Calculus M211

Jörg Endrullis

Indiana University Bloomington

2013

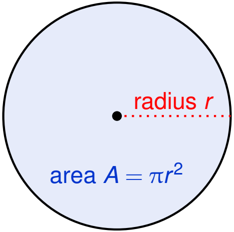
Functions

Example

The area A of a circle depends on its radius r. The rule is

$$A = \pi r^2$$

We say that A is a **function** of r.



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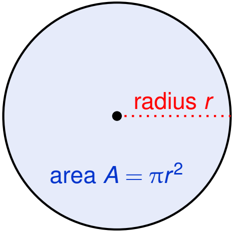
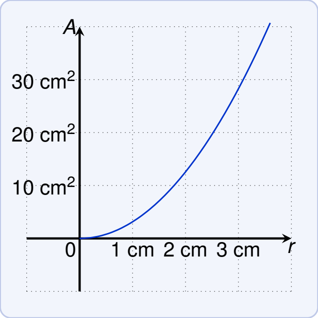
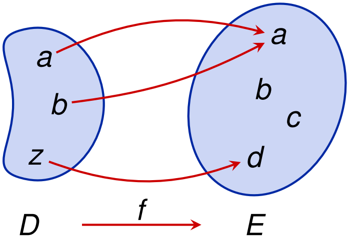




Functions

A **function** f from D to E is a rule that assigns to each element x in a set D exactly one element, called $f(x)$, in a set E.

Visualizing functions as **arrow diagrams**:



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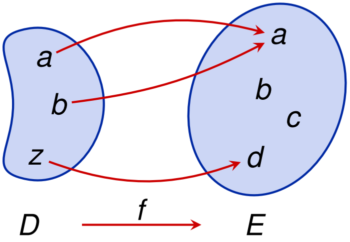

Terminology:

- $f(x)$ is the value of f at x

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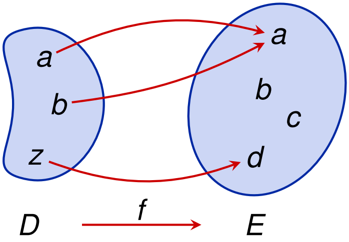

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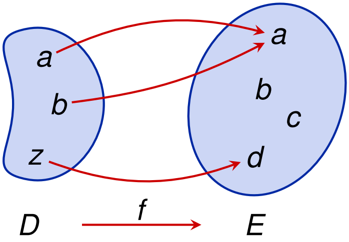

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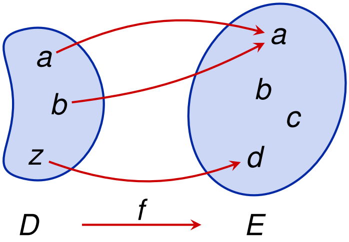

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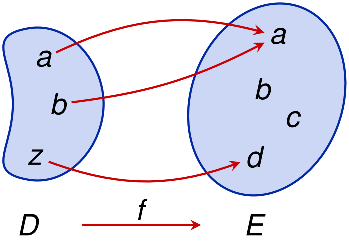

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- domain $D = \{ a, b, z \}$

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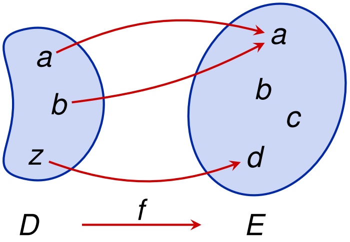

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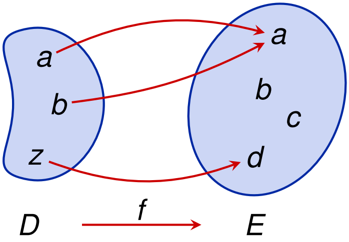

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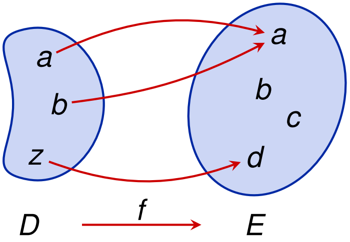

