \\ &= \sin x \cdot \lim_{h\to 0}\left(\frac{\cos h - 1}{h} \cdot \frac{\cos h + 1}{\cos h + 1}\right) + \cos x\end{aligned}$$

Derivatives of Trigonometric Functions

We will prove that

$$\frac{d}{dx}\sin(x) = \cos(x)$$

We have

$$\begin{aligned}
\frac{d}{dx}\sin(x) &= \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \cos x \\
&= \sin x \cdot \lim_{h\to 0} \left(\frac{\cos h - 1}{h} \cdot \frac{\cos h + 1}{\cos h + 1} \right) + \cos x \\
&= \sin x \cdot \lim_{h\to 0} \left(\frac{(\cos h)^2 - 1}{h(\cos h + 1)} \right) + \cos x
\end{aligned}$$

Derivatives of Trigonometric Functions

We will prove that

$$\frac{d}{dx}\sin(x) \;=\; \cos(x)$$

We have

$$\begin{aligned}
\frac{d}{dx}\sin(x) &= \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \cos x \\
&= \sin x \cdot \lim_{h\to 0} \left(\frac{\cos h - 1}{h} \cdot \frac{\cos h + 1}{\cos h + 1} \right) + \cos x \\
&= \sin x \cdot \lim_{h\to 0} \left(\frac{(\cos h)^2 - 1}{h(\cos h + 1)} \right) + \cos x \\
&= \sin x \cdot \lim_{h\to 0} \left(\frac{-(\sin h)^2}{h(\cos h + 1)} \right) + \cos x
\end{aligned}$$

Derivatives of Trigonometric Functions

We will prove that

$$\frac{d}{dx}\sin(x) \;=\; \cos(x)$$

We have

$$\begin{aligned}
\frac{d}{dx}\sin(x) &= \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \cos x \\
&= \sin x \cdot \lim_{h\to 0} \left(\frac{\cos h - 1}{h} \cdot \frac{\cos h + 1}{\cos h + 1} \right) + \cos x \\
&= \sin x \cdot \lim_{h\to 0} \left(\frac{(\cos h)^2 - 1}{h(\cos h + 1)} \right) + \cos x \\
&= \sin x \cdot \lim_{h\to 0} \left(\frac{-(\sin h)^2}{h(\cos h + 1)} \right) + \cos x \\
&= \sin x \cdot \left(\lim_{h\to 0} \frac{\sin h}{h} \cdot \lim_{h\to 0} \frac{-\sin h}{\cos h + 1} \right) + \cos x
\end{aligned}$$

Derivatives of Trigonometric Functions

We will prove that

$$\frac{d}{dx}\sin(x) = \cos(x)$$

We have

$$\begin{aligned}
\frac{d}{dx}\sin(x) &= \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \cos x \\
&= \sin x \cdot \lim_{h\to 0} \left(\frac{\cos h - 1}{h} \cdot \frac{\cos h + 1}{\cos h + 1} \right) + \cos x \\
&= \sin x \cdot \lim_{h\to 0} \left(\frac{(\cos h)^2 - 1}{h(\cos h + 1)} \right) + \cos x \\
&= \sin x \cdot \lim_{h\to 0} \left(\frac{-(\sin h)^2}{h(\cos h + 1)} \right) + \cos x \\
&= \sin x \cdot \left(\lim_{h\to 0} \frac{\sin h}{h} \cdot \lim_{h\to 0} \frac{-\sin h}{\cos h + 1} \right) + \cos x \\
&= \sin x \cdot 1 \cdot 0 + \cos x
\end{aligned}$$

