Calculus M211

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Differentiation Rules: Product Rule

Lets f and g be linear functions:

$$f(x) = ax + b \qquad\qquad g(x) = cx + d$$

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&= a(cx + d) + c(ax + b)
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\end{aligned}$$

We will now see that this also holds for general f and g.

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Assume that f and g are differentiable at x, and define

$$h(x) = f(x) \cdot g(x)$$

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$$h'(x) = \lim_{\Delta x \to 0} \frac{\Delta h}{\Delta x} \qquad \text{where} \qquad \begin{aligned} \Delta h &= h(x + \Delta x) - h(x) \\ \Delta f &= f(x + \Delta x) - f(x) \\ \Delta g &= g(x + \Delta x) - g(x) \end{aligned}$$

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$$\Delta h = f(x + \Delta x) \cdot g(x + \Delta x) - f(x) \cdot g(x)$$

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$$\begin{aligned} \Delta h &= f(x + \Delta x) \cdot g(x + \Delta x) - f(x) \cdot g(x) \\ &= (f(x) + \Delta f) \cdot (g(x) + \Delta g) - f(x) \cdot g(x) \end{aligned}$$

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Then

$$\begin{aligned} \Delta h &= f(x + \Delta x) \cdot g(x + \Delta x) - f(x) \cdot g(x) \\ &= (f(x) + \Delta f) \cdot (g(x) + \Delta g) - f(x) \cdot g(x) \\ &= \Delta f \cdot g(x) + f(x) \cdot \Delta g + \Delta f \cdot \Delta g \end{aligned}$$

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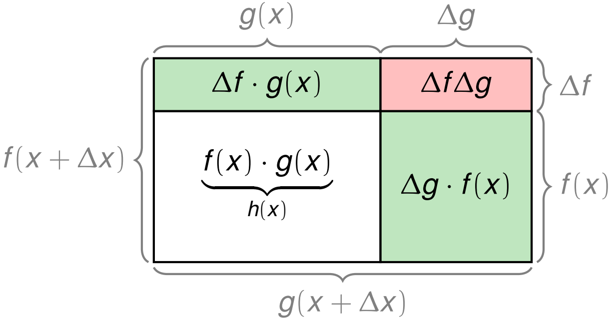

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We compute the limit:

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&= \lim_{\Delta x \to 0} \frac{\Delta f \cdot g(x)}{\Delta x} + \lim_{\Delta x \to 0} \frac{f(x) \cdot \Delta g}{\Delta x} + \lim_{\Delta x \to 0} \frac{\Delta f \cdot \Delta g}{\Delta x} \\
&= g(x) \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} + f(x) \lim_{\Delta x \to 0} \frac{\Delta g}{\Delta x} + \lim_{\Delta x \to 0} \left(\frac{\Delta f}{\Delta x} \cdot \Delta g \right) \\
&= g(x) f'(x) + f(x) g'(x) + \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} \cdot \lim_{\Delta x \to 0} \Delta g \\
&= g(x) f'(x) + f(x) g'(x) + f'(x) \cdot 0
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&= g(x) \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} + f(x) \lim_{\Delta x \to 0} \frac{\Delta g}{\Delta x} + \lim_{\Delta x \to 0} \left(\frac{\Delta f}{\Delta x} \cdot \Delta g \right) \\
&= g(x) f'(x) + f(x) g'(x) + \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} \cdot \lim_{\Delta x \to 0} \Delta g \\
&= g(x) f'(x) + f(x) g'(x) + f'(x) \cdot 0 \\
&= g(x) f'(x) + f(x) g'(x)
\end{aligned}$$

Differentiation Rules: Product Rule

Product Rule

If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) \cdot g(x)] \; = \; f(x)\frac{d}{dx}(g(x)) + g(x) \cdot \frac{d}{dx}(f(x))$$

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In different notation

$$(f \cdot g)'(x) \;=\; f(x) \cdot g'(x) + f'(x) \cdot g(x)$$

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In different notation

$$(f \cdot g)'(x) \; = \; f(x) \cdot g'(x) + f'(x) \cdot g(x)$$

In words:

> The derivative of the product of two function is the first function times the derivative of the second function plus the second function times the derivative of the first.

