Calculus M211

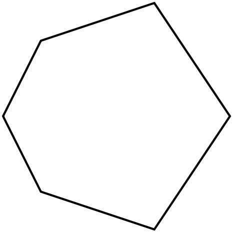
Jörg Endrullis

Indiana University Bloomington

2013

Motivation and Overview

How to compute the area of a polygon?



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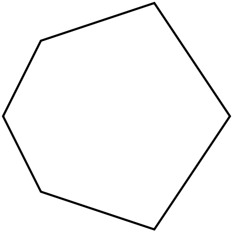

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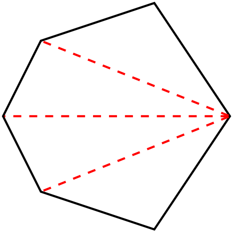

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- divide the polygon into triangles

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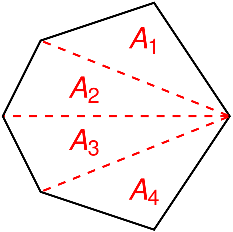

The ancient Greek did it as follows:

- divide the polygon into triangles
- compute the area of the triangles (and sum them up)

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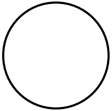
But what did the ancient Greek do with curved figures?



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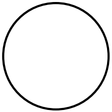

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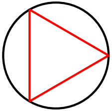

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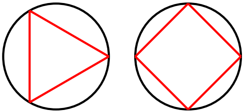

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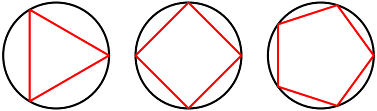

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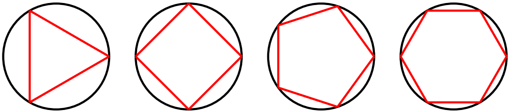

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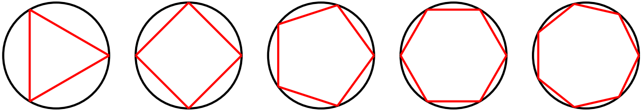

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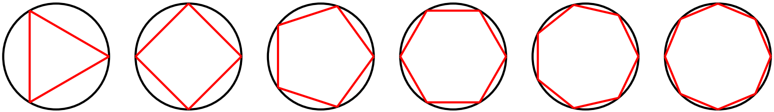

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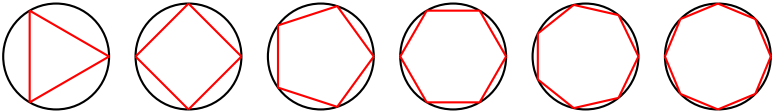

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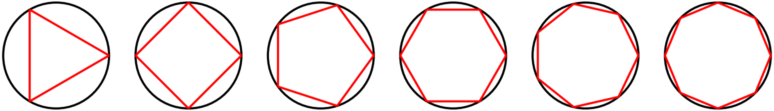

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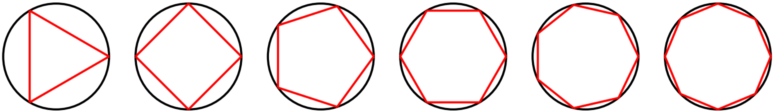

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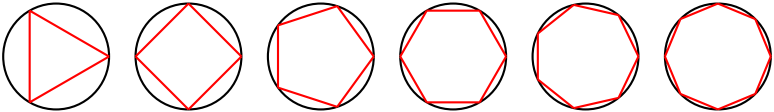

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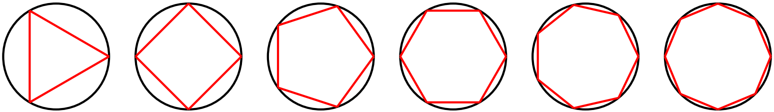

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$$\mathrm{A} = \lim_{n\to\infty} A_n$$

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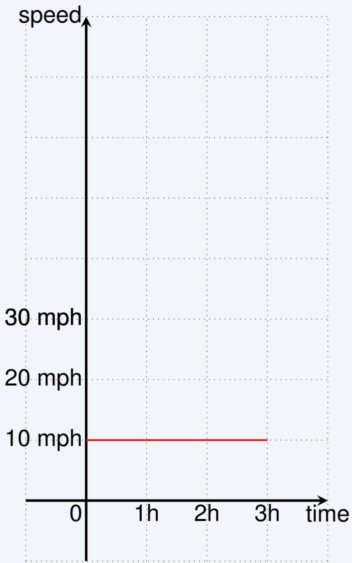

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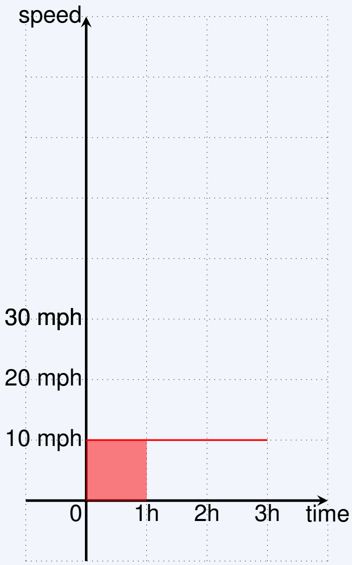

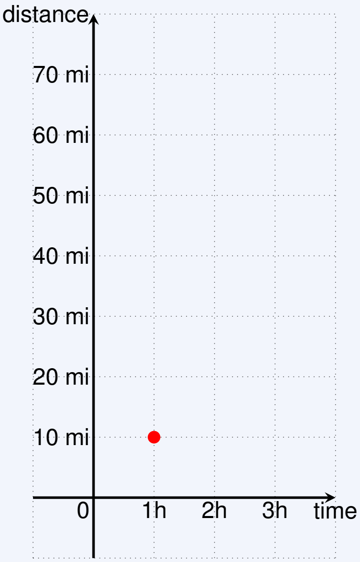

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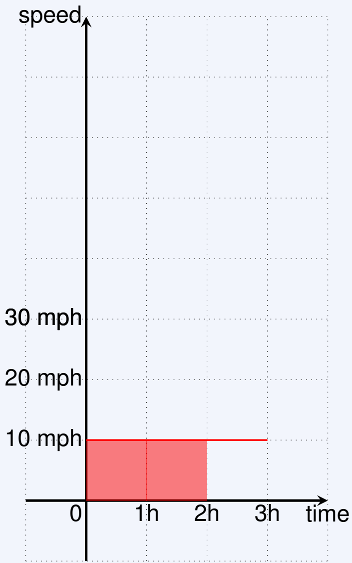

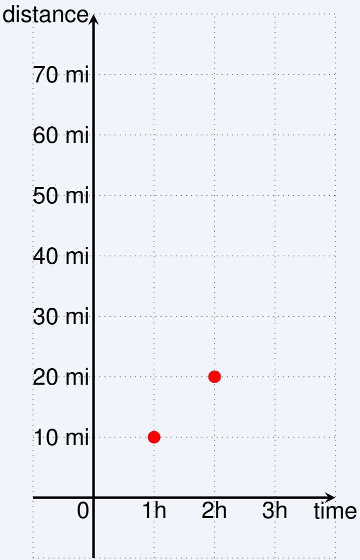

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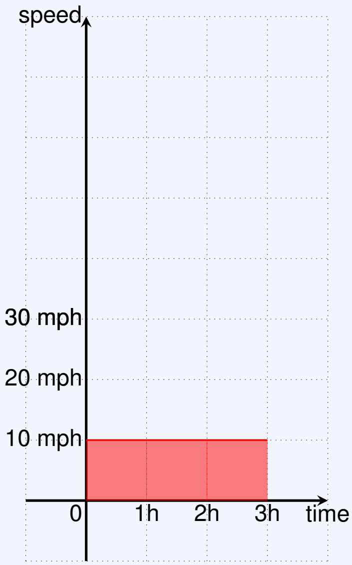