Derivatives of Trigonometric Functions

We will prove that

$$\frac{d}{dx}\sin(x) \;=\; \cos(x)$$

We have

$$\begin{aligned}
\frac{d}{dx}\sin(x) &= \sin x \cdot \lim_{h\to 0}\frac{\cos h - 1}{h} + \cos x \\
&= \sin x \cdot \lim_{h\to 0}\left(\frac{\cos h - 1}{h}\cdot\frac{\cos h + 1}{\cos h + 1}\right) + \cos x \\
&= \sin x \cdot \lim_{h\to 0}\left(\frac{(\cos h)^2 - 1}{h(\cos h + 1)}\right) + \cos x \\
&= \sin x \cdot \lim_{h\to 0}\left(\frac{-(\sin h)^2}{h(\cos h + 1)}\right) + \cos x \\
&= \sin x \cdot \left(\lim_{h\to 0}\frac{\sin h}{h}\cdot\lim_{h\to 0}\frac{-\sin h}{\cos h + 1}\right) + \cos x \\
&= \sin x \cdot 1 \cdot 0 + \cos x = \cos x
\end{aligned}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate

$$f(x) = x^2 \sin x$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate

$$f(x) = x^2 \sin x$$

We have

$$f'(x) =$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate

$$f(x) = x^2 \sin x$$

We have

$$f'(x) = x^2 \frac{d}{dx}\sin x + \sin x \frac{d}{dx}x^2 \quad \text{product rule}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate

$$f(x) = x^2 \sin x$$

We have

$$\begin{aligned} f'(x) &= x^2 \frac{d}{dx}\sin x + \sin x \frac{d}{dx}x^2 \quad \text{product rule} \\ &= x^2 \cos x + 2x \sin x \end{aligned}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate $\tan x$:

$$\frac{d}{dx}\tan x =$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate $\tan x$:

$$\frac{d}{dx}\tan x = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate $\tan x$:

$$\begin{aligned}\frac{d}{dx}\tan x &= \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)\\ &= \frac{\cos x \cdot \frac{d}{dx}\sin x - \sin x \cdot \frac{d}{dx}\cos x}{(\cos x)^2}\end{aligned}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad\qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate $\tan x$:

$$\begin{aligned}
\frac{d}{dx}\tan x &= \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) \\
&= \frac{\cos x \cdot \frac{d}{dx}\sin x - \sin x \cdot \frac{d}{dx}\cos x}{(\cos x)^2} \\
&= \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x}
\end{aligned}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate $\tan x$:

$$\begin{aligned}\frac{d}{dx}\tan x &= \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)\\ &= \frac{\cos x \cdot \frac{d}{dx}\sin x - \sin x \cdot \frac{d}{dx}\cos x}{(\cos x)^2}\\ &= \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x}\\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}\end{aligned}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate $\tan x$:

$$\begin{aligned}
\frac{d}{dx}\tan x &= \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) \\
&= \frac{\cos x \cdot \frac{d}{dx}\sin x - \sin x \cdot \frac{d}{dx}\cos x}{(\cos x)^2} \\
&= \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x} \\
&= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\
&= \frac{1}{\cos^2 x}
\end{aligned}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad\qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate $\tan x$:

$$\begin{aligned}
\frac{d}{dx}\tan x &= \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) \\
&= \frac{\cos x \cdot \frac{d}{dx}\sin x - \sin x \cdot \frac{d}{dx}\cos x}{(\cos x)^2} \\
&= \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x} \\
&= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\
&= \frac{1}{\cos^2 x} = \sec^2 x
\end{aligned}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate the **secant** $\sec x = \frac{1}{\cos x}$:

$$\frac{d}{dx}\sec x = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate the **secant** $\sec x = \frac{1}{\cos x}$:

$$\frac{d}{dx}\sec x = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$$

$$= \frac{\cos x \cdot \frac{d}{dx}1 - 1 \cdot \frac{d}{dx}\cos x}{(\cos x)^2}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate the **secant** $\sec x = \frac{1}{\cos x}$:

$$\begin{aligned}\frac{d}{dx}\sec x &= \frac{d}{dx}\left(\frac{1}{\cos x}\right) \\ &= \frac{\cos x \cdot \frac{d}{dx}1 - 1 \cdot \frac{d}{dx}\cos x}{(\cos x)^2} \\ &= \frac{\sin x}{\cos^2 x}\end{aligned}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate the **secant** $\sec x = \frac{1}{\cos x}$:

