Automata Theory :: CYK Parsing

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Bottom-up Parsing

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Cocke-Younger-Kasami algorithm (1965)

The **CYK algorithm** is a bottom-up parsing technique for grammars in Chomsky normal form.

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as follows:

- if $|u| = 1$, then $V_u = \{ A \in V \mid A \to u \in P \}$
- if $|u| > 1$, then V_u is the set of all $A \in V$ such that
  - $u = u_1 u_2$ for some non-empty words u_1, u_2, and
  - $A \to BC \in P$ with $B \in V_{u_1}$ and $C \in V_{u_2}$.

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Finally, $w \in L(G) \Leftrightarrow S \in V_w$.

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Finally, $w \in L(G) \Leftrightarrow S \in V_w$.

Worst-case time complexity: $O(n^3)$
(There are $n(n+1)/2$ sets V_u, and computation of V_u is $O(n)$.)

Exercise

Use the CYK algorithm to check whether *abbb* is generated by

$$S \to AB \qquad A \to BB \mid a \qquad B \to AB \mid b$$

We have

$$\begin{aligned}
V_a &= \\
V_b &= \\
V_{ab} &= \\
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