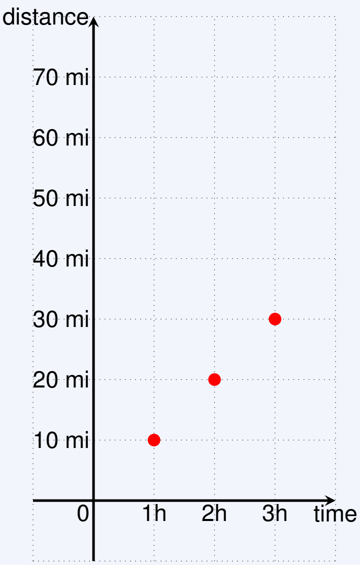

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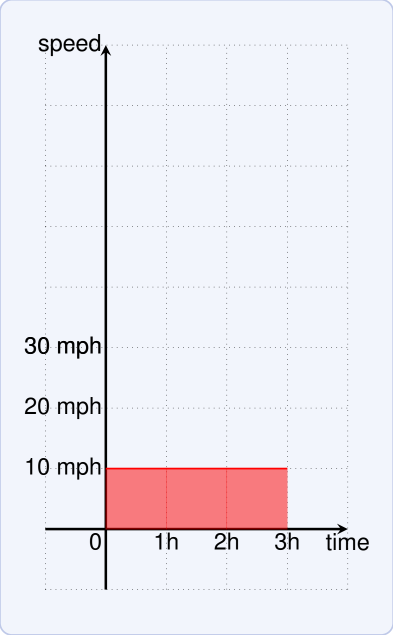

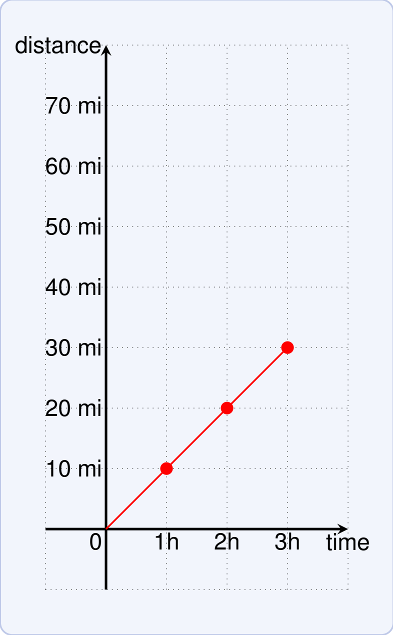

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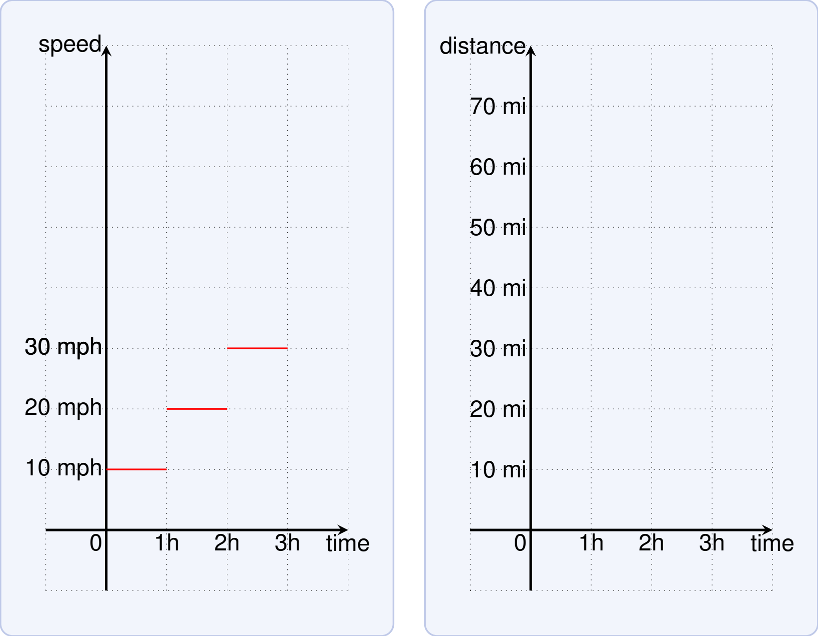

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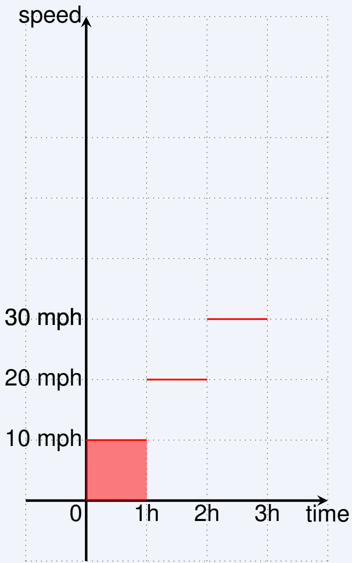

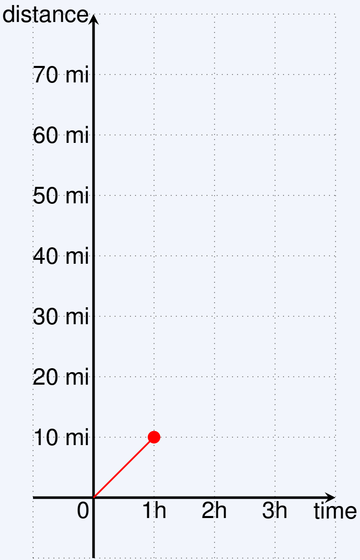

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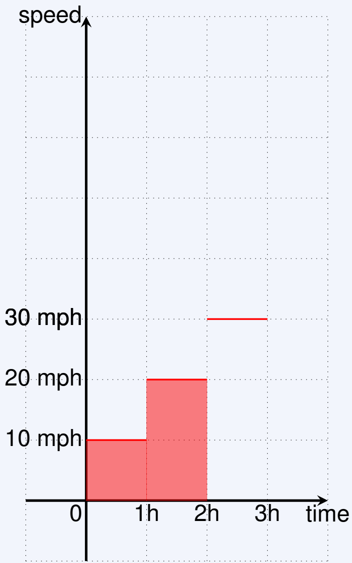

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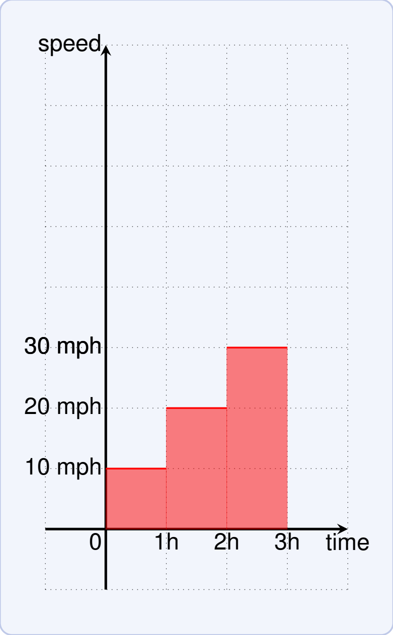

