Calculus M211

Jörg Endrullis

Indiana University Bloomington

2013

Derivatives of Basic Functions

The derivative of a constant function

$$\frac{d}{dx}(c) = 0$$

Derivatives of Basic Functions

The derivative of a constant function

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x) = 1$$

Derivatives of Basic Functions

The derivative of a constant function

$$\frac{d}{dx}(c) \;=\; 0$$

$$\frac{d}{dx}(x) \;=\; 1$$

If n is any real number, then

$$\frac{d}{dx}(x^n) \;=\; n\,x^{n-1}$$

Derivatives of Basic Functions

The derivative of a constant function

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x) = 1$$

If n is any real number, then

$$\frac{d}{dx}(x^n) = n\,x^{n-1}$$

Differentiate the following functions:

Derivatives of Basic Functions

The derivative of a constant function

$$\frac{d}{dx}(c) \;=\; 0$$

$$\frac{d}{dx}(x) \;=\; 1$$

If n is any real number, then

$$\frac{d}{dx}(x^n) \;=\; n\,x^{n-1}$$

Differentiate the following functions:

- $\frac{d}{dx}(x^7) \quad =$

Derivatives of Basic Functions

The derivative of a constant function

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x) = 1$$

If n is any real number, then

$$\frac{d}{dx}(x^n) = n\,x^{n-1}$$

Differentiate the following functions:

- $\frac{d}{dx}(x^7) = 7x^6$

Derivatives of Basic Functions

The derivative of a constant function

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x) = 1$$

If n is any real number, then

$$\frac{d}{dx}(x^n) = n\,x^{n-1}$$

Differentiate the following functions:

- $\frac{d}{dx}(x^7) = 7x^6$
- $\frac{d}{dx}\left(\frac{1}{x^2}\right) =$

Derivatives of Basic Functions

The derivative of a constant function

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x) = 1$$

If n is any real number, then

$$\frac{d}{dx}(x^n) = n\,x^{n-1}$$

Differentiate the following functions:

- $\frac{d}{dx}(x^7) = 7x^6$
- $\frac{d}{dx}\left(\frac{1}{x^2}\right) = \frac{d}{dx}(x^{-2}) =$

Derivatives of Basic Functions

The derivative of a constant function

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x) = 1$$

If n is any real number, then

$$\frac{d}{dx}(x^n) = n\,x^{n-1}$$

Differentiate the following functions:

- $\frac{d}{dx}(x^7) = 7x^6$
- $\frac{d}{dx}(\frac{1}{x^2}) = \frac{d}{dx}(x^{-2}) = -2\,x^{-3}$

Derivatives of Basic Functions

The derivative of a constant function

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x) = 1$$

If n is any real number, then

$$\frac{d}{dx}(x^n) = n\,x^{n-1}$$

Differentiate the following functions:

- $\frac{d}{dx}(x^7) = 7x^6$
- $\frac{d}{dx}\left(\frac{1}{x^2}\right) = \frac{d}{dx}(x^{-2}) = -2\,x^{-3} = -\frac{2}{x^3}$

Derivatives of Basic Functions

The derivative of a constant function

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x) = 1$$

If n is any real number, then

$$\frac{d}{dx}(x^n) = n\,x^{n-1}$$

Differentiate the following functions:

- $\frac{d}{dx}(x^7) = 7x^6$
- $\frac{d}{dx}\left(\frac{1}{x^2}\right) = \frac{d}{dx}(x^{-2}) = -2\,x^{-3} = -\frac{2}{x^3}$
- $\frac{d}{dx}(\sqrt[3]{x^2}) =$

Derivatives of Basic Functions

The derivative of a constant function

$$\frac{d}{dx}(c) \;=\; 0$$

$$\frac{d}{dx}(x) \;=\; 1$$

If n is any real number, then

$$\frac{d}{dx}(x^n) \;=\; n\,x^{n-1}$$

Differentiate the following functions:

- $\frac{d}{dx}(x^7) \;=\; 7x^6$
- $\frac{d}{dx}(\frac{1}{x^2}) \;=\; \frac{d}{dx}(x^{-2}) \;=\; -2\,x^{-3} \;=\; -\frac{2}{x^3}$
- $\frac{d}{dx}(\sqrt[3]{x^2}) \;=\; \frac{d}{dx}(x^{\frac{2}{3}}) \;=$