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Let $f(x) = xe^x$. Find $f'(x)$.

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$$f'(x) = \frac{d}{dx}(x \cdot e^x)$$

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Let $f(x) = xe^x$. Find $f'(x)$.

$$\begin{aligned} f'(x) &= \frac{d}{dx}(x \cdot e^x) \\ &= x\frac{d}{dx}(e^x) + e^x\frac{d}{dx}(x) \end{aligned}$$

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$$\begin{aligned} f'(x) &= \frac{d}{dx}(x \cdot e^x) \\ &= x\frac{d}{dx}(e^x) + e^x\frac{d}{dx}(x) \\ &= xe^x + e^x \end{aligned}$$

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Let $f(x) = xe^x$. Find the n-th derivative $f^{(n)}(x)$.

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Let $f(x) = xe^x$. Find the n-th derivative $f^{(n)}(x)$.

$$f''(x)$$

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$$f'''(x) = \frac{d}{dx}(xe^x + 2e^x)$$

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$$f'''(x) = \frac{d}{dx}(xe^x + 2e^x) = (x+1)e^x + 2e^x = (x+3)e^x$$

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$$f'''(x) = \frac{d}{dx}(xe^x + 2e^x) = (x+1)e^x + 2e^x = (x+3)e^x$$

Thus obviously we have

$$f^{(n)}(x) = (x+n)e^x$$

Differentiation Rules: Product Rule

Differentiate $f(t) = \sqrt{t}(a + bt)$.

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$$f'(t) = \sqrt{t}\frac{d}{dt}(a + bt) + (a + bt)\frac{d}{dt}(\sqrt{t})$$

Differentiation Rules: Product Rule

Differentiate $f(t) = \sqrt{t}(a + bt)$.

$$\begin{aligned} f'(t) &= \sqrt{t}\frac{d}{dt}(a + bt) + (a + bt)\frac{d}{dt}(\sqrt{t}) \\ &= \sqrt{t}b + (a + bt)\frac{1}{2}t^{-\frac{1}{2}}) \end{aligned}$$

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&= \sqrt{t}b + (a + bt)\frac{1}{2}t^{-\frac{1}{2}}) \\
&= b\sqrt{t} + \frac{a + bt}{2\sqrt{t}}
\end{aligned}
$$

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Differentiate $f(t) = \sqrt{t}(a + bt)$.

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f'(t) &= \sqrt{t}\frac{d}{dt}(a + bt) + (a + bt)\frac{d}{dt}(\sqrt{t}) \\
&= \sqrt{t}b + (a + bt)\frac{1}{2}t^{-\frac{1}{2}}) \\
&= b\sqrt{t} + \frac{a + bt}{2\sqrt{t}} \\
&= \frac{2bt}{2\sqrt{t}} + \frac{a + bt}{2\sqrt{t}} \\
&= \frac{a + 3bt}{2\sqrt{t}}
\end{aligned}$$

Differentiation Rules: Product Rule

Differentiate $f(t) = \sqrt{t}(a + bt)$.

$$\begin{aligned} f'(t) &= \sqrt{t}\frac{d}{dt}(a + bt) + (a + bt)\frac{d}{dt}(\sqrt{t}) \\ &= \sqrt{t}b + (a + bt)\frac{1}{2}t^{-\frac{1}{2}}) \\ &= b\sqrt{t} + \frac{a + bt}{2\sqrt{t}} \\ &= \frac{2bt}{2\sqrt{t}} + \frac{a + bt}{2\sqrt{t}} \\ &= \frac{a + 3bt}{2\sqrt{t}} \end{aligned}$$

Alternative solution: first simplify $f(t) = \sqrt{t}(a + bt) = at^{\frac{1}{2}} + bt^{\frac{3}{2}}$.