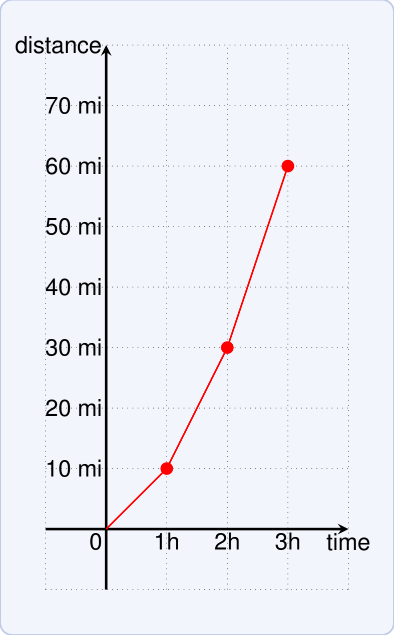

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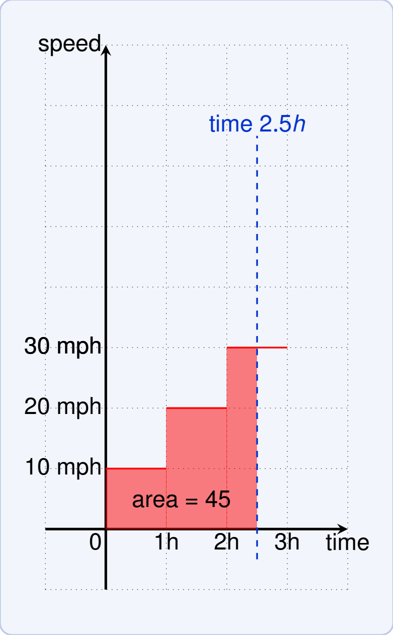

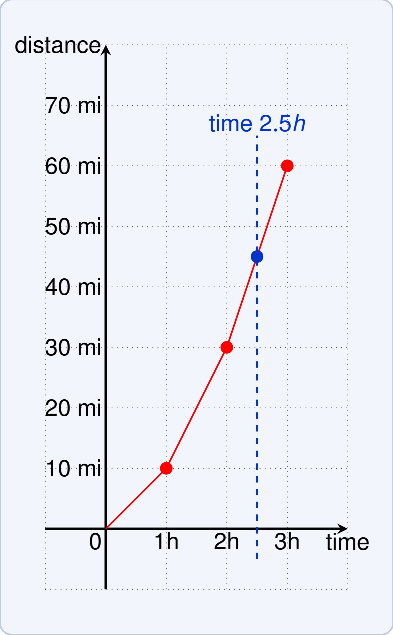

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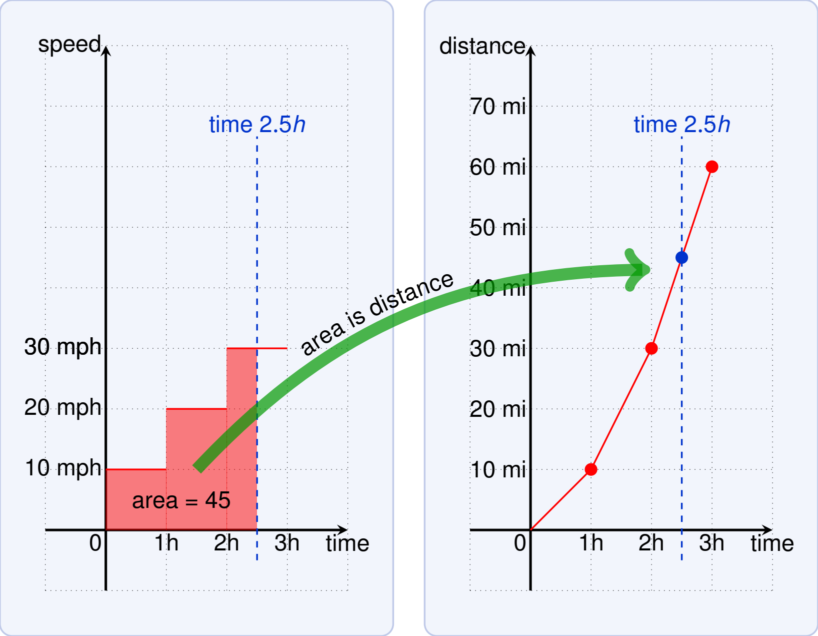

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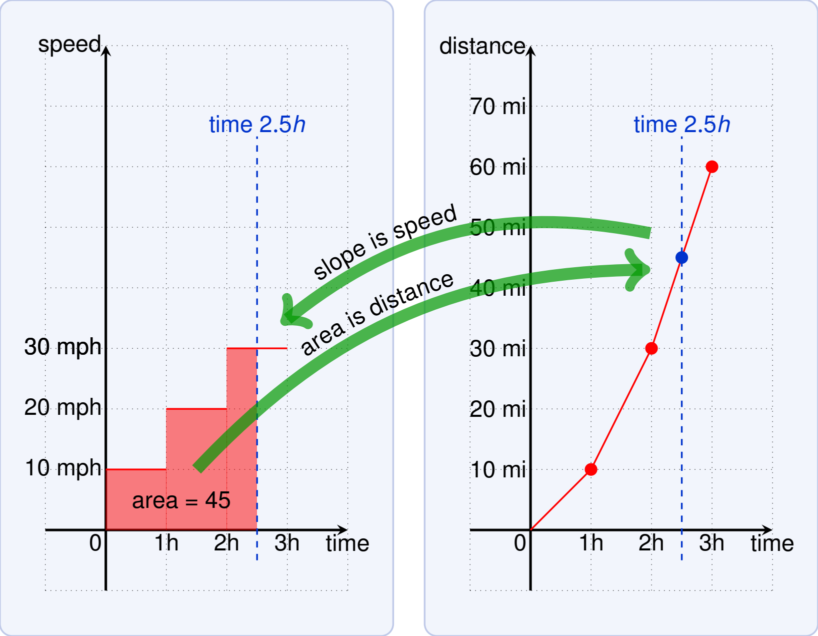

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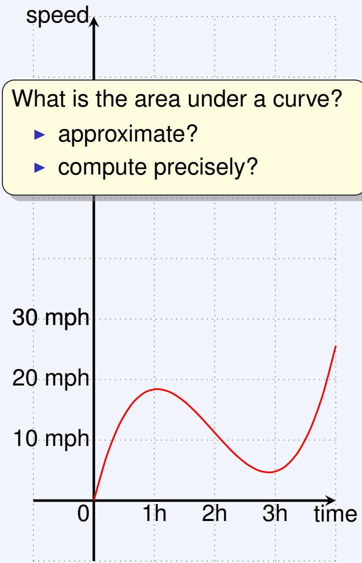

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- approximate?
- compute precisely?

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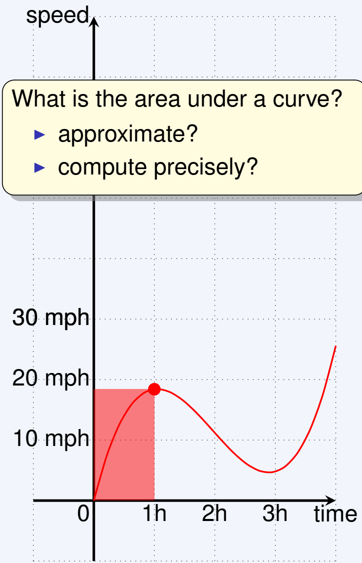

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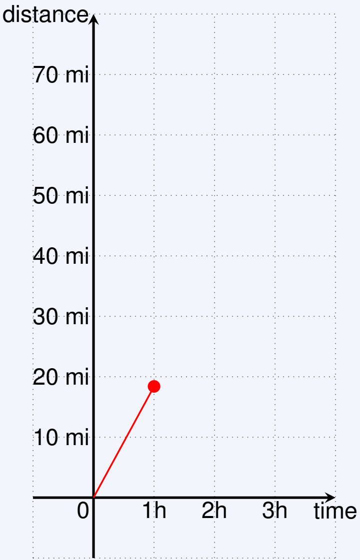