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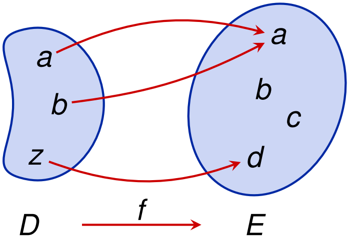

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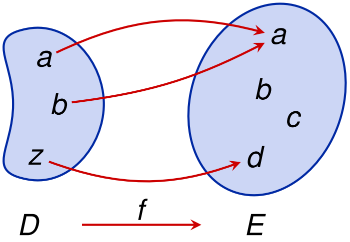

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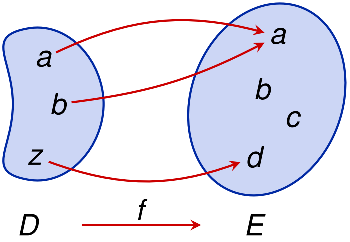

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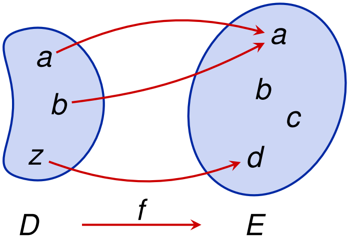

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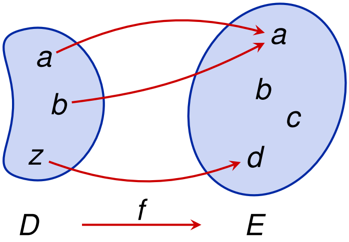

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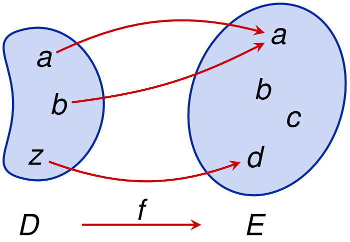

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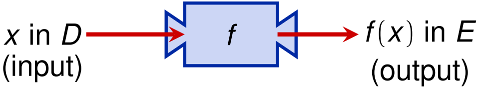
Functions as Machines

A function as a **machine**:



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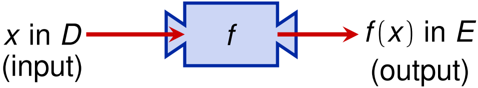

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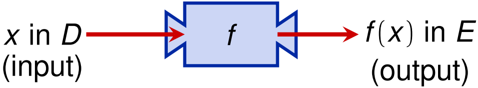

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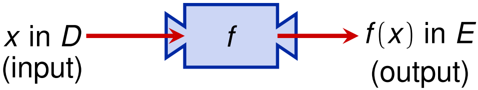

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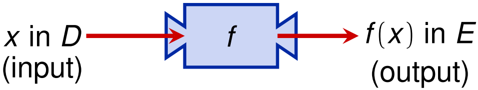

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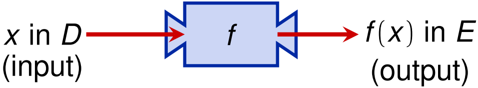

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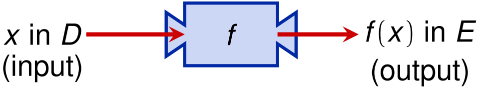

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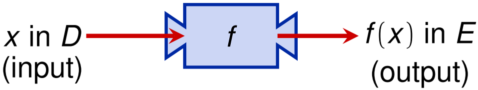

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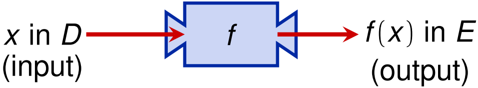

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Square root $f(x) = \sqrt{x}$ (over real numbers):

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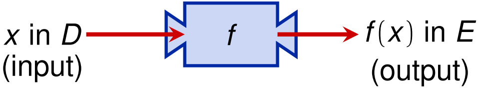