$$\begin{aligned}
\frac{d}{dx}\sec x &= \frac{d}{dx}\left(\frac{1}{\cos x}\right) \\
&= \frac{\cos x \cdot \frac{d}{dx}1 - 1 \cdot \frac{d}{dx}\cos x}{(\cos x)^2} \\
&= \frac{\sin x}{\cos^2 x} = \sec x \cdot \tan x
\end{aligned}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate the **cosecant** $\csc x = \frac{1}{\sin x}$:

$$\frac{d}{dx}\csc x = \frac{d}{dx}\left(\frac{1}{\sin x}\right)$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad\qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate the **cosecant** $\csc x = \frac{1}{\sin x}$:

$$\frac{d}{dx}\csc x = \frac{d}{dx}\left(\frac{1}{\sin x}\right)$$

$$= \frac{\sin x \cdot \frac{d}{dx}1 - 1 \cdot \frac{d}{dx}\sin x}{(\sin x)^2}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate the **cosecant** $\csc x = \frac{1}{\sin x}$:

$$\begin{aligned}\frac{d}{dx}\csc x &= \frac{d}{dx}\left(\frac{1}{\sin x}\right)\\ &= \frac{\sin x \cdot \frac{d}{dx}1 - 1 \cdot \frac{d}{dx}\sin x}{(\sin x)^2}\\ &= \frac{-\cos x}{\sin^2 x}\end{aligned}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

Differentiate the **cosecant** $\csc x = \frac{1}{\sin x}$:

$$\begin{aligned}\frac{d}{dx}\csc x &= \frac{d}{dx}\left(\frac{1}{\sin x}\right) \\ &= \frac{\sin x \cdot \frac{d}{dx}1 - 1 \cdot \frac{d}{dx}\sin x}{(\sin x)^2} \\ &= \frac{-\cos x}{\sin^2 x} = -\csc x \cdot \cot x\end{aligned}$$

Derivatives of Trigonometric Functions

Summary: derivatives of trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \frac{1}{\cos^2 x} = \sec^2 x \qquad \frac{d}{dx}\cot x = -\frac{1}{\sin^2 x} = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \cdot \tan x \qquad \frac{d}{dx}\csc x = -\csc x \cdot \cot x$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $g(x) = \sin x$ and $h(x) = x^2$:

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $g(x) = \sin x$ and $h(x) = x^2$:

$$g'(x)$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $g(x) = \sin x$ and $h(x) = x^2$:

$$g'(x) = \cos x$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $g(x) = \sin x$ and $h(x) = x^2$:

$$g'(x) = \cos x$$
$$h'(x)$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $g(x) = \sin x$ and $h(x) = x^2$:

$$g'(x) = \cos x$$
$$h'(x) = 2x$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $g(x) = \sin x$ and $h(x) = x^2$:

$$g'(x) = \cos x$$

$$h'(x) = 2x$$

$$f'(x)$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $g(x) = \sin x$ and $h(x) = x^2$:

$$g'(x) = \cos x$$
$$h'(x) = 2x$$
$$f'(x) = g'(h(x)) \cdot h'(x)$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $g(x) = \sin x$ and $h(x) = x^2$:

$$g'(x) = \cos x$$

$$h'(x) = 2x$$

$$f'(x) = g'(h(x)) \cdot h'(x) = \cos(x^2) \cdot 2x$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $g(x) = \sin x$ and $h(x) = x^2$:

$$g'(x) = \cos x$$

$$h'(x) = 2x$$

$$f'(x) = g'(h(x)) \cdot h'(x) = \cos(x^2) \cdot 2x = 2x \cos(x^2)$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $g(x) = \sin x$ and $h(x) = x^2$:

$$g'(x) = \cos x$$
$$h'(x) = 2x$$
$$f'(x) = g'(h(x)) \cdot h'(x) = \cos(x^2) \cdot 2x = 2x\cos(x^2)$$

Differentiate $g(x) = \sin^2 x$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $g(x) = \sin x$ and $h(x) = x^2$:

$$g'(x) = \cos x$$
$$h'(x) = 2x$$
$$f'(x) = g'(h(x)) \cdot h'(x) = \cos(x^2) \cdot 2x = 2x \cos(x^2)$$

Differentiate $g(x) = \sin^2 x = (\sin x)^2$.