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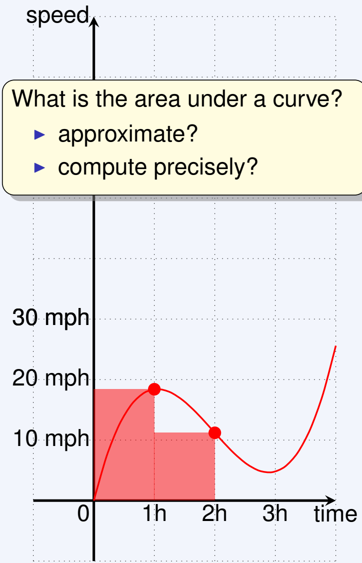

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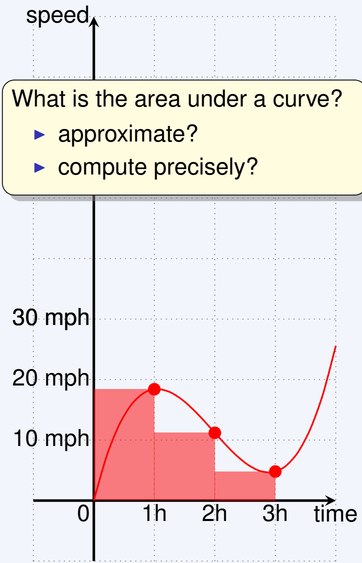

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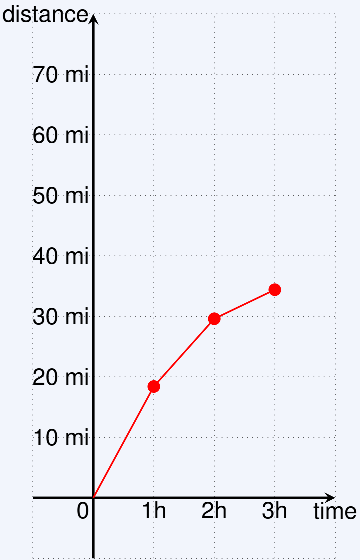

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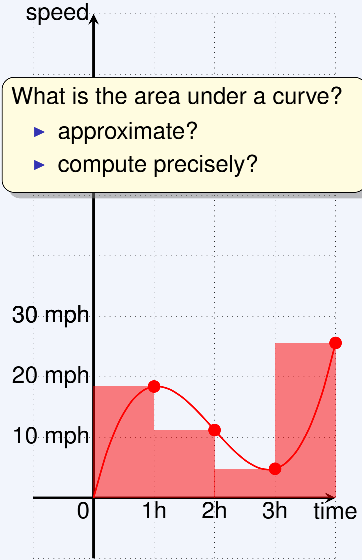

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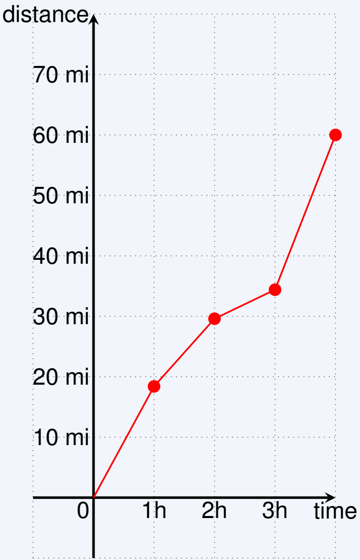

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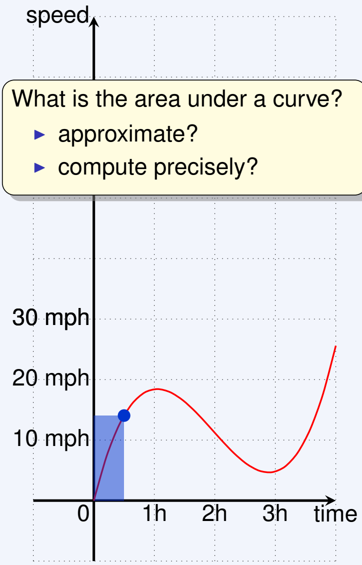

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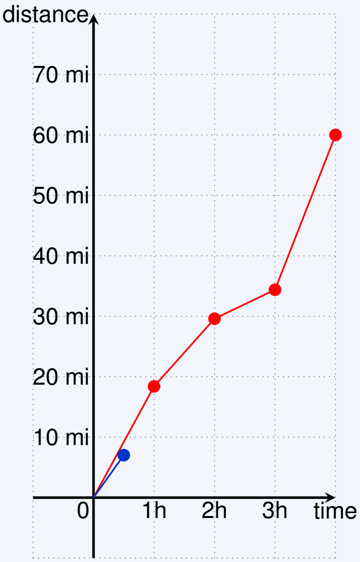

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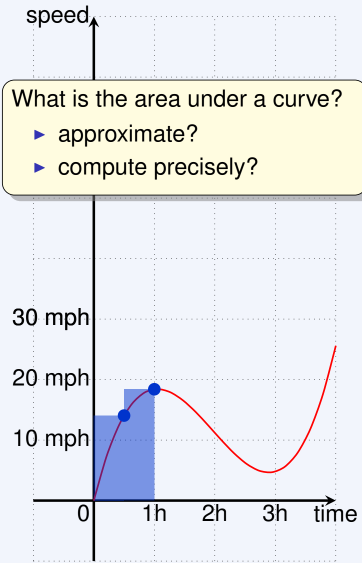

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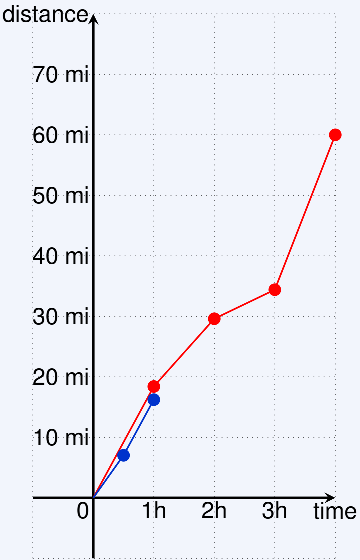

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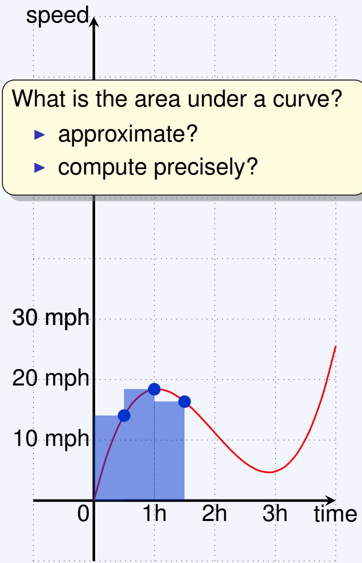

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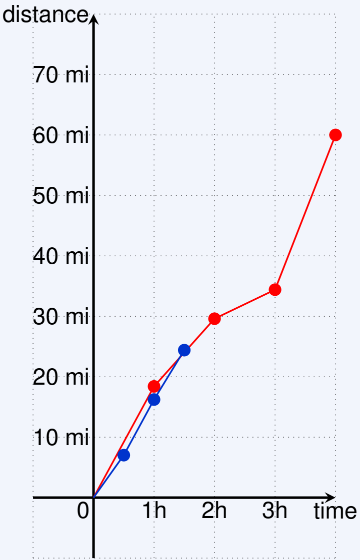

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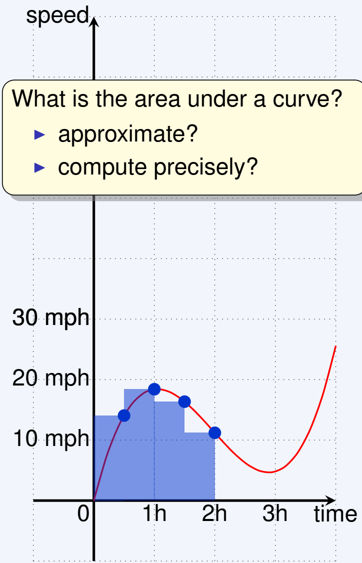