Derivatives of Basic Functions

The derivative of a constant function

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x) = 1$$

If n is any real number, then

$$\frac{d}{dx}(x^n) = n\,x^{n-1}$$

Differentiate the following functions:

- $\frac{d}{dx}(x^7) = 7x^6$
- $\frac{d}{dx}\left(\frac{1}{x^2}\right) = \frac{d}{dx}(x^{-2}) = -2\,x^{-3} = -\frac{2}{x^3}$
- $\frac{d}{dx}(\sqrt[3]{x^2}) = \frac{d}{dx}(x^{\frac{2}{3}}) = \frac{2}{3}\,x^{\frac{2}{3}-1} = \frac{2}{3}\,x^{-\frac{1}{3}}$

Derivatives of Basic Functions

The **normal line** is perpendicular to the tangent.

Derivatives of Basic Functions

The **normal line** is perpendicular to the tangent.

If the tangent has slope m, then the normal line has slope $-\frac{1}{m}$.

Derivatives of Basic Functions

The **normal line** is perpendicular to the tangent.

If the tangent has slope m, then the normal line has slope $-\frac{1}{m}$.

Find equations for the tangent and normal line to $x\sqrt{x}$ at $(1, 1)$.

The **normal line** is perpendicular to the tangent.

If the tangent has slope m, then the normal line has slope $-\frac{1}{m}$.

Find equations for the tangent and normal line to $x\sqrt{x}$ at $(1, 1)$.

$$f'(x) = \frac{d}{dx}(x\sqrt{x})$$

Derivatives of Basic Functions

The **normal line** is perpendicular to the tangent.

If the tangent has slope m, then the normal line has slope $-\frac{1}{m}$.

Find equations for the tangent and normal line to $x\sqrt{x}$ at $(1, 1)$.

$$f'(x) = \frac{d}{dx}(x\sqrt{x}) = \frac{d}{dx}(x^{1.5})$$

Derivatives of Basic Functions

The **normal line** is perpendicular to the tangent.

If the tangent has slope m, then the normal line has slope $-\frac{1}{m}$.

Find equations for the tangent and normal line to $x\sqrt{x}$ at $(1, 1)$.

$$f'(x) = \frac{d}{dx}(x\sqrt{x}) = \frac{d}{dx}(x^{1.5}) = 1.5x^{.5}$$

Derivatives of Basic Functions

The **normal line** is perpendicular to the tangent.

If the tangent has slope m, then the normal line has slope $-\frac{1}{m}$.

Find equations for the tangent and normal line to $x\sqrt{x}$ at $(1, 1)$.

$$f'(x) = \frac{d}{dx}(x\sqrt{x}) = \frac{d}{dx}(x^{1.5}) = 1.5x^{.5} = \frac{3}{2}\sqrt{x}$$

Derivatives of Basic Functions

The **normal line** is perpendicular to the tangent.

If the tangent has slope m, then the normal line has slope $-\frac{1}{m}$.

Find equations for the tangent and normal line to $x\sqrt{x}$ at $(1, 1)$.

$$f'(x) = \frac{d}{dx}(x\sqrt{x}) = \frac{d}{dx}(x^{1.5}) = 1.5x^{.5} = \frac{3}{2}\sqrt{x}$$

The slope of the tangent at $(1, 1)$ is $\frac{3}{2}$.

Derivatives of Basic Functions

The **normal line** is perpendicular to the tangent.

If the tangent has slope m, then the normal line has slope $-\frac{1}{m}$.

Find equations for the tangent and normal line to $x\sqrt{x}$ at $(1, 1)$.

$$f'(x) = \frac{d}{dx}(x\sqrt{x}) = \frac{d}{dx}(x^{1.5}) = 1.5x^{.5} = \frac{3}{2}\sqrt{x}$$

The slope of the tangent at $(1, 1)$ is $\frac{3}{2}$. Hence the tangent is

$$y - 1 = \frac{3}{2}(x - 1) \qquad\qquad y = \frac{3}{2}x - \frac{1}{2}$$

Derivatives of Basic Functions

The **normal line** is perpendicular to the tangent.