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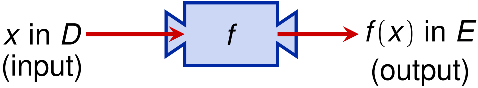

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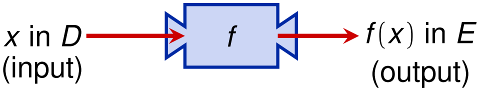

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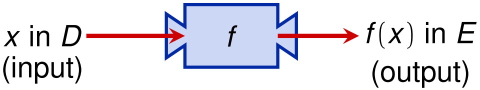

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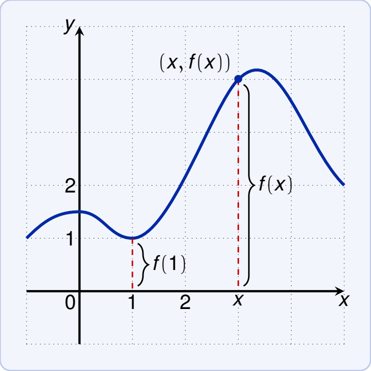

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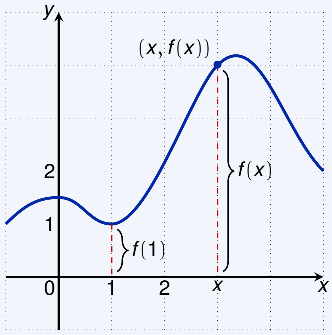
The **graph** of a function f is the set of pairs $\{\, (x, f(x)) \mid x \in D \,\}$

- set of all points (x, y) in the coordinate plane such that $y = f(x)$ and x is in the domain



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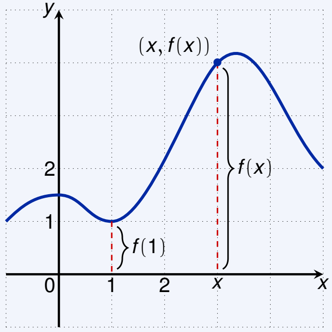

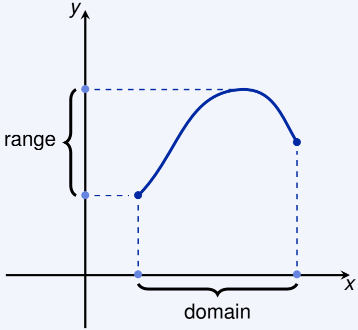

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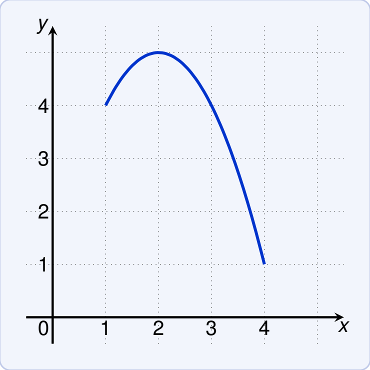

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- domain $= \{x \mid 1 \leq x \leq 4\} = [1, 4]$
- range $=$

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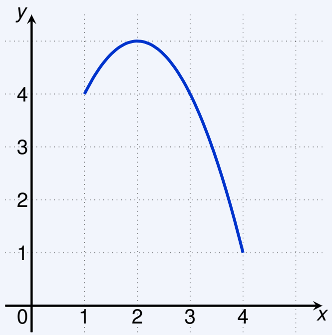

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- range $= \{y \mid 1 \leq x \leq 5\} = [1, 5]$

Functions: Examples

What is the domain and range of $f(x) = \sqrt{x+2}$?