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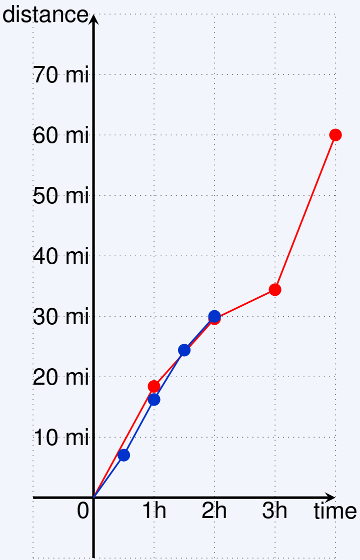

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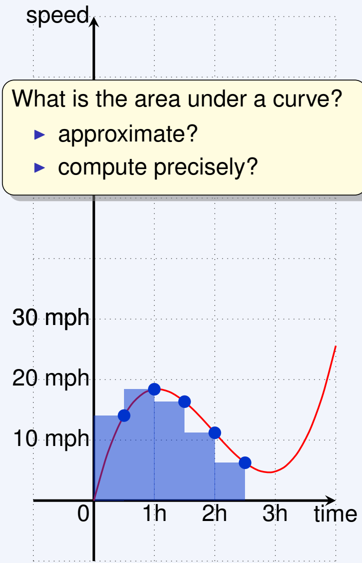

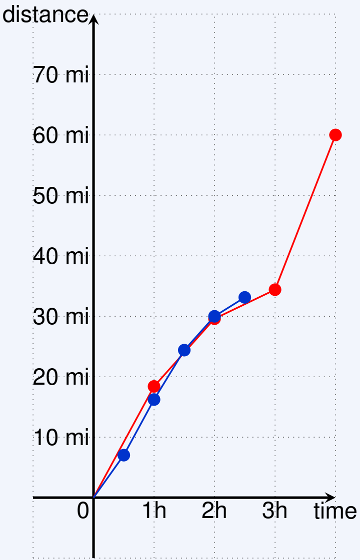

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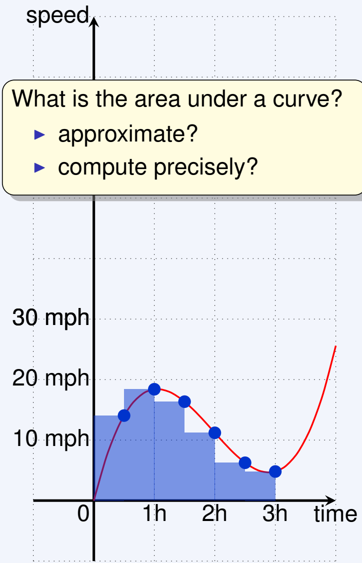

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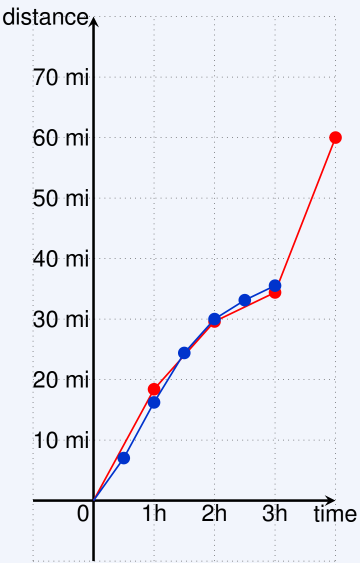

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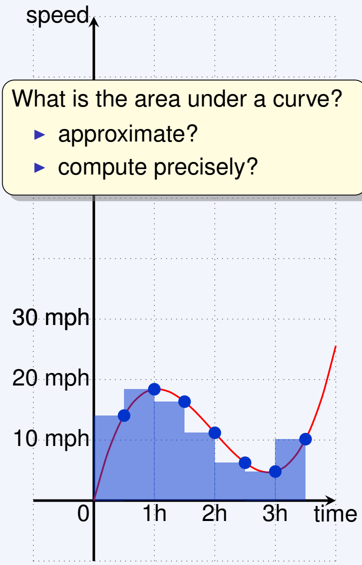

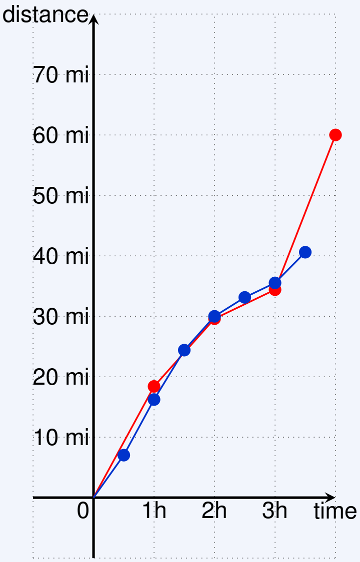

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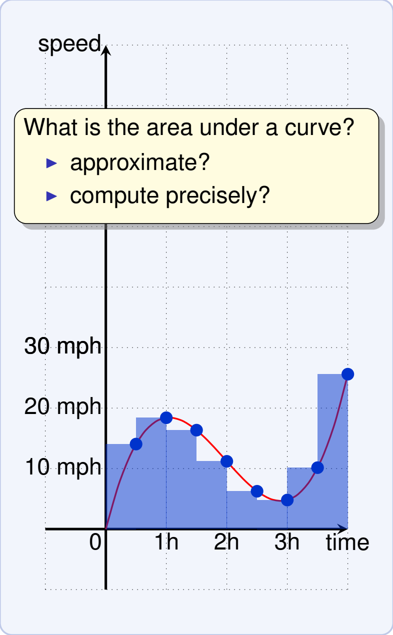

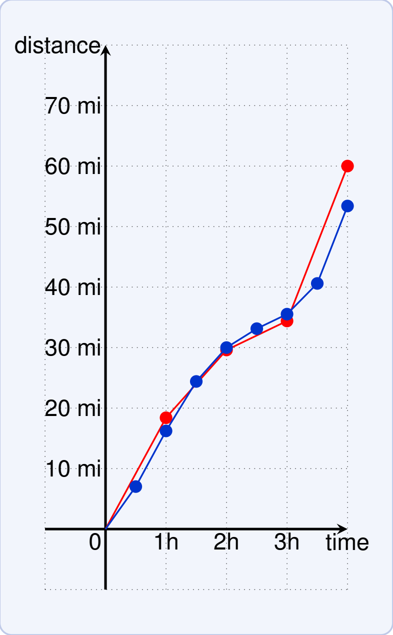

Motivation and Overview

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- approximate?
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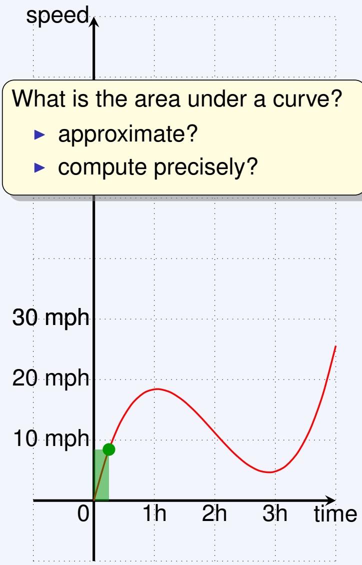