If the tangent has slope m, then the normal line has slope $-\frac{1}{m}$.

Find equations for the tangent and normal line to $x\sqrt{x}$ at $(1,1)$.

$$f'(x) = \frac{d}{dx}(x\sqrt{x}) = \frac{d}{dx}(x^{1.5}) = 1.5x^{.5} = \frac{3}{2}\sqrt{x}$$

The slope of the tangent at $(1,1)$ is $\frac{3}{2}$. Hence the tangent is

$$y - 1 = \frac{3}{2}(x-1) \qquad\qquad y = \frac{3}{2}x - \frac{1}{2}$$

The slope of the normal at $(1,1)$ is $-1/\frac{3}{2} = -\frac{2}{3}$.

Derivatives of Basic Functions

The **normal line** is perpendicular to the tangent.

If the tangent has slope m, then the normal line has slope $-\frac{1}{m}$.

Find equations for the tangent and normal line to $x\sqrt{x}$ at $(1, 1)$.

$$f'(x) = \frac{d}{dx}(x\sqrt{x}) = \frac{d}{dx}(x^{1.5}) = 1.5x^{.5} = \frac{3}{2}\sqrt{x}$$

The slope of the tangent at $(1, 1)$ is $\frac{3}{2}$. Hence the tangent is

$$y - 1 = \frac{3}{2}(x - 1) \qquad\qquad y = \frac{3}{2}x - \frac{1}{2}$$

The slope of the normal at $(1, 1)$ is $-1/\frac{3}{2} = -\frac{2}{3}$. Hence the normal is

$$y - 1 = -\frac{2}{3}(x - 1) \qquad\qquad y = -\frac{2}{3}x + \frac{5}{3}$$

Derivatives of Basic Functions

Constant Multiple Rule

If c is a constant and f is differentiable, then

$$\frac{d}{dx}[c\,f(x)] \;=\; c \cdot \frac{d}{dx} f(x)$$

Derivatives of Basic Functions

Constant Multiple Rule

If c is a constant and f is differentiable, then

$$\frac{d}{dx}[c\,f(x)] \;=\; c \cdot \frac{d}{dx} f(x)$$

Sum Rule

If f and g are differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] \;=\; \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$$

Derivatives of Basic Functions

Constant Multiple Rule

If c is a constant and f is differentiable, then

$$\frac{d}{dx}[c\,f(x)] \;=\; c \cdot \frac{d}{dx} f(x)$$

Sum Rule

If f and g are differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] \;=\; \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$$

Difference Rule

If f and g are differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] \;=\; \frac{d}{dx} f(x) - \frac{d}{dx} g(x)$$

Derivatives of Basic Functions

Compute the following derivative:

$$\frac{d}{dx}(12x^5 - 10x^3 - 6x + 5)$$

Derivatives of Basic Functions

Compute the following derivative:

$$\frac{d}{dx}(12x^5 - 10x^3 - 6x + 5)$$
$$= 12\frac{d}{dx}(x^5) - 10\frac{d}{dx}(x^3) - 6\frac{d}{dx}(x) + \frac{d}{dx}(5)$$

Derivatives of Basic Functions

Compute the following derivative:

$$\frac{d}{dx}(12x^5 - 10x^3 - 6x + 5)$$
$$= 12\frac{d}{dx}(x^5) - 10\frac{d}{dx}(x^3) - 6\frac{d}{dx}(x) + \frac{d}{dx}(5)$$
$$= 12 \cdot 5x^4 - 10 \cdot 3x^2 - 6 \cdot 1 + 0$$

Derivatives of Basic Functions

Compute the following derivative:

$$\begin{aligned}
&\frac{d}{dx}(12x^5 - 10x^3 - 6x + 5) \\
&\qquad = 12\frac{d}{dx}(x^5) - 10\frac{d}{dx}(x^3) - 6\frac{d}{dx}(x) + \frac{d}{dx}(5) \\
&\qquad = 12 \cdot 5x^4 - 10 \cdot 3x^2 - 6 \cdot 1 + 0 = 60x^4 - 30x^2 - 6
\end{aligned}$$

