Automata & Complexity

Jörg Endrullis

Vrije Universiteit Amsterdam

2018

Information

Lecturer:

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Teaching assistant:

- **Christopher Esterhuyse**
- **Johannes van der Meer**
- **Petar Vukmirovic**

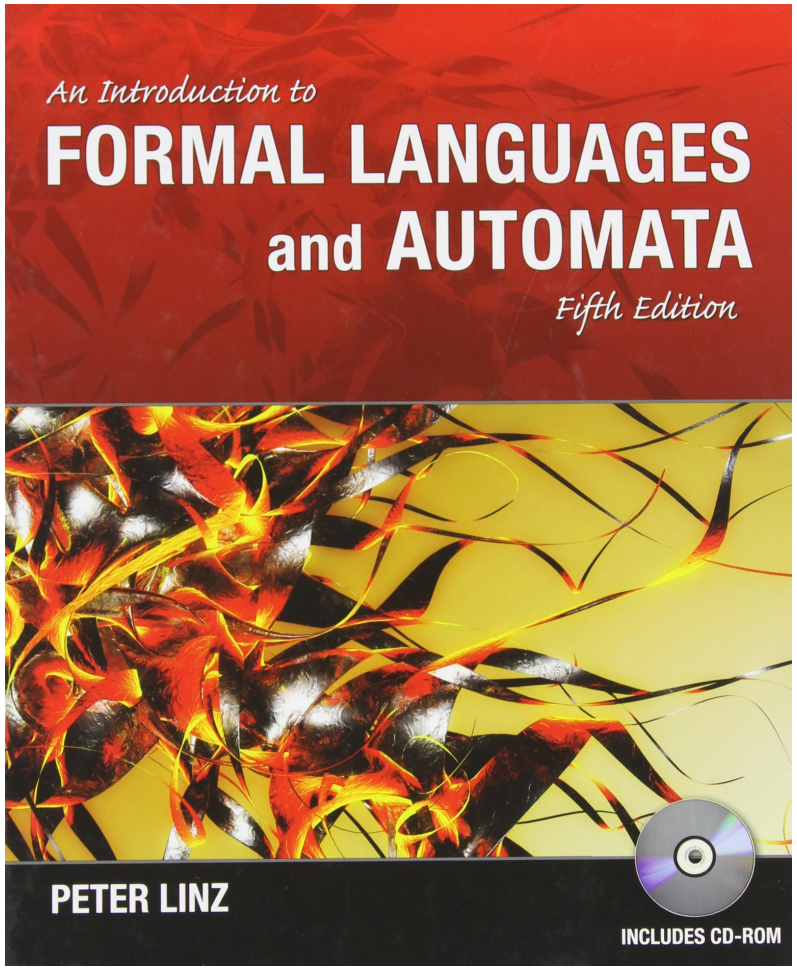
Course information:

- all relevant course information is on Canvas
- link to the **slides** on Canvas

Course Structure

- **2 lectures per week** (Tuesday and Wednesday)
- **2 exercise classes per week** (Wednesday and Thursday)
- **Homework**:
  - 80% of the homework in an online system
    (you will receive an email with activation link today)
  - 20% of the homework written assignments
    (one written assignment at the end of the course)
  - **50% of homework points to qualify for the exam**
  - **90% of homework points for 0.5 points bonus**
- **Written exam**

Material



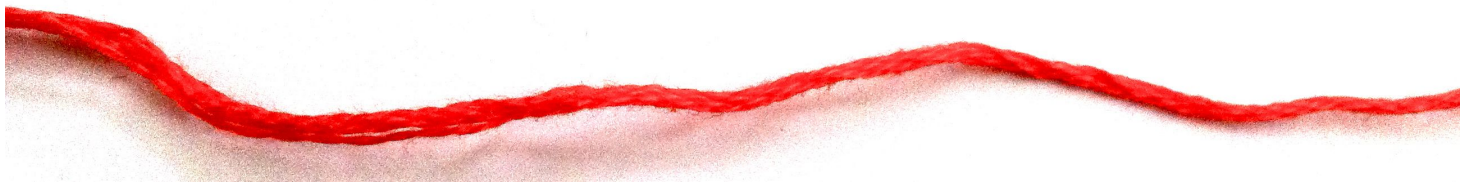
Peter Linz

An Introduction to Formal Languages and Automata (5th edition)

Jones and Bartlett

Course Motivation

Computer is based on a **universal computation mechanism**.