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $g(x) = \sin x$ and $h(x) = x^2$:

$$g'(x) = \cos x$$
$$h'(x) = 2x$$
$$f'(x) = g'(h(x)) \cdot h'(x) = \cos(x^2) \cdot 2x = 2x\cos(x^2)$$

Differentiate $g(x) = \sin^2 x = (\sin x)^2$.

We have $f = g \circ h$ where $g(x) = x^2$ and $h(x) = \sin x$:

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $g(x) = \sin x$ and $h(x) = x^2$:

$$g'(x) = \cos x$$
$$h'(x) = 2x$$
$$f'(x) = g'(h(x)) \cdot h'(x) = \cos(x^2) \cdot 2x = 2x\cos(x^2)$$

Differentiate $g(x) = \sin^2 x = (\sin x)^2$.

We have $f = g \circ h$ where $g(x) = x^2$ and $h(x) = \sin x$:

$$g'(x)$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $g(x) = \sin x$ and $h(x) = x^2$:

$$g'(x) = \cos x$$
$$h'(x) = 2x$$
$$f'(x) = g'(h(x)) \cdot h'(x) = \cos(x^2) \cdot 2x = 2x\cos(x^2)$$

Differentiate $g(x) = \sin^2 x = (\sin x)^2$.

We have $f = g \circ h$ where $g(x) = x^2$ and $h(x) = \sin x$:

$$g'(x) = 2x$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $g(x) = \sin x$ and $h(x) = x^2$:

$$g'(x) = \cos x$$
$$h'(x) = 2x$$
$$f'(x) = g'(h(x)) \cdot h'(x) = \cos(x^2) \cdot 2x = 2x\cos(x^2)$$

Differentiate $g(x) = \sin^2 x = (\sin x)^2$.

We have $f = g \circ h$ where $g(x) = x^2$ and $h(x) = \sin x$:

$$g'(x) = 2x$$
$$h'(x)$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $\quad g(x) = \sin x \quad$ and $\quad h(x) = x^2 \quad$:

$$g'(x) = \cos x$$
$$h'(x) = 2x$$
$$f'(x) = g'(h(x)) \cdot h'(x) = \cos(x^2) \cdot 2x = 2x\cos(x^2)$$

Differentiate $g(x) = \sin^2 x = (\sin x)^2$.

We have $f = g \circ h$ where $\quad g(x) = x^2 \quad$ and $\quad h(x) = \sin x \quad$:

$$g'(x) = 2x$$
$$h'(x) = \cos x$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $g(x) = \sin x$ and $h(x) = x^2$:

$$g'(x) = \cos x$$
$$h'(x) = 2x$$
$$f'(x) = g'(h(x)) \cdot h'(x) = \cos(x^2) \cdot 2x = 2x\cos(x^2)$$

Differentiate $g(x) = \sin^2 x = (\sin x)^2$.

We have $f = g \circ h$ where $g(x) = x^2$ and $h(x) = \sin x$:

$$g'(x) = 2x$$
$$h'(x) = \cos x$$
$$f'(x)$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $g(x) = \sin x$ and $h(x) = x^2$:

$$g'(x) = \cos x$$
$$h'(x) = 2x$$
$$f'(x) = g'(h(x)) \cdot h'(x) = \cos(x^2) \cdot 2x = 2x\cos(x^2)$$

Differentiate $g(x) = \sin^2 x = (\sin x)^2$.

We have $f = g \circ h$ where $g(x) = x^2$ and $h(x) = \sin x$:

$$g'(x) = 2x$$
$$h'(x) = \cos x$$
$$f'(x) = g'(h(x)) \cdot h'(x)$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(x^2)$.