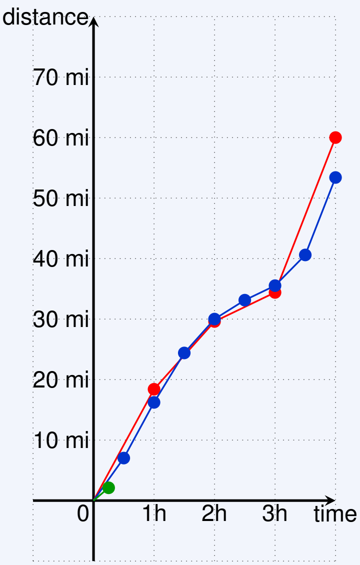

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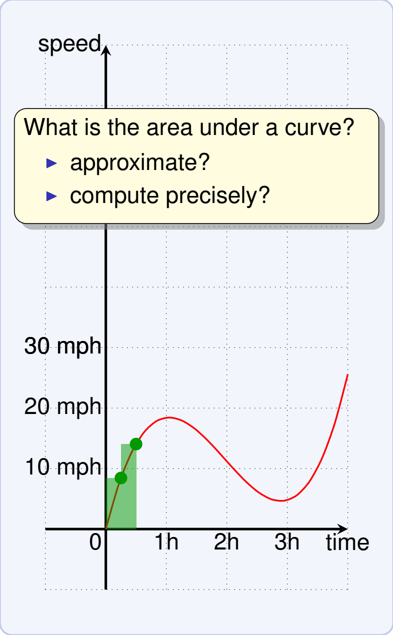

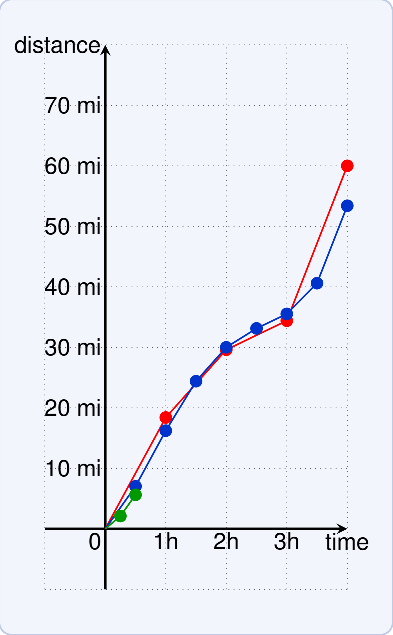

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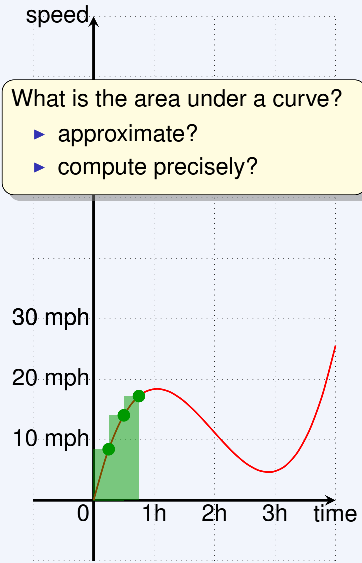

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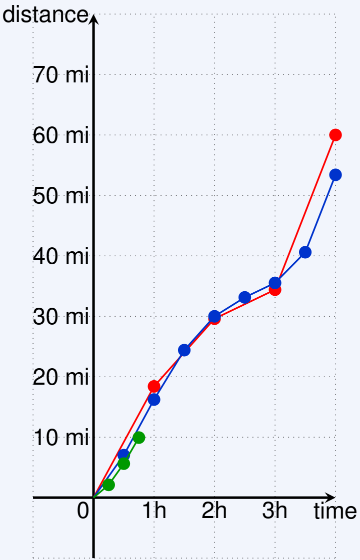

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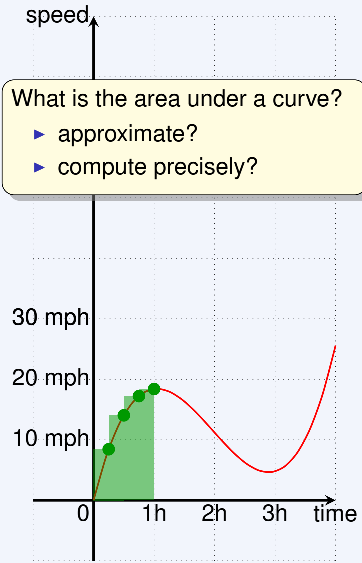

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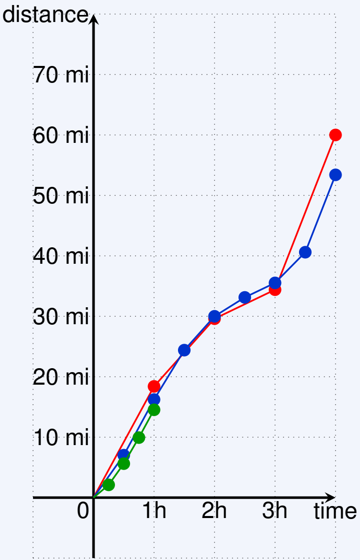

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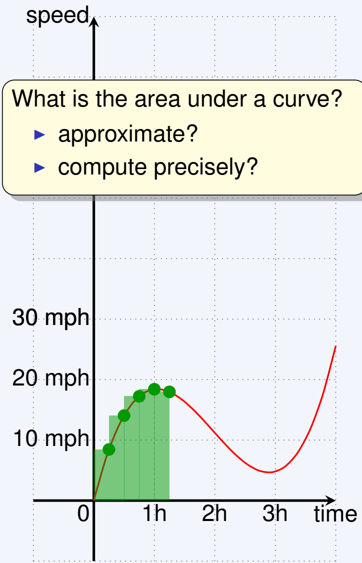

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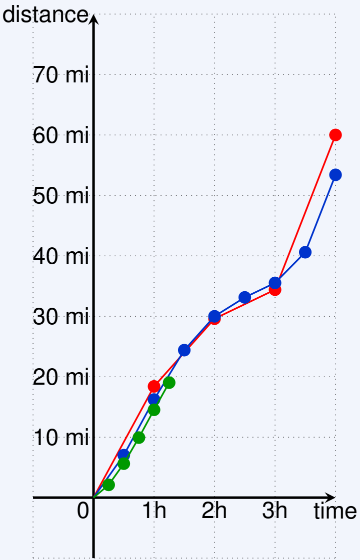

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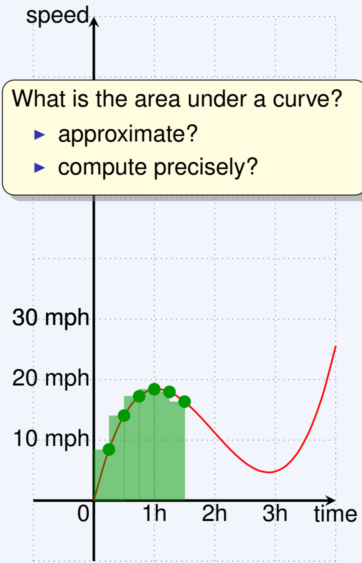

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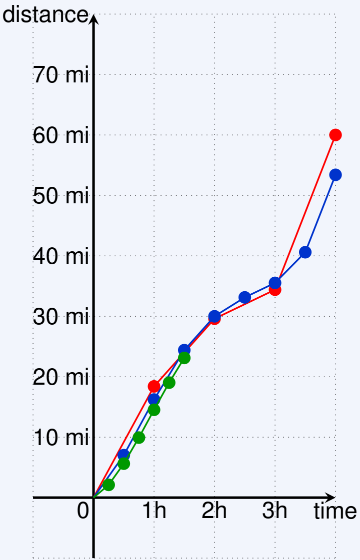

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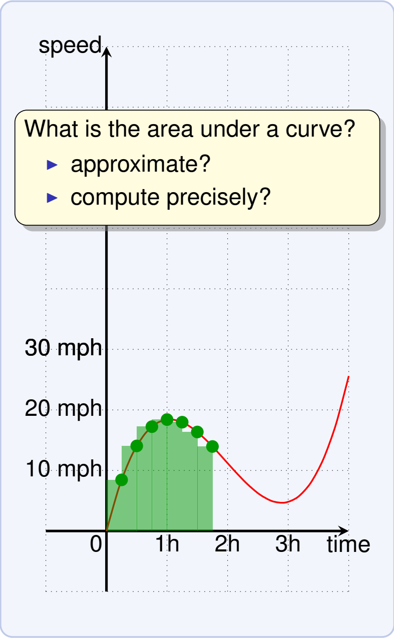