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$$g(x) = \frac{1}{x^2 - x} = \frac{1}{x(x-1)}$$

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$$g(x) = \frac{1}{x^2 - x} = \frac{1}{x(x-1)}$$

Thus $g(x)$ is **not** defined if

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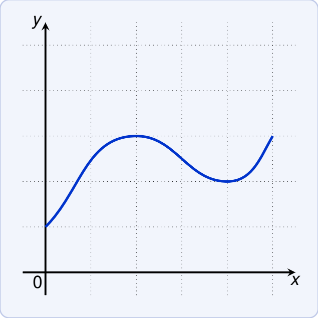
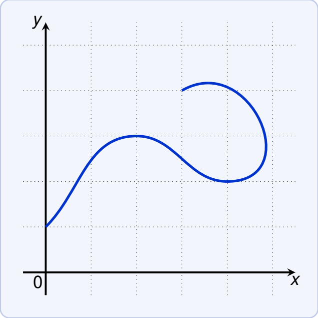
which can also be written as

$$(-\infty, 0) \cup (0, 1) \cup (1, \infty)$$

Vertical Line Test

When does a curve represent a function?





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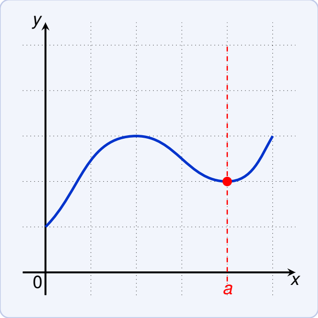
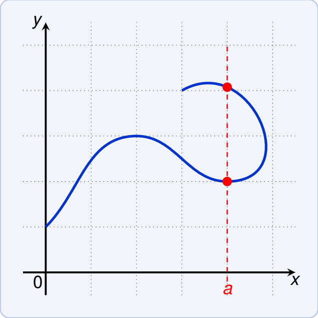
Vertical Line Test

A curve in the *xy*-plane represents a function of *x* if and only if no vertical line intersects the curve more than once.



corresponds to a function of *x*



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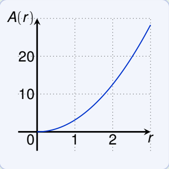

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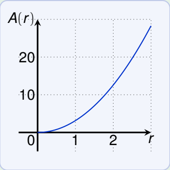

- algebraically (an explicit formula)

$$A(r) = \pi r^2$$

Piecewise Defined Functions

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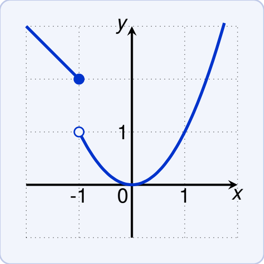

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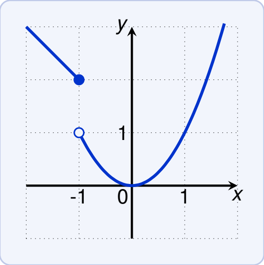
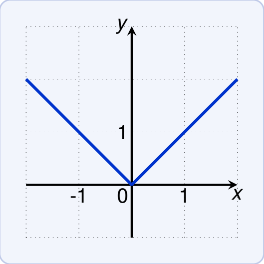


- ● point belongs to the graph
- ○ point is not in the graph

Piecewise Defined Functions: Example

The **absolute value function** $f(x) = |x|$ is piecewise defined:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



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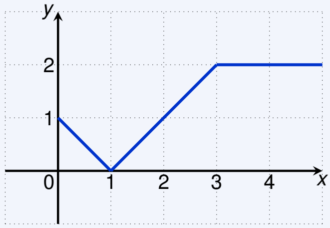

Find a formula for the function f with the graph above.