We have $f = g \circ h$ where $\quad g(x) = \sin x \quad$ and $\quad h(x) = x^2 \quad$:

$$g'(x) = \cos x$$
$$h'(x) = 2x$$
$$f'(x) = g'(h(x)) \cdot h'(x) = \cos(x^2) \cdot 2x = 2x\cos(x^2)$$

Differentiate $g(x) = \sin^2 x = (\sin x)^2$.

We have $f = g \circ h$ where $\quad g(x) = x^2 \quad$ and $\quad h(x) = \sin x \quad$:

$$g'(x) = 2x$$
$$h'(x) = \cos x$$
$$f'(x) = g'(h(x)) \cdot h'(x) = 2\sin x \cdot \cos x$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = e^{\sin x}$.

Derivatives of Trigonometric Functions

Differentiate $f(x) = e^{\sin x}$.

We have $f = g \circ h$ where $g(x) = e^x$ and $h(x) = \sin x$:

Derivatives of Trigonometric Functions

Differentiate $f(x) = e^{\sin x}$.

We have $f = g \circ h$ where $g(x) = e^x$ and $h(x) = \sin x$:

$$g'(x)$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = e^{\sin x}$.

We have $f = g \circ h$ where $g(x) = e^x$ and $h(x) = \sin x$:

$$g'(x) = e^x$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = e^{\sin x}$.

We have $f = g \circ h$ where $g(x) = e^x$ and $h(x) = \sin x$:

$$g'(x) = e^x$$
$$h'(x)$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = e^{\sin x}$.

We have $f = g \circ h$ where $g(x) = e^x$ and $h(x) = \sin x$:

$$g'(x) = e^x$$
$$h'(x) = \cos x$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = e^{\sin x}$.

We have $f = g \circ h$ where $g(x) = e^x$ and $h(x) = \sin x$:

$$g'(x) = e^x$$
$$h'(x) = \cos x$$
$$f'(x)$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = e^{\sin x}$.

We have $f = g \circ h$ where $g(x) = e^x$ and $h(x) = \sin x$:

$$g'(x) = e^x$$

$$h'(x) = \cos x$$

$$f'(x) = g'(h(x)) \cdot h'(x)$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = e^{\sin x}$.

We have $f = g \circ h$ where $g(x) = e^x$ and $h(x) = \sin x$:

$$g'(x) = e^x$$

$$h'(x) = \cos x$$

$$f'(x) = g'(h(x)) \cdot h'(x) = e^{\sin x} \cdot \cos x$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(\cos(\tan x))$.

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(\cos(\tan x))$.

$$f'(x) =$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(\cos(\tan x))$.

$$f'(x) = \cos(\ \cos(\tan x)\ ) \cdot \frac{d}{dx}\cos(\tan x)$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(\cos(\tan x))$.

$$f'(x) = \cos(\ \cos(\tan x)\ ) \cdot \frac{d}{dx} \cos(\tan x)$$

$$= \cos(\cos(\tan x)) \cdot (-\sin(\tan x)) \cdot \frac{d}{dx} \tan x$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(\cos(\tan x))$.

$$
\begin{aligned}
f'(x) &= \cos(\ \ \cos(\tan x)\ \ ) \cdot \frac{d}{dx}\cos(\tan x) \\
&= \cos(\cos(\tan x)) \cdot (-\sin(\tan x)) \cdot \frac{d}{dx}\tan x \\
&= -\cos(\cos(\tan x)) \cdot \sin(\tan x) \cdot \frac{1}{\cos^2 x}
\end{aligned}
$$

Derivatives of Trigonometric Functions

Differentiate $f(x) = \sin(\cos(\tan x))$.

$$
\begin{aligned}
f'(x) &= \cos(\ \cos(\tan x)\ ) \cdot \frac{d}{dx} \cos(\tan x) \\
&= \cos(\cos(\tan x)) \cdot (-\sin(\tan x)) \cdot \frac{d}{dx} \tan x \\
&= -\cos(\cos(\tan x)) \cdot \sin(\tan x) \cdot \frac{1}{\cos^2 x}
\end{aligned}
$$

Note that we have applied the chain rule twice!