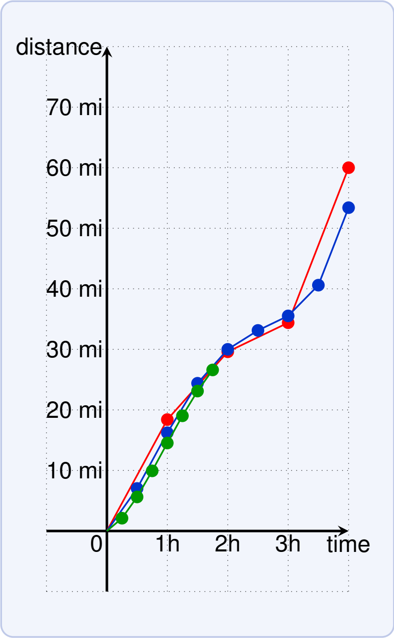

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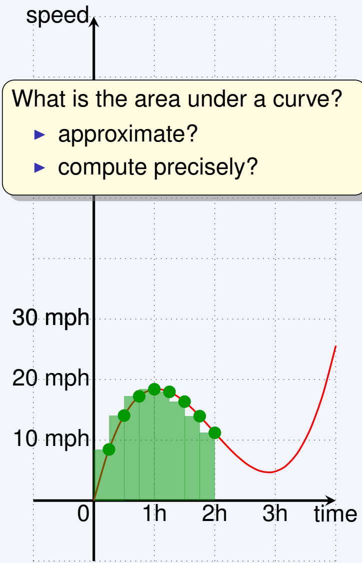

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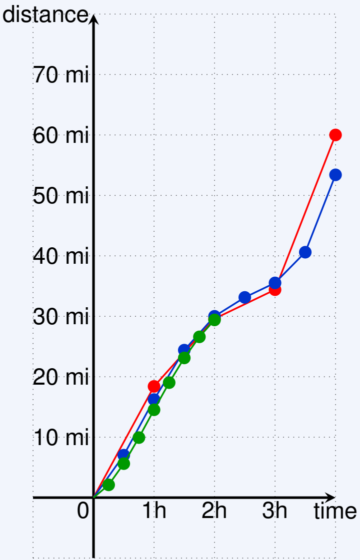

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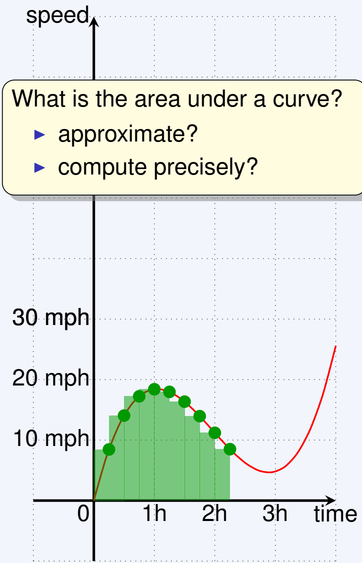

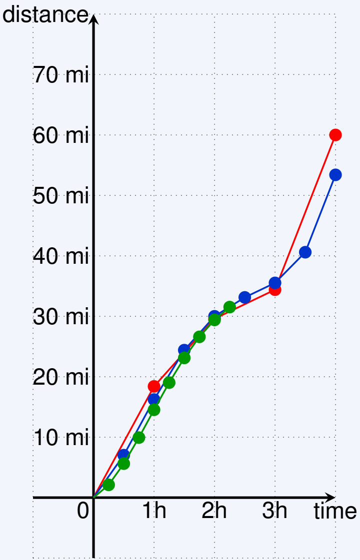

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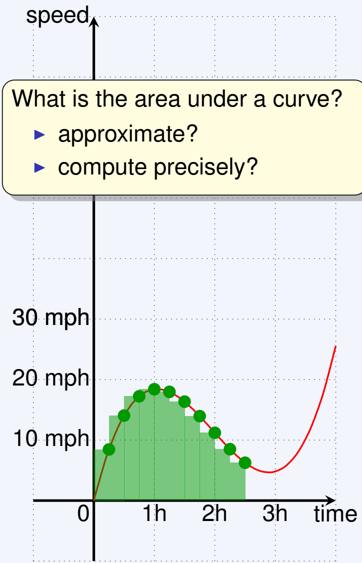

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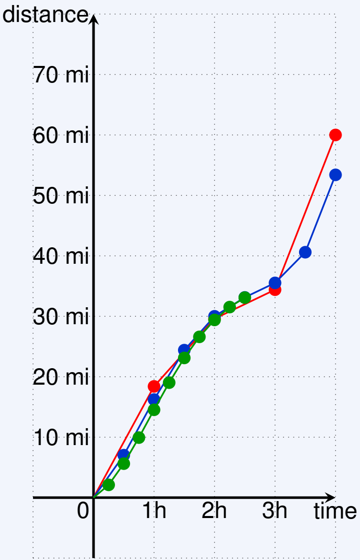

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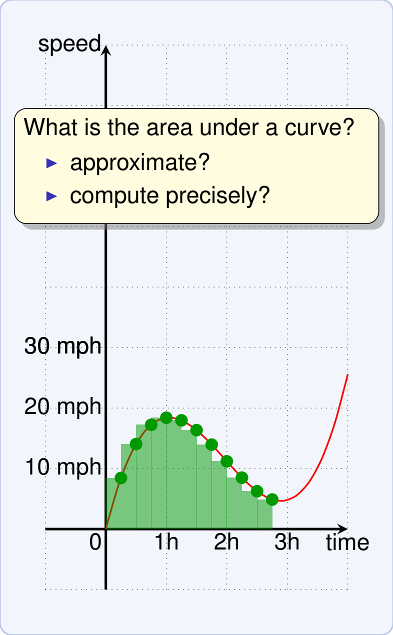

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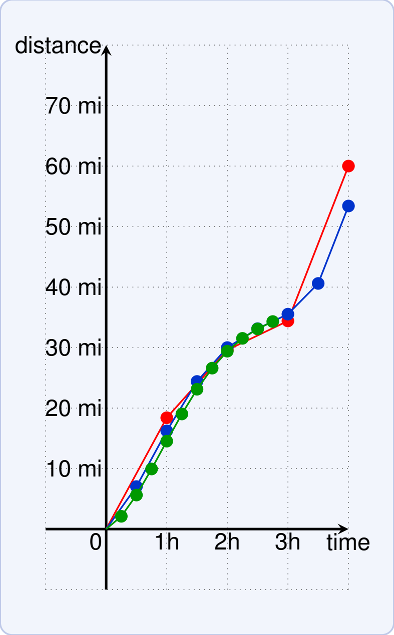

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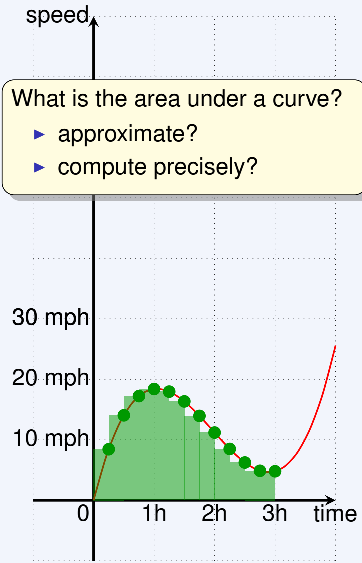