Derivatives of Basic Functions

Compute the following derivative:

$$\frac{d}{dx}(12x^5 - 10x^3 - 6x + 5)$$
$$= 12\frac{d}{dx}(x^5) - 10\frac{d}{dx}(x^3) - 6\frac{d}{dx}(x) + \frac{d}{dx}(5)$$
$$= 12 \cdot 5x^4 - 10 \cdot 3x^2 - 6 \cdot 1 + 0 = 60x^4 - 30x^2 - 6$$

The motion of a particle is given by:

- $s(t) = 2t^3 - 5t^2 + 3t + 4$ (t is in seconds, and $s(t)$ in cm)

Find the acceleration function, and the acceleration after $2s$.

Derivatives of Basic Functions

Compute the following derivative:

$$\frac{d}{dx}(12x^5 - 10x^3 - 6x + 5)$$
$$= 12\frac{d}{dx}(x^5) - 10\frac{d}{dx}(x^3) - 6\frac{d}{dx}(x) + \frac{d}{dx}(5)$$
$$= 12 \cdot 5x^4 - 10 \cdot 3x^2 - 6 \cdot 1 + 0 = 60x^4 - 30x^2 - 6$$

The motion of a particle is given by:

- $s(t) = 2t^3 - 5t^2 + 3t + 4$ (t is in seconds, and $s(t)$ in cm)

Find the acceleration function, and the acceleration after $2s$.

$$v(t) = \frac{d}{dt}s(t) =$$

Derivatives of Basic Functions

Compute the following derivative:

$$\frac{d}{dx}(12x^5 - 10x^3 - 6x + 5)$$
$$= 12\frac{d}{dx}(x^5) - 10\frac{d}{dx}(x^3) - 6\frac{d}{dx}(x) + \frac{d}{dx}(5)$$
$$= 12 \cdot 5x^4 - 10 \cdot 3x^2 - 6 \cdot 1 + 0 = 60x^4 - 30x^2 - 6$$

The motion of a particle is given by:

- $s(t) = 2t^3 - 5t^2 + 3t + 4$ (t is in seconds, and $s(t)$ in cm)

Find the acceleration function, and the acceleration after $2s$.

$$v(t) = \frac{d}{dt}s(t) = 6t^2 - 10t + 3$$

Derivatives of Basic Functions

Compute the following derivative:

$$\frac{d}{dx}(12x^5 - 10x^3 - 6x + 5)$$
$$= 12\frac{d}{dx}(x^5) - 10\frac{d}{dx}(x^3) - 6\frac{d}{dx}(x) + \frac{d}{dx}(5)$$
$$= 12 \cdot 5x^4 - 10 \cdot 3x^2 - 6 \cdot 1 + 0 = 60x^4 - 30x^2 - 6$$

The motion of a particle is given by:

- $s(t) = 2t^3 - 5t^2 + 3t + 4$ (t is in seconds, and $s(t)$ in cm)

Find the acceleration function, and the acceleration after $2s$.

$$v(t) = \frac{d}{dt}s(t) = 6t^2 - 10t + 3$$
$$a(t) = \frac{d}{dt}v(t) =$$

Derivatives of Basic Functions

Compute the following derivative:

$$\frac{d}{dx}(12x^5 - 10x^3 - 6x + 5)$$
$$= 12\frac{d}{dx}(x^5) - 10\frac{d}{dx}(x^3) - 6\frac{d}{dx}(x) + \frac{d}{dx}(5)$$
$$= 12 \cdot 5x^4 - 10 \cdot 3x^2 - 6 \cdot 1 + 0 = 60x^4 - 30x^2 - 6$$

The motion of a particle is given by:

- $s(t) = 2t^3 - 5t^2 + 3t + 4$ (t is in seconds, and $s(t)$ in cm)

Find the acceleration function, and the acceleration after $2s$.

$$v(t) = \frac{d}{dt}s(t) = 6t^2 - 10t + 3$$
$$a(t) = \frac{d}{dt}v(t) = 12t - 10$$