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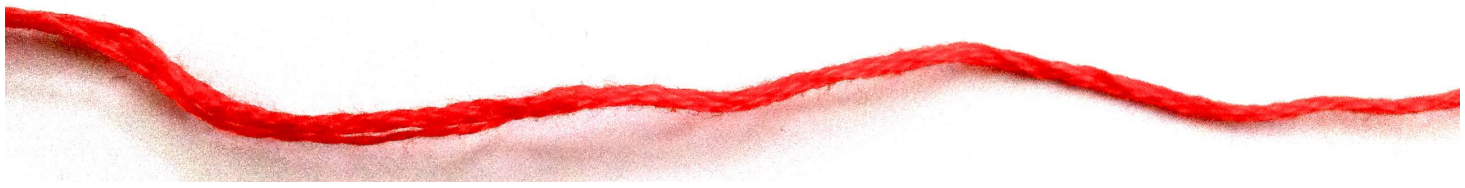

Different applications $\Longrightarrow$ different formalisms:

- **Pattern recognition**: regular languages
- **Parsing**: context-free languages, grammars
- **Computation**: Turing machines

Course Motivation

What can a computer do?

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Some (at first glance simple) problems are undecidable.

For example:

- program termination,
- Post correspondence problem,
- validity in predicate logic.

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Some (at first glance simple) problems are undecidable.

For example:

- program termination,
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- validity in predicate logic.

Some problems (NP-complete problems) are (probably) not efficiently solvable by a computer.

For example:

- travelling salesman problem,
- satisfiability in propositional logic.

Course Motivation

Typical questions that we will answer

- What is a (programming) **language**?
- How can languages be recognised by computers? (**automata**, **regular expressions**, **grammars**)
- What problems can be solved by what **types of automata**?
- **How much time/memory** is needed for solving a problem?

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Aspects of languages:

- **syntax**: the form of the words in the language
- **semantics**: the meaning of the words in the language

We will focus on the syntax.

Words

Words

Word = finite sequence of **symbols** from a finite **alphabet** Σ.

- notation for symbols: $a, b, c, \ldots$
- symbol from the alphabet: $a \in \Sigma$
- notation for words: u, v, w, x, y, z
- **empty word:** λ

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A computer program:

- is itself a **word** (a finite sequence of bits/bytes)
- takes **input word**
- produces **output word**

Operations on Words (1)

Concatenation

If $v = a_1 \cdots a_n$ and $w = b_1 \cdots b_m$, then

$$vw = a_1 \cdots a_n b_1 \cdots b_m$$

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Length

If $v = a_1 \cdots a_n$, then $|v| = n$.

The length can be defined inductively:

$$|\lambda| = 0$$
$$|va| = |v| + 1$$

Operations on Words (2)

Power

The power v^k consists of k concatenations of v's:

$$v^0 = \lambda$$

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Reverse

The reverse of $a_1 \cdots a_n$ is

$$(a_1 \cdots a_n)^R = a_n \cdots a_1$$

The reverse can be inductively defined

$$\lambda^R = \lambda$$
$$(va)^R = a(v^R)$$

Formal Languages

Formal language = **set** of **words**

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Example

All parseable C programs form a language.

Formal Languages

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Example

All parseable C programs form a language.

More precisely:

- Σ^* is the set of all words over Σ.

Formal language

A **(formal) language** L is a subset of Σ^*, that is, $L \subseteq \Sigma^*$.

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$\{ ab^n a \mid n \geq 1 \}$ is an infinite language:

$$\{ aba,\ abba,\ abbba,\ abbbba,\ \ldots \}$$

$\{a^n b^n \mid n \geq 0\}$ is an infinite language:

$$\{ \lambda,\ ab,\ aabb,\ aaabbb,\ aaaabbbb,\ \ldots \}$$

Operations on Languages (1)

A language is a **set** of words.

So the usual set operations have meaning for languages:

$$\in,\ \subseteq,\ \cap,\ \cup,\ \setminus,\ \ldots$$

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$$\overline{L} = \Sigma^* \setminus L$$

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The reverse of a language L is

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Kleene star

$$L^* = \bigcup_{i=0}^{\infty} L^i = L^0 \cup L^1 \cup L^2 \cup L^3 \cup \cdots$$
$$L^+ = \bigcup_{i=1}^{\infty} L^i = L^1 \cup L^2 \cup L^3 \cup \cdots$$

Thus $L^* = L^+ \cup \{\lambda\}$.