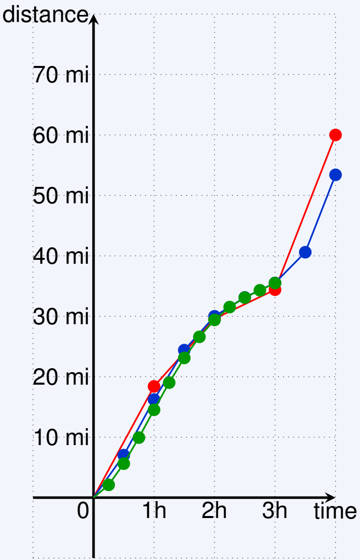

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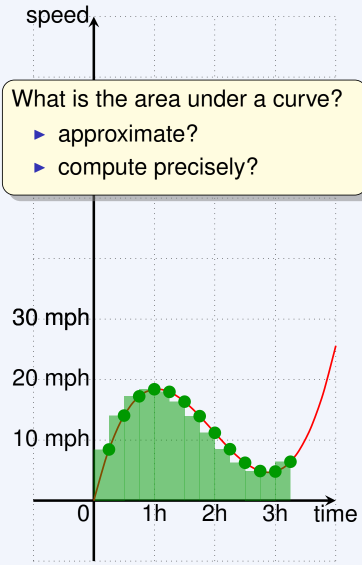

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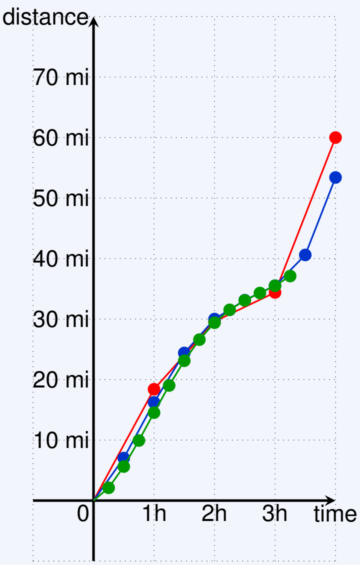

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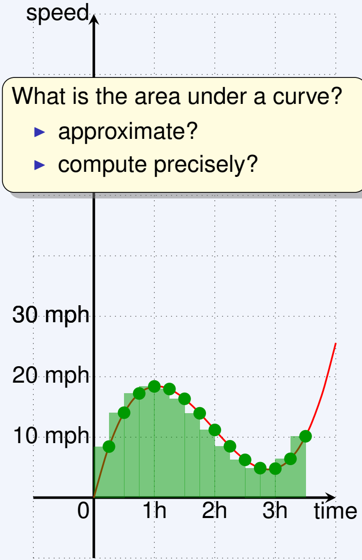

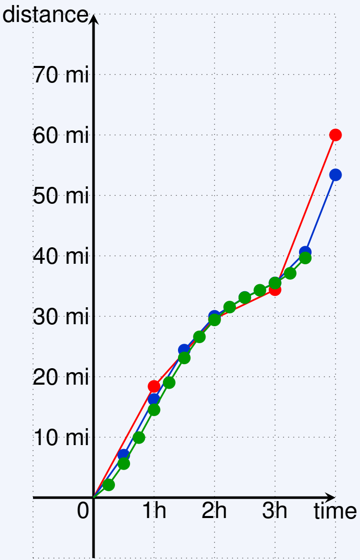

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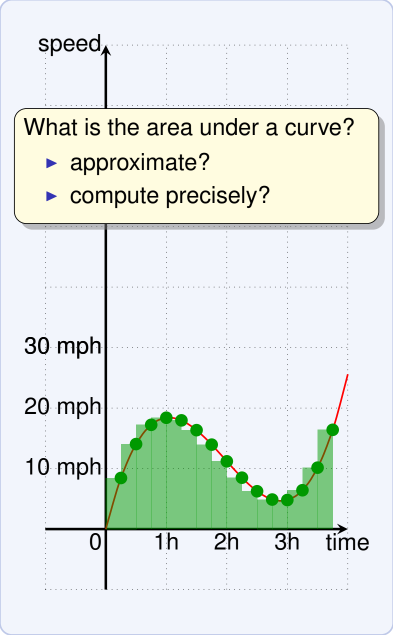

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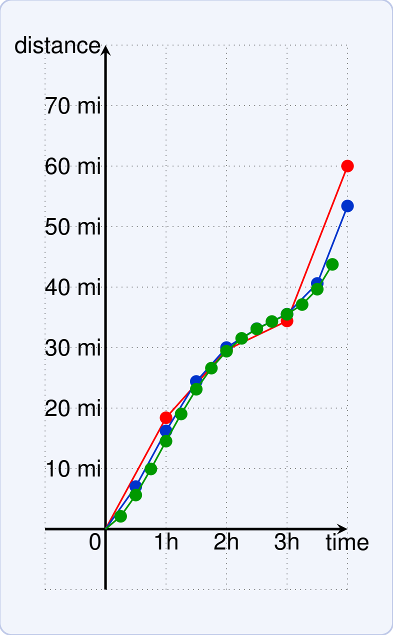

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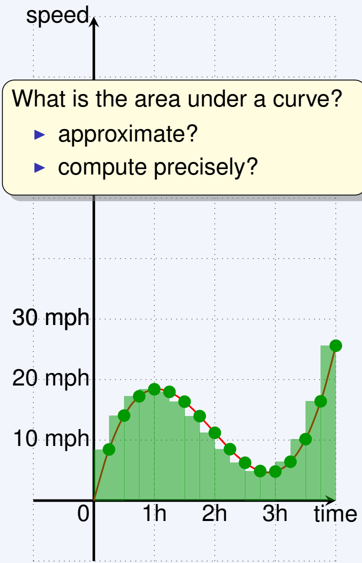

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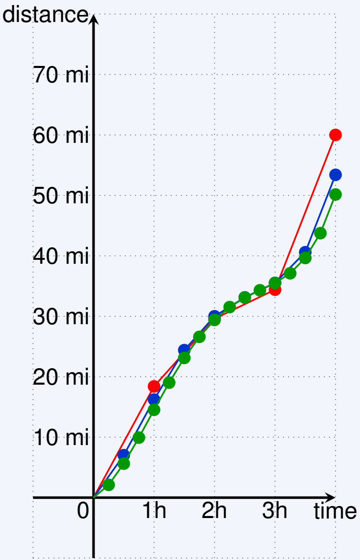

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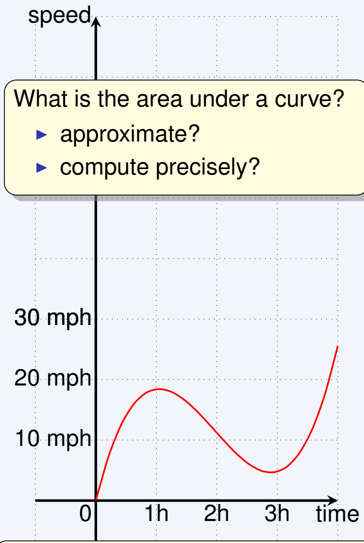

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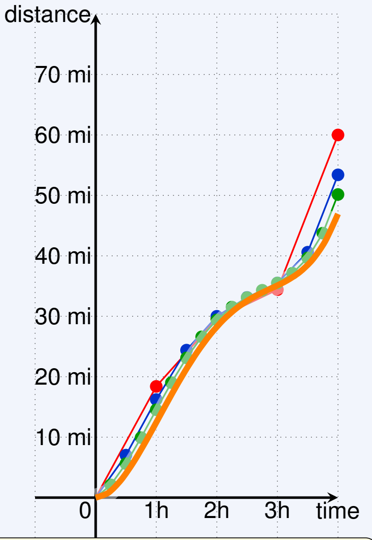

The finer approximations get closer and closer to the precise solution .