Derivatives of Basic Functions

Compute the following derivative:

$$\frac{d}{dx}(12x^5 - 10x^3 - 6x + 5)$$
$$= 12\frac{d}{dx}(x^5) - 10\frac{d}{dx}(x^3) - 6\frac{d}{dx}(x) + \frac{d}{dx}(5)$$
$$= 12 \cdot 5x^4 - 10 \cdot 3x^2 - 6 \cdot 1 + 0 = 60x^4 - 30x^2 - 6$$

The motion of a particle is given by:

- $s(t) = 2t^3 - 5t^2 + 3t + 4$ (t is in seconds, and $s(t)$ in cm)

Find the acceleration function, and the acceleration after $2s$.

$$v(t) = \frac{d}{dt}s(t) = 6t^2 - 10t + 3 \qquad \text{in cm/s}$$
$$a(t) = \frac{d}{dt}v(t) = 12t - 10$$

Derivatives of Basic Functions

Compute the following derivative:

$$\frac{d}{dx}(12x^5 - 10x^3 - 6x + 5)$$
$$= 12\frac{d}{dx}(x^5) - 10\frac{d}{dx}(x^3) - 6\frac{d}{dx}(x) + \frac{d}{dx}(5)$$
$$= 12 \cdot 5x^4 - 10 \cdot 3x^2 - 6 \cdot 1 + 0 = 60x^4 - 30x^2 - 6$$

The motion of a particle is given by:

- $s(t) = 2t^3 - 5t^2 + 3t + 4$ (t is in seconds, and $s(t)$ in cm)

Find the acceleration function, and the acceleration after $2s$.

$$v(t) = \frac{d}{dt}s(t) = 6t^2 - 10t + 3 \qquad \text{in cm/s}$$
$$a(t) = \frac{d}{dt}v(t) = 12t - 10 \qquad \text{in cm/s}^2$$

Derivatives of Basic Functions

Compute the following derivative:

$$\frac{d}{dx}(12x^5 - 10x^3 - 6x + 5)$$
$$= 12\frac{d}{dx}(x^5) - 10\frac{d}{dx}(x^3) - 6\frac{d}{dx}(x) + \frac{d}{dx}(5)$$
$$= 12 \cdot 5x^4 - 10 \cdot 3x^2 - 6 \cdot 1 + 0 = 60x^4 - 30x^2 - 6$$

The motion of a particle is given by:

- $s(t) = 2t^3 - 5t^2 + 3t + 4$ (t is in seconds, and $s(t)$ in cm)

Find the acceleration function, and the acceleration after $2s$.

$$v(t) = \frac{d}{dt}s(t) = 6t^2 - 10t + 3 \qquad \text{in cm/s}$$
$$a(t) = \frac{d}{dt}v(t) = 12t - 10 \qquad \text{in cm/s}^2$$

The acceleration after $2s$ is $14\text{cm}/\text{s}^2$.

Derivatives of Basic Functions

Find the points of $f(x) = x^4 - 6x^2 + 4$ with horizontal tangent.

Derivatives of Basic Functions

Find the points of $f(x) = x^4 - 6x^2 + 4$ with horizontal tangent.

Horizontal tangent means that the slope (the derivative) is 0:

$$\frac{d}{dx} f(x) \; =$$

Derivatives of Basic Functions

Find the points of $f(x) = x^4 - 6x^2 + 4$ with horizontal tangent.

Horizontal tangent means that the slope (the derivative) is 0:

$$\frac{d}{dx} f(x) \; = \; 4x^3 - 12x$$

Derivatives of Basic Functions

Find the points of $f(x) = x^4 - 6x^2 + 4$ with horizontal tangent.

Horizontal tangent means that the slope (the derivative) is 0:

$$\frac{d}{dx}f(x) \;=\; 4x^3 - 12x \;=\; 4x(x^2 - 3)$$

Derivatives of Basic Functions

Find the points of $f(x) = x^4 - 6x^2 + 4$ with horizontal tangent.

Horizontal tangent means that the slope (the derivative) is 0:

$$\frac{d}{dx} f(x) \;=\; 4x^3 - 12x \;=\; 4x(x^2 - 3)$$

Thus $f'(x) = 0$ when $x = 0$ or $x = \sqrt{3}$ or $x = -\sqrt{3}$.