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Alternative solution: first simplify $f(t) = \sqrt{t}(a+bt) = at^{\frac{1}{2}} + bt^{\frac{3}{2}}$. Then compute the derivative.

Differentiation Rules: Product Rule

Let $f(x) = \sqrt{x} \cdot g(x)$ where $g(4) = 2$ and $g'(4) = 3$. Find $f'(4)$.

$$f'(x)$$

Differentiation Rules: Product Rule

Let $f(x) = \sqrt{x} \cdot g(x)$ where $g(4) = 2$ and $g'(4) = 3$. Find $f'(4)$.

$$f'(x) = \sqrt{x} \cdot \frac{d}{dx} g(x) + g(x) \cdot \frac{d}{dx} \sqrt{x}$$

Differentiation Rules: Product Rule

Let $f(x) = \sqrt{x} \cdot g(x)$ where $g(4) = 2$ and $g'(4) = 3$. Find $f'(4)$.

$$
\begin{aligned}
f'(x) &= \sqrt{x} \cdot \frac{d}{dx} g(x) + g(x) \cdot \frac{d}{dx} \sqrt{x} \\
&= \sqrt{x} \cdot g'(x) + g(x) \cdot \frac{1}{2} x^{-\frac{1}{2}}
\end{aligned}
$$

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Thus

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Thus

$$f'(4) = \sqrt{4} \cdot g'(4) + g(4) \cdot \frac{1}{2\sqrt{4}}$$

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Thus

$$\begin{aligned} f'(4) &= \sqrt{4} \cdot g'(4) + g(4) \cdot \frac{1}{2\sqrt{4}} \\ &= 2 \cdot 3 + 2 \cdot \frac{1}{2 \cdot 2} \end{aligned}$$

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Differentiation Rules: Quotient Rule

Assume that f and g are differentiable at x, and define

$$h(x) = \frac{f(x)}{g(x)}$$

Differentiation Rules: Quotient Rule

Assume that f and g are differentiable at x, and define

$$h(x) = \frac{f(x)}{g(x)} \qquad \begin{aligned} \Delta h &= h(x + \Delta x) - h(x) \\ \Delta f &= f(x + \Delta x) - f(x) \\ \Delta g &= g(x + \Delta x) - g(x) \end{aligned}$$

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We try to find the derivative of h at x:

$$\Delta h = h(x + \Delta x) - h(x)$$

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Differentiation Rules: Quotient Rule

Quotient Rule

If f and g are both differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\cdot\frac{d}{dx}(f(x)) - f(x)\cdot\frac{d}{dx}(g(x))}{[g(x)]^2}$$

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In different notation

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{g(x)^2}$$

Differentiation Rules: Quotient Rule

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In different notation

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{g(x)^2}$$

In words:

> The derivative of a quotient is the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

Differentiation Rules: Quotient Rule

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{g(x)^2}$$

Differentiation Rules: Quotient Rule

$$\left(\frac{f}{g}\right)'(x) \;=\; \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{g(x)^2}$$

Let

$$f(x) = \frac{x^2 + x - 2}{x^3 + 6}$$

Then

$$f'(x)$$

Differentiation Rules: Quotient Rule

$$\left(\frac{f}{g}\right)'(x) \; = \; \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{g(x)^2}$$

Let

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Then

$$f'(x) = \frac{(x^3 + 6) \cdot \frac{d}{dx}(x^2 + x - 2) - (x^2 + x - 2) \cdot \frac{d}{dx}(x^3 + 6)}{(x^3 + 6)^2}$$

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Let

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Then

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Differentiation Rules: Quotient Rule

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{g(x)^2}$$

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Then

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Differentiation Rules: Quotient Rule

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{g(x)^2}$$

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Differentiation Rules: Quotient Rule

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{g(x)^2}$$

Find an equation to the tangent line to

$$f(x) = \frac{e^x}{1 + x^2}$$

at point $(1, \frac{e}{2})$.