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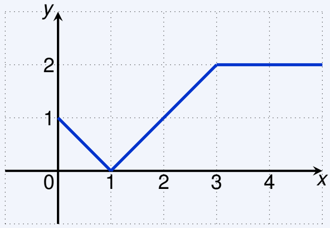

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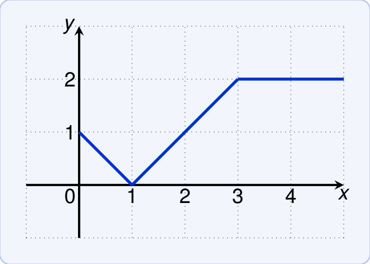

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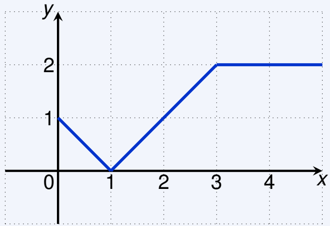


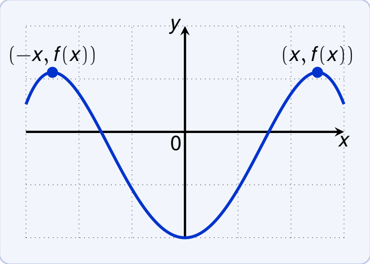
Find a formula for the function f with the graph above.

$$f(x) = \begin{cases} 1 - x & \text{if } 0 \leq x \leq 1 \\ x - 1 & \text{if } 1 < x \leq 3 \\ 2 & \text{if } x > 3 \end{cases}$$

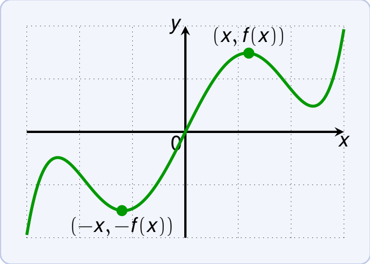
Symmetry

A function f is called

- **even** if $f(-x) = f(x)$ for every x in its domain, and
- **odd** if $f(-x) = -f(x)$ for every x in its domain.



an even function



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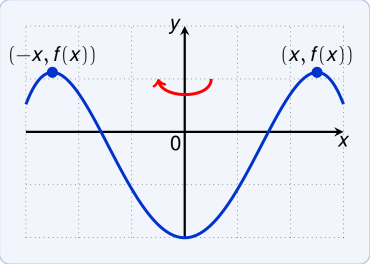

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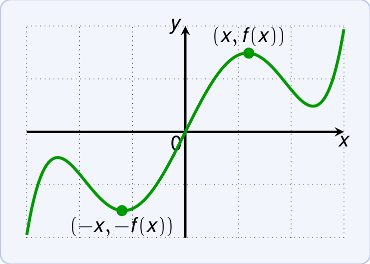

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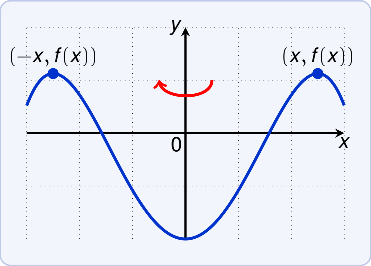

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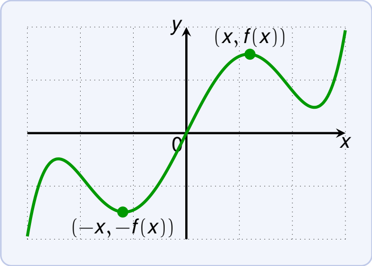

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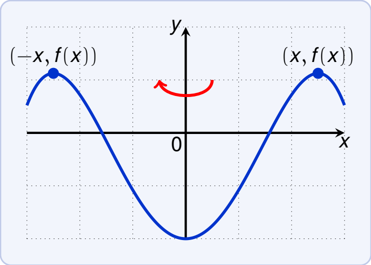

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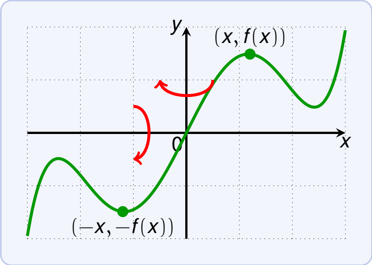

