Calculus M211

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2013

Express the domain of the function

$$f(x) = \frac{x + \log(x + 1) + \sqrt{5 - x}}{x - 2}$$

as a union of intervals.

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&= -1
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Thus the inverse function is $f(y) = 10^{\frac{1-5y}{2y-1}}$.

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The limit $\lim_{x \to 5} \frac{x-5}{|x-5|}$ does not exist since the left- and the right-limit are different.

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has a solution for x on the interval $[0, 1]$.

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We know that the range of cos is $[-1, 1]$. Thus

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Thus we can apply the Squeeze Theorem with

- lower bound $-x^{12}$ (i.e. $\leq g(x)$), and
- upper bound x^{12} (i.e. $\geq g(x)$)

and it follows that

$$\lim_{x\to 0} g(x) = 0$$

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$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 3^x - k = 9 - k$$

$$f(2) = 3^2 - k =$$

1st Midterm Exam - Review

For what value of k is the following function continuous?

$$f(x) = \begin{cases} x^2 + 2k & \text{for } x < 2 \\ 3^x - k & \text{for } x \geq 2 \end{cases}$$

For any k, the function is continuous at all $x \neq 2$ since

- $x^2 + 2k$ is continuous, and
- $3^x - k$ is continuous.

(Both are compositions of continuous functions)

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We have continuity at 2 if $4 + 2k = 9 - k$. Thus $k = \frac{5}{3}$.

Departmental Help Sessions

- 6:00 - 8:00 PM in WY 115 on Monday, Tuesday, Wednesday and Thursday.