Differentiation Rules: Quotient Rule

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$$= \frac{x^2e^x - 2xe^x + e^x}{(1+x^2)^2}$$

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$$= \frac{x^2e^x - 2xe^x + e^x}{(1+x^2)^2} = \frac{(x-1)^2e^x}{(1+x^2)^2}$$

Differentiation Rules: Quotient Rule

$$\left(\frac{f}{g}\right)'(x) \;=\; \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{g(x)^2}$$

Find an equation to the tangent line to

$$f(x) = \frac{e^x}{1+x^2}$$

at point $(1, \frac{e}{2})$. We have

$$f'(x) = \frac{(1+x^2)\cdot \frac{d}{dx}(e^x) - e^x \frac{d}{dx}(1+x^2)}{(1+x^2)^2} = \frac{(1+x^2)e^x - e^x\cdot 2x}{(1+x^2)^2}$$

$$= \frac{x^2e^x - 2xe^x + e^x}{(1+x^2)^2} = \frac{(x-1)^2e^x}{(1+x^2)^2}$$

Thus the slope of the tangent is $f'(1) = 0$.

Differentiation Rules: Quotient Rule

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Thus the slope of the tangent is $f'(1) = 0$. Hence the tangent is

$$y = \frac{e}{2}$$

Differentiation Rules: Quotient Rule

Sometimes it is easier to simplify than apply the quotient rule:

$$f(x) = \frac{3x^2 + 2\sqrt{x}}{x}$$

Differentiation Rules: Quotient Rule

Sometimes it is easier to simplify than apply the quotient rule:

$$f(x) = \frac{3x^2 + 2\sqrt{x}}{x}$$

Instead of applying the quotient rule, we can simplify to

$$f(x) = 3x + 2x^{-\frac{1}{2}}$$

which is easier to differentiate.

Differentiation Rules: Chain Rule

Suppose we want to differentiate

$$f(x) = \sqrt{x^2 + 1}$$

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That is:

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We need a rule that gives us f' from g' and h'...

Differentiation Rules: Chain Rule

Chain Rule

If g is differentiable at x and f at $g(x)$, then

$$h = f \circ g \qquad \text{or equivalently} \qquad h(x) = f(g(x))$$

is differentiable at x and

$$h'(x) \;=\; (f \circ g)'(x) \;=\; f'(g(x)) \cdot g'(x)$$

Differentiation Rules: Chain Rule

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$$h'(x) \;=\; (f \circ g)'(x) \;=\; f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx} \underbrace{f}_{\text{outer function}} \underbrace{(\;g(x)\;)}_{\text{evaluated at inner function}} = \underbrace{f'}_{\text{derivative of outer function}} \underbrace{(\;g(x)\;)}_{\text{evaluated at inner function}} \cdot \underbrace{g'(x)}_{\text{derivative of inner function}}$$

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In words:

The derivative of the composition of f and g is the derivative of f at $g(x)$ times the derivative of g at x.

Differentiation Rules: Chain Rule

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Intuition with rates of change:

Differentiation Rules: Chain Rule

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Intuition with rates of change:

- If $g'(x) = N$. Then $g(x)$ changes N times as much as x.

Differentiation Rules: Chain Rule

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Intuition with rates of change:

- If $g'(x) = N$. Then $g(x)$ changes N times as much as x.
- If $f'(g(x)) = M$. Then $f(x)$ changes M times as much as $g(x)$.

Differentiation Rules: Chain Rule

Chain Rule

If g is differentiable at x and f at $g(x)$, then

$$h = f \circ g \qquad \text{or equivalently} \qquad h(x) = f(g(x))$$

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Intuition with rates of change:

- If $g'(x) = N$. Then $g(x)$ changes N times as much as x.
- If $f'(g(x)) = M$. Then $f(x)$ changes M times as much as $g(x)$.
- Thus $(f \circ g)(x) = f(g(x))$ changes $N \cdot M$ times as much as x.

Differentiation Rules: Chain Rule

$$(f \circ g)'(x) \;=\; f'(g(x)) \cdot g'(x)$$

Let $f(x) = \sqrt{x^2 + 1}$. Find $f'(x)$.