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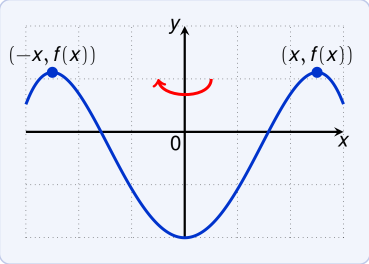

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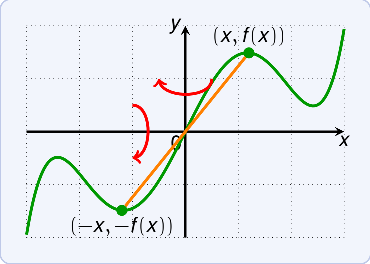

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How to remember what is even and odd?

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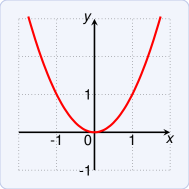
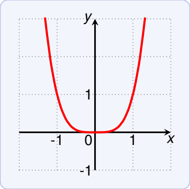
x^2



x^4



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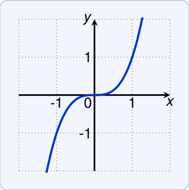
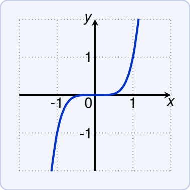


x^5



Thick of **power functions** x^n with n a natural number:

- x^n is even if n is even
- x^n is odd if n is odd

Symmetry

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Note that:

- The sum of even functions is even (e.g. $1 + x^4$).
- The sum of odd functions is odd (e.g. $x^5 + x$).

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$$f(x_1) < f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ and } x_1, x_2 \in I$$

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This function is:

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- decreasing on $[3, 4]$

Increasing and Decreasing Functions

A function f is **increasing** on an interval I if

$$f(x_1) < f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ and } x_1, x_2 \in I$$

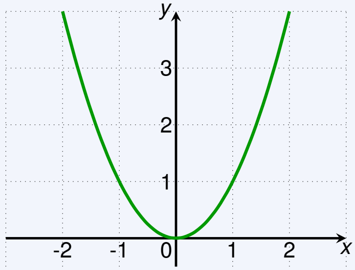
The function is **decreasing** on an interval I if

$$f(x_1) > f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ and } x_1, x_2 \in I$$

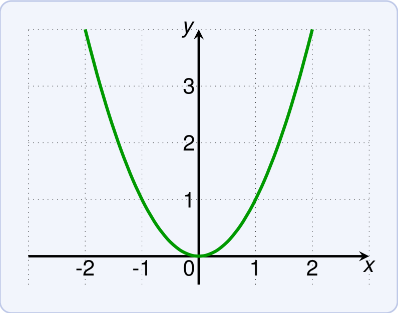
This function is:

- increasing on $[0, 3]$
- decreasing on $[3, 4]$
- increasing on $[4, 6]$

Increasing and Decreasing Functions

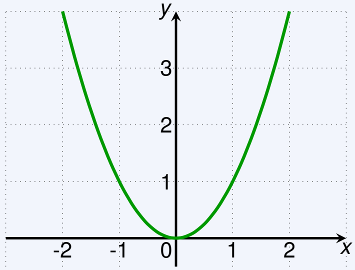


Increasing and Decreasing Functions



The function $f(x) = x^2$ is:

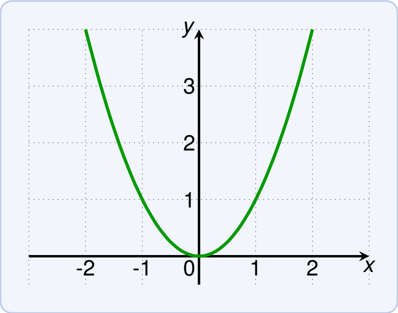
Increasing and Decreasing Functions



The function $f(x) = x^2$ is:

- increasing on $[0, \infty)$

Increasing and Decreasing Functions



The function $f(x) = x^2$ is:

- increasing on $[0, \infty)$
- decreasing on $(-\infty, 0]$