Calculus M211

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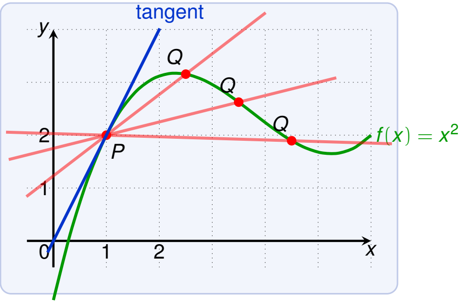
Indiana University Bloomington

2013

Finding a Tangent

We move Q closer and closer to P.



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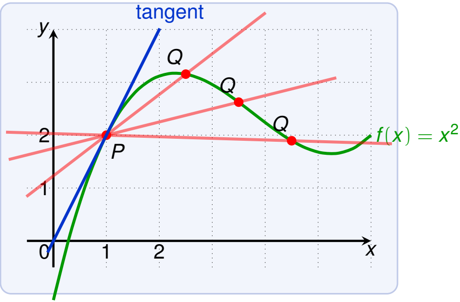

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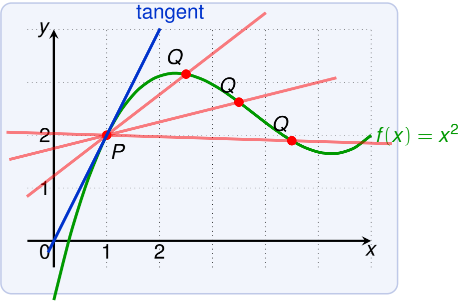

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The **tangent line** to the curve $f(x)$ at point $P = (a, f(a))$ is the line through P with slope

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

provided that the limit exists.

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Thus $y - 1 = 2(x - 1)$, that is, the tangent is $y = 2x - 1$.

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An equivalent way of defining the derivative (take $x = a + h$):

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(Note that large derivative $f'(x_1)$ means rapid change.)

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- Typically $f'(500) < f'(50)$ since usually the cost of production per yard will decrease the more you produce (due to fixed costs: you have already bought and installed the machines...)

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- in biology: rate of change of the population of bacteria with respect to time