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$$f(x) = g(h(x)) \qquad \text{where} \qquad g(x) = \sqrt{x} \qquad h(x) = x^2 + 1$$

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Differentiation Rules: Chain Rule

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Hence:

$$f'(x) = (g \circ h)'(x) =$$

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Hence:

$$f'(x) = (g \circ h)'(x) = \frac{1}{2\sqrt{x^2+1}} \cdot 2x = \frac{x}{\sqrt{x^2+1}}$$

Differentiation Rules: Chain Rule

$$(f \circ g)'(x) \;=\; f'(g(x)) \cdot g'(x)$$

Differentiate $f(x) = (x^3 - 1)^{100}$.

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Differentiation Rules: Chain Rule

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Differentiate $f(x) = (x^3 - 1)^{100}$.

We have that

$$f(x) = g(h(x)) \quad \text{where} \quad g(x) = x^{100} \quad h(x) = x^3 - 1$$

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$$g'(x) = 100x^{99} \qquad h'(x) =$$

Differentiation Rules: Chain Rule

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Hence:

$$f'(x) = (g \circ h)'(x) = 100(x^3 - 1)^{99} \cdot 3x^2$$

Differentiation Rules: Chain Rule

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Differentiate $f(x) = (x^3 - 1)^{100}$.

We have that

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and

$$g'(x) = 100x^{99} \qquad h'(x) = 3x^2$$

Hence:

$$\begin{aligned} f'(x) = (g \circ h)'(x) &= 100(x^3 - 1)^{99} \cdot 3x^2 \\ &= 300x^2 \cdot (x^3 - 1)^{99} \end{aligned}$$

Differentiation Rules: Chain Rule

$$(f \circ g)'(x) \;=\; f'(g(x)) \cdot g'(x)$$

Differentiation Rules: Chain Rule

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

In general (combining the power and chain rule) we have:

$$\frac{d}{dx}[g(x)]^n =$$

if $g(x)$ is differentiable.

Differentiation Rules: Chain Rule

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In general (combining the power and chain rule) we have:

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Differentiate

$$f(x) = \frac{1}{\sqrt[3]{x^2 + x + 1}}$$

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Differentiate

$$f(x) = \frac{1}{\sqrt[3]{x^2 + x + 1}}$$

We have

$$f(x) = (x^2 + x + 1)^{-\frac{1}{3}}$$

$$f'(x) = -\frac{1}{3} \cdot (x^2 + x + 1)^{-\frac{4}{3}} \cdot (2x + 1)$$

Differentiation Rules: Chain Rule

$$(f \circ g)'(x) \;=\; f'(g(x)) \cdot g'(x)$$

Differentiate

$$f(x) = \left(\frac{x-2}{2x+1}\right)^9$$

Differentiation Rules: Chain Rule

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

Differentiate

$$f(x) = \left(\frac{x-2}{2x+1}\right)^9$$

We have

$$f'(x) =$$

Differentiation Rules: Chain Rule

$$(f \circ g)'(x) \;=\; f'(g(x)) \cdot g'(x)$$

Differentiate

$$f(x) = \left(\frac{x-2}{2x+1}\right)^9$$

We have

$$f'(x) = 9\left(\frac{x-2}{2x+1}\right)^8 \frac{d}{dx}\frac{x-2}{2x+1}$$

Differentiation Rules: Chain Rule

$$(f \circ g)'(x) \;=\; f'(g(x)) \cdot g'(x)$$

Differentiate

$$f(x) = \left(\frac{x-2}{2x+1}\right)^9$$

We have

$$\begin{aligned} f'(x) &= 9\left(\frac{x-2}{2x+1}\right)^8 \frac{d}{dx}\frac{x-2}{2x+1} \\ &= 9\left(\frac{x-2}{2x+1}\right)^8 \frac{(2x+1)\cdot 1-(x-2)\cdot 2}{(2x+1)^2} \end{aligned}$$