Derivatives of Basic Functions

Find the points of $f(x) = x^4 - 6x^2 + 4$ with horizontal tangent.

Horizontal tangent means that the slope (the derivative) is 0:

$$\frac{d}{dx} f(x) \; = \; 4x^3 - 12x \; = \; 4x(x^2 - 3)$$

Thus $f'(x) = 0$ when $x = 0$ or $x = \sqrt{3}$ or $x = -\sqrt{3}$.

Thus the corresponding points are $(0, 4)$, $(\sqrt{3}, -5)$, $(-\sqrt{3}, -5)$.

Derivatives of Basic Functions

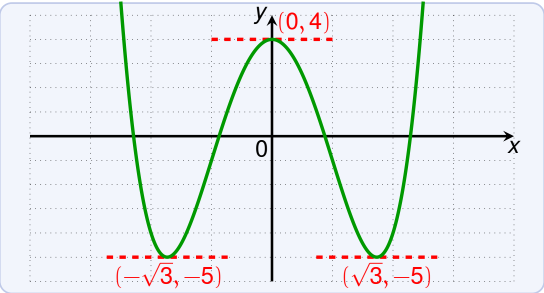
Find the points of $f(x) = x^4 - 6x^2 + 4$ with horizontal tangent.

Horizontal tangent means that the slope (the derivative) is 0:

$$\frac{d}{dx} f(x) \; = \; 4x^3 - 12x \; = \; 4x(x^2 - 3)$$

Thus $f'(x) = 0$ when $x = 0$ or $x = \sqrt{3}$ or $x = -\sqrt{3}$.

Thus the corresponding points are $(0, 4)$, $(\sqrt{3}, -5)$, $(-\sqrt{3}, -5)$.



Derivatives of Basic Functions

Sum Rule

If f and g are differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] \;=\; \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

Derivatives of Basic Functions

Sum Rule

If f and g are differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

Proof.

$$\frac{d}{dx}[f(x) + g(x)]$$

□

Derivatives of Basic Functions

Sum Rule

If f and g are differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] \;=\; \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

Proof.

$$\frac{d}{dx}[f(x) + g(x)] = \lim_{h \to 0} \frac{[f(x+h) + g(x+h)] - [f(x) + g(x)]}{h}$$

□

Derivatives of Basic Functions

Sum Rule

If f and g are differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] \; = \; \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

Proof.

$$\begin{aligned}\frac{d}{dx}[f(x) + g(x)] &= \lim_{h \to 0} \frac{[f(x+h) + g(x+h)] - [f(x) + g(x)]}{h} \\ &= \lim_{h \to 0} \frac{[f(x+h) - f(x)] + [g(x+h) - g(x)]}{h}\end{aligned}$$

□

Derivatives of Basic Functions

Sum Rule

If f and g are differentiable, then

$$\frac{d}{dx}[f(x)+g(x)] \;=\; \frac{d}{dx}f(x)+\frac{d}{dx}g(x)$$

Proof.

$$\begin{aligned}
\frac{d}{dx}[f(x)+g(x)] &= \lim_{h\to 0}\frac{[f(x+h)+g(x+h)]-[f(x)+g(x)]}{h} \\
&= \lim_{h\to 0}\frac{[f(x+h)-f(x)]+[g(x+h)-g(x)]}{h} \\
&= \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}+\frac{g(x+h)-g(x)}{h}\right)
\end{aligned}$$

□

Derivatives of Basic Functions

Sum Rule

If f and g are differentiable, then

$$\frac{d}{dx}[f(x)+g(x)] \;=\; \frac{d}{dx}f(x)+\frac{d}{dx}g(x)$$

Proof.

$$\begin{aligned}
\frac{d}{dx}[f(x)+g(x)] &= \lim_{h\to 0}\frac{[f(x+h)+g(x+h)]-[f(x)+g(x)]}{h} \\
&= \lim_{h\to 0}\frac{[f(x+h)-f(x)]+[g(x+h)-g(x)]}{h} \\
&= \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}+\frac{g(x+h)-g(x)}{h}\right) \\
&= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}
\end{aligned}$$