Exercise

Let

- $\Sigma = \{a, b\}$
- $L_1 = \{a,\ bb\}$
- $L_2 = \{ab,\ ba\}$

(In groups of two, 2 minutes)

Describe the following languages as sets:

- $L_1 L_2 =$
- $L_1^2 =$
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Conclusion

Sets are not ideal to describe languages.

Deterministic Finite Automata

A **deterministic finite automaton**, short **DFA**, consists of:

- a finite set Q of states
- a finite input alphabet Σ
- a transition function $\delta : Q \times \Sigma \to Q$
- a starting state $q_0 \in Q$
- a set $F \subseteq Q$ of final states

Understanding the transition function $\delta : Q \times \Sigma \to Q$

If the automaton in state q reads the symbol a, then the resulting state is $\delta(q, a)$.

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Example DFA

Let $M = (Q, \Sigma, \delta, q_0, F)$ with $Q = \{q_0, q_1\}$, $\Sigma = \{a, b\}$, $F = \{q_0\}$,

$$\delta(q_0, a) = q_0 \qquad \delta(q_1, a) = q_1$$
$$\delta(q_0, b) = q_1 \qquad \delta(q_1, b) = q_0$$

DFA's Reading Words

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA. We write

$$(q, aw) \vdash (q', w) \quad \text{if } \delta(q, a) = q'$$

Here, (q, w) indicates that M is in state q and reads word w.

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We define $\vdash^*$ as the **reflexive transitive closure of** $\vdash$.

Continuing the above example, we have

$$(q_0, abba) \vdash^* (q_0, \lambda)$$

Transition Function in Table Notation

Let $M = (Q, \Sigma, \delta, q_0, F)$ with $Q = \{q_0, q_1\}$, $\Sigma = \{a, b\}$, $F = \{q_0\}$,

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Hint: transition function δ can be written in the form of a **table**:

δ	q_0	q_1
a	q_0	q_1
b	q_1	q_0

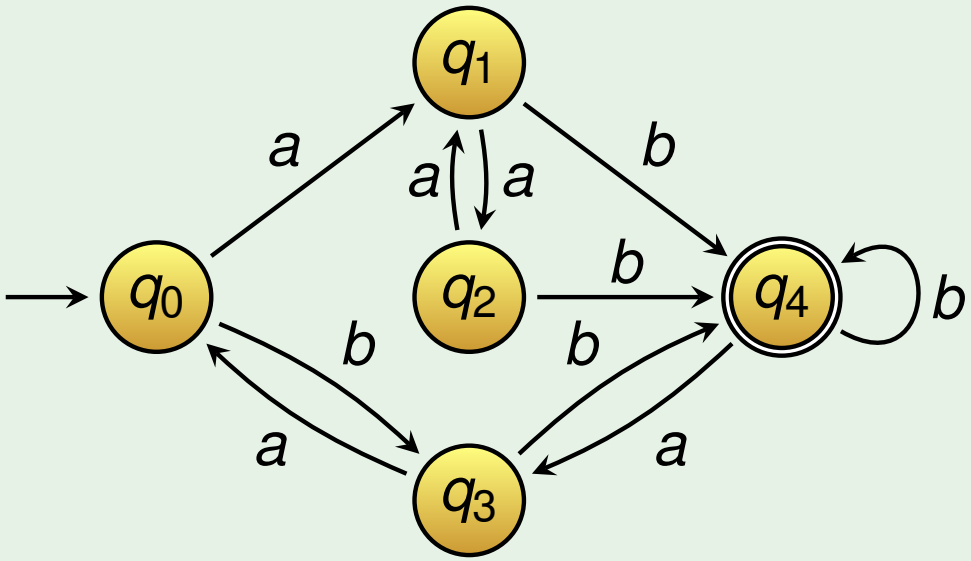
DFA's as Transition Graphs

A DFA can be visualised as a **transition graph**, consisting of:

- **states** are the **nodes** of the graph
- **arrows** with labels from Σ
- **starting state** indicated by an **extra incoming arrow**
- **final states** indicated by **double circle**

We have an arrow from q to q' with label a if $\delta(q, a) = q'$.

$\Sigma = \{a, b\}$



Exercise (1)

We define a DFA $M = (Q, \Sigma, \delta, q_0, F)$ with

$$Q = \{q_0, q_1\} \qquad \Sigma = \{a, b\} \qquad F = \{q_0\}$$

and the transition function given by

δ	q_0	q_1
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(In groups of two, 1 minute)

Visualise M as a transition graph.