Differentiation Rules: Chain Rule

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Differentiation Rules: Chain Rule

$$(f \circ g)'(x) \;=\; f'(g(x)) \cdot g'(x)$$

Differentiate

$$f(x) = \left(\frac{x-2}{2x+1}\right)^9$$

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Differentiation Rules: Chain Rule

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

Differentiate

$$f(x) = (2x + 1)^5 \cdot (x^3 - x + 1)^4$$

Differentiation Rules: Chain Rule

$$(f \circ g)'(x) \;=\; f'(g(x)) \cdot g'(x)$$

Differentiate

$$f(x) = (2x+1)^5 \cdot (x^3 - x + 1)^4$$

We have

$$f'(x) =$$

Differentiation Rules: Chain Rule

$$(f \circ g)'(x) \;=\; f'(g(x)) \cdot g'(x)$$

Differentiate

$$f(x) = (2x+1)^5 \cdot (x^3 - x + 1)^4$$

We have

$$\begin{aligned} f'(x) = (2x+1)^5 \cdot \frac{d}{dx}[(x^3 - x + 1)^4] \\ + (x^3 - x + 1)^4 \cdot \frac{d}{dx}[(2x+1)^5] \end{aligned}$$

Differentiation Rules: Chain Rule

$$(f \circ g)'(x) \;=\; f'(g(x)) \cdot g'(x)$$

Differentiate

$$f(x) = (2x+1)^5 \cdot (x^3 - x + 1)^4$$

We have

$$\begin{aligned} f'(x) &= (2x+1)^5 \cdot \frac{d}{dx}[(x^3-x+1)^4] \\ &\quad + (x^3-x+1)^4 \cdot \frac{d}{dx}[(2x+1)^5] \\ &= (2x+1)^5 \cdot 4(x^3-x+1)^3 \cdot (3x^2-1) \\ &\quad + (x^3-x+1)^4 \cdot 5(2x+1)^4 \cdot 2 \end{aligned}$$

Differentiation Rules: Chain Rule

$$(f \circ g)'(x) \;=\; f'(g(x)) \cdot g'(x)$$

Use

$$\frac{d}{dx} e^x = e^x$$

and the chain rule to prove

$$\frac{d}{dx} a^x = \ln a \cdot a^x$$

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We have

$$a^x \; = \; (e^{\ln a})^x$$

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We have

$$a^x \;=\; (e^{\ln a})^x = e^{\ln a \cdot x}$$

and $f = g \circ h$ where $g(x) =$ and $h(x) =$.

Differentiation Rules: Chain Rule

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and $f = g \circ h$ where $g(x) = e^x$ and $h(x) = \ln a \cdot x$. Thus

$$f'(x) = g'(h(x)) \cdot h'(x) =$$

Differentiation Rules: Chain Rule

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We have

$$a^x \; = \; (e^{\ln a})^x = e^{\ln a \cdot x}$$

and $f = g \circ h$ where $g(x) = e^x$ and $h(x) = \ln a \cdot x$. Thus

$$f'(x) = g'(h(x)) \cdot h'(x) = e^{\ln a \cdot x} \cdot \ln a$$

Differentiation Rules: Chain Rule

$$(f \circ g)'(x) \; = \; f'(g(x)) \cdot g'(x)$$

Use

$$\frac{d}{dx} e^x = e^x$$

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$$\frac{d}{dx} a^x = \ln a \cdot a^x$$

We have

$$a^x \; = \; (e^{\ln a})^x = e^{\ln a \cdot x}$$

and $f = g \circ h$ where $g(x) = e^x$ and $h(x) = \ln a \cdot x$. Thus

$$f'(x) = g'(h(x)) \cdot h'(x) = e^{\ln a \cdot x} \cdot \ln a = \ln a \cdot a^x$$

Summary of Differentiation Rules

$$\frac{d}{dx}(c) = 0 \qquad \frac{d}{dx}(x^r) = r\,x^{r-1}$$

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(a^x) = \ln a \cdot a^x$$

$$(f+g)' = f' + g' \qquad (f-g)' = f' - g'$$

$$(cf)' = cf'$$

$$(fg)' = f'g + fg' \qquad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$