□

Derivatives of Basic Functions

Sum Rule

If f and g are differentiable, then

$$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x)+\frac{d}{dx}g(x)$$

Proof.

$$\begin{aligned}
\frac{d}{dx}[f(x)+g(x)] &= \lim_{h\to 0}\frac{[f(x+h)+g(x+h)]-[f(x)+g(x)]}{h} \\
&= \lim_{h\to 0}\frac{[f(x+h)-f(x)]+[g(x+h)-g(x)]}{h} \\
&= \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}+\frac{g(x+h)-g(x)}{h}\right) \\
&= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+\lim_{h\to 0}\frac{g(x+h)-g(x)}{h} \\
&= \frac{d}{dx}f(x)+\frac{d}{dx}g(x)
\end{aligned}$$

□

Derivatives of Exponential Functions

We compute the derivative of $f(x) = a^x$:

Derivatives of Exponential Functions

We compute the derivative of $f(x) = a^x$:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivatives of Exponential Functions

We compute the derivative of $f(x) = a^x$:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{a^{x+h} - a^x}{h}$$

Derivatives of Exponential Functions

We compute the derivative of $f(x) = a^x$:

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h} \\ &= \lim_{h\to 0} \frac{a^x \cdot a^h - a^x}{h} \end{aligned}$$

Derivatives of Exponential Functions

We compute the derivative of $f(x) = a^x$:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{a^{x+h} - a^x}{h}$$

$$= \lim_{h \to 0} \frac{a^x \cdot a^h - a^x}{h} = \lim_{h \to 0} a^x \frac{a^h - 1}{h}$$

Derivatives of Exponential Functions

We compute the derivative of $f(x) = a^x$:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{a^{x+h} - a^x}{h}$$

$$= \lim_{h \to 0} \frac{a^x \cdot a^h - a^x}{h} = \lim_{h \to 0} a^x \frac{a^h - 1}{h} = a^x \cdot \lim_{h \to 0} \frac{a^h - 1}{h}$$

Derivatives of Exponential Functions

We compute the derivative of $f(x) = a^x$:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h}$$

$$= \lim_{h\to 0} \frac{a^x \cdot a^h - a^x}{h} = \lim_{h\to 0} a^x \frac{a^h - 1}{h} = a^x \cdot \lim_{h\to 0} \frac{a^h - 1}{h}$$

Note that

$$\lim_{h\to 0} \frac{a^h - 1}{h} = f'(0)$$

Derivatives of Exponential Functions

We compute the derivative of $f(x) = a^x$:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h}$$

$$= \lim_{h\to 0} \frac{a^x \cdot a^h - a^x}{h} = \lim_{h\to 0} a^x \frac{a^h - 1}{h} = a^x \cdot \lim_{h\to 0} \frac{a^h - 1}{h}$$

Note that

$$\lim_{h\to 0} \frac{a^h - 1}{h} = f'(0)$$

For $f(x) = a^x$ we have

$$f'(x) = f'(0) \cdot a^x$$

Derivatives of Exponential Functions

We compute the derivative of $f(x) = a^x$:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h}$$

$$= \lim_{h\to 0} \frac{a^x \cdot a^h - a^x}{h} = \lim_{h\to 0} a^x \frac{a^h - 1}{h} = a^x \cdot \lim_{h\to 0} \frac{a^h - 1}{h}$$

Note that

$$\lim_{h\to 0} \frac{a^h - 1}{h} = f'(0)$$

For $f(x) = a^x$ we have

$$f'(x) = f'(0) \cdot a^x$$

Note that slope is proportional to the function itself.