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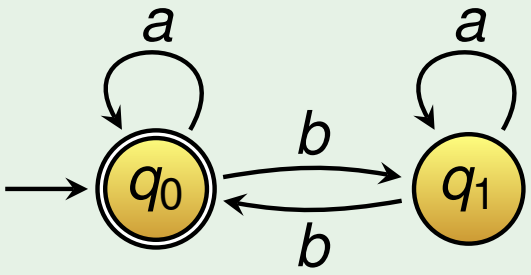

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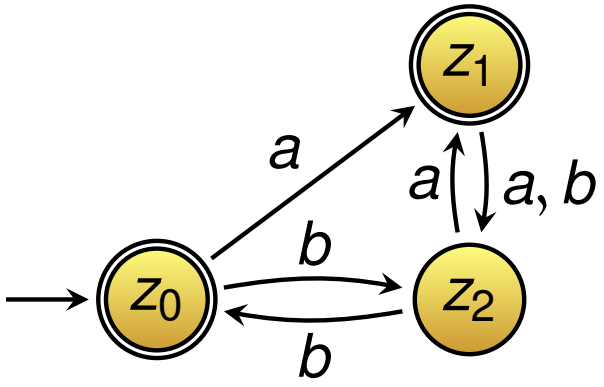

Note that the transition with label a, b is shorthand for two transitions: one with label a and one with label b.

(In groups of two, 1 minute)

Translate the graph to a deterministic finite automaton:

- states $Q = ?$
- alphabet $\Sigma = ?$
- transition function $\delta : Q \times \Sigma \to Q$ is given by

δ	z_0	z_1	z_2
a			
b			

- starting state ?
- final states $F = ?$

DFA's Reading Words continued...

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DFA's Reading Words continued...

If $(q, w) \vdash^* (q', \lambda)$, we also write

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In other words,

$$q \xrightarrow{w} q'$$

means that there is a path from q to q' in the automaton with transitions labelled $a_1, \ldots, a_n$ such that $w = a_1 \cdots a_n$.

Regular Languages

A DFA defines (accepts) a language!

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The **language accepted by** DFA $M = (Q, \Sigma, \delta, q_0, F)$ is

$$L(M) = \{ w \in \Sigma^* \mid (q_0, w) \vdash^* (q, \lambda) \text{ with } q \in F \}$$

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$$\delta(q_0, a) = q_0 \qquad \delta(q_1, a) = q_1$$
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Then we have

$$(q_0, abba) \vdash (q_0, bba) \vdash (q_1, ba) \vdash (q_0, a) \vdash (q_0, \lambda)$$

The word $abba$ is accepted by M, that is, $abba \in L(M)$.

Regular Languages

A DFA defines (accepts) a language!

The **language accepted by** DFA $M = (Q, \Sigma, \delta, q_0, F)$ is

$$L(M) = \{ w \in \Sigma^* \mid (q_0, w) \vdash^* (q, \lambda) \text{ with } q \in F \}$$
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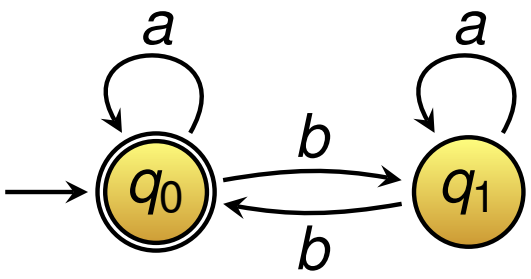
A language L is **regular** if there exists a DFA M with $L(M) = L$.

Exercise

Let M be the following DFA:



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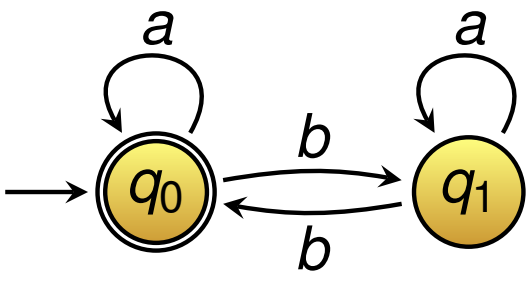

(In groups of two, 2 minutes)

What is the language accepted by M? (describe in words)

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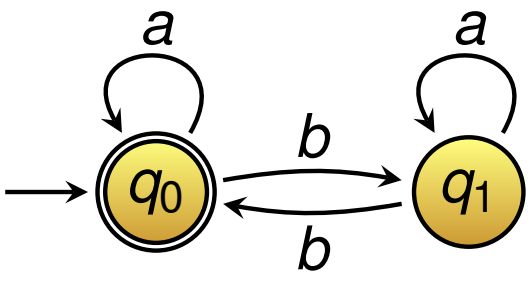

(In groups of two, 2 minutes)

What is the language accepted by M? (describe in words)

$L(M)$ consists of all words that contain an even number of b's.

Determinism

DFAs are deterministic

For every state $q \in Q$ and every symbol $a \in \Sigma$, the state q has **precisely one outgoing arrow** with label a.

Recall that δ is a function from $Q \times \Sigma$ to Q.

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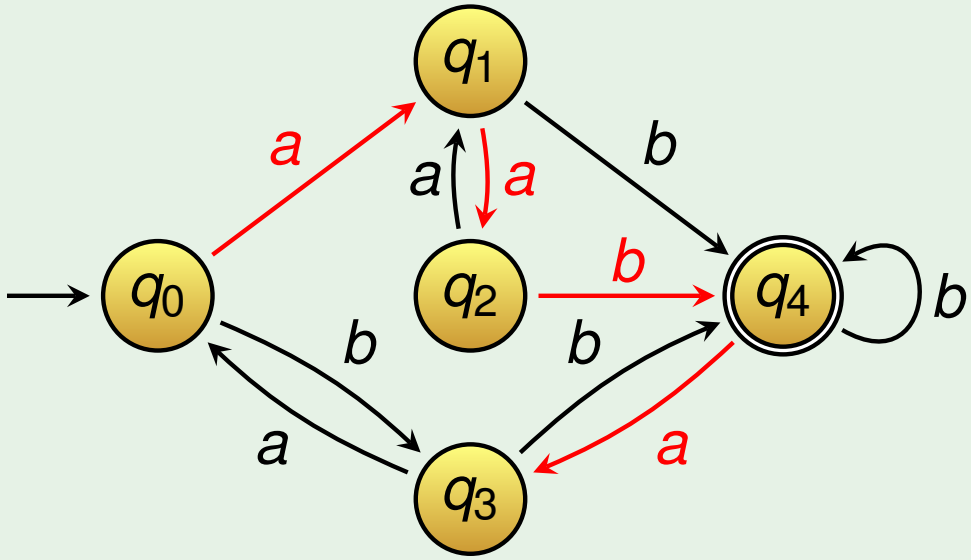
Recall that δ is a function from $Q \times \Sigma$ to Q.

Hence, for every input word, there is precisely one path from the starting state through the transition graph.

The following picture shows the path for *aaba*:



Exercise (1)

(Individually, 10 seconds)

Show that the following language is regular.

Construct a deterministic finite automaton for the language:

$$\{\lambda\}$$

Exercise (2)

(Individually, 1 minutes)

Show that the following language is regular.

Construct a deterministic finite automaton for the language:

$$\{ a^n b \mid n \geq 0 \}$$

Exercise (3)

(Individually, 2 minutes)

Show that the following language is regular.

Construct a deterministic finite automaton for the language:

$$\{ a^{2n+1} \mid n \geq 0 \} \cup \{ b^{2n} \mid n \geq 0 \}$$

Exercise (4)

(Individually, 4 minutes)

Show that the following languages are regular.

Construct deterministic finite automata for the languages:

$$\{ w \in \{a, b\}^* \mid w \text{ contains the subword } bab \}$$

and

$$\{ w \in \{a, b\}^* \mid w \text{ does } \textbf{not} \text{ contain the subword } bab \}$$

Theorems about Regular Languages (1)

Theorem

If L is a regular language, then $\overline{L}$ is also regular.

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Then it follows that $L(N) = L(M_1) \cup L(M_2) = L_1 \cup L_2$.

Theorems about Regular Languages (3)

Question

Let L_1 and L_2 be regular languages.

How to change the product construction to show that

- $L_1 \cap L_2$ is regular, and
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This proof will be an exercise.

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Question

Is every **finite** language regular ?

Nondeterministic Finite Automata

Nondeterministic finite automaton (NFA)

- A state can have **zero or more outgoing arrows** with the same label.
- Allows for **empty steps**: arrows with label λ that do not 'eat' a symbol from the input word.

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DFA's and NFA's are used everywhere:

- in software engineering,
- for modelling of hardware circuits,
- in compilers, and
- in network protocols.

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NFA's are defined like DFA's, except for the transition function!

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A **nondeterministic finite automaton**, short **NFA**, consists of:

- a finite set Q of states
- a finite input alphabet Σ
- a transition function $\delta : Q \times (\Sigma \cup \{\lambda\}) \to 2^Q$
- a starting state $q_0 \in Q$
- a set $F \subseteq Q$ of final states