Derivatives of Exponential Functions

For $f(x) = a^x$ we have

$$f'(x) = f'(0) \cdot a^x$$

Derivatives of Exponential Functions

For $f(x) = a^x$ we have

$$f'(x) = f'(0) \cdot a^x$$

Using the calculator we can estimate that:

Derivatives of Exponential Functions

For $f(x) = a^x$ we have

$$f'(x) = f'(0) \cdot a^x$$

Using the calculator we can estimate that:

$$\text{for } a = 2 \qquad\qquad f'(0) = \lim_{h \to 0} \frac{2^h - 1}{h} \approx 0.69$$

Derivatives of Exponential Functions

For $f(x) = a^x$ we have

$$f'(x) = f'(0) \cdot a^x$$

Using the calculator we can estimate that:

$$\text{for } a = 2 \qquad f'(0) = \lim_{h \to 0} \frac{2^h - 1}{h} \approx 0.69$$

$$\text{for } a = 3 \qquad f'(0) = \lim_{h \to 0} \frac{3^h - 1}{h} \approx 1.10$$

Derivatives of Exponential Functions

For $f(x) = a^x$ we have

$$f'(x) = f'(0) \cdot a^x$$

Using the calculator we can estimate that:

$$\text{for } a = 2 \qquad f'(0) = \lim_{h \to 0} \frac{2^h - 1}{h} \approx 0.69$$

$$\text{for } a = 3 \qquad f'(0) = \lim_{h \to 0} \frac{3^h - 1}{h} \approx 1.10$$

There is a number a between 2 and 3 such that $f'(0) = 1$:

Derivatives of Exponential Functions

For $f(x) = a^x$ we have

$$f'(x) = f'(0) \cdot a^x$$

Using the calculator we can estimate that:

$$\text{for } a = 2 \qquad f'(0) = \lim_{h \to 0} \frac{2^h - 1}{h} \approx 0.69$$

$$\text{for } a = 3 \qquad f'(0) = \lim_{h \to 0} \frac{3^h - 1}{h} \approx 1.10$$

There is a number a between 2 and 3 such that $f'(0) = 1$:

$$e \text{ is the number such that } \quad \lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

Derivatives of Exponential Functions

For $f(x) = a^x$ we have

$$f'(x) = f'(0) \cdot a^x$$

Using the calculator we can estimate that:

$$\text{for } a = 2 \qquad f'(0) = \lim_{h \to 0} \frac{2^h - 1}{h} \approx 0.69$$

$$\text{for } a = 3 \qquad f'(0) = \lim_{h \to 0} \frac{3^h - 1}{h} \approx 1.10$$

There is a number a between 2 and 3 such that $f'(0) = 1$:

$$e \text{ is the number such that} \quad \lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

The function e^x is the only exponential with slope 1 at $(0, 1)$.

Derivatives of Exponential Functions

$$\frac{d}{dx}(e^x) = e^x$$

Derivatives of Exponential Functions

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = \ln a \cdot a^x$$

Derivatives of Exponential Functions

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = \ln a \cdot a^x$$

At what point on the curve e^x is the tangent parallel to $y = 2x$?

Derivatives of Exponential Functions

$$\frac{d}{dx}(e^x) \;=\; e^x$$

$$\frac{d}{dx}(a^x) \;=\; \ln a \cdot a^x$$

At what point on the curve e^x is the tangent parallel to $y = 2x$?

Let $f(x) = e^x$

$$f'(a) = e^a = 2$$

Derivatives of Exponential Functions

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = \ln a \cdot a^x$$

At what point on the curve e^x is the tangent parallel to $y = 2x$?

Let $f(x) = e^x$

$$f'(a) = e^a = 2$$

Thus $a = \ln 2$, that is, the point is $(a, e^a) = (\ln 2, 2)$.

Derivatives of Exponential Functions

Let $f(x) = e^x - x$. Find f' and f''.

Derivatives of Exponential Functions

Let $f(x) = e^x - x$. Find f' and f''.

$$f'(x) =$$

Derivatives of Exponential Functions

Let $f(x) = e^x - x$. Find f' and f''.

$$f'(x) = e^x - 1$$

Derivatives of Exponential Functions

Let $f(x) = e^x - x$. Find f' and f''.

$$f'(x) = e^x - 1 \qquad\qquad f''(x) =$$

Derivatives of Exponential Functions

Let $f(x) = e^x - x$. Find f' and f''.

$$f'(x) = e^x - 1 \qquad f''(x) = e^x$$

Derivatives of Exponential Functions

Let $f(x) = e^x - x$. Find f' and f''.

$$f'(x) = e^x - 1 \qquad\qquad f''(x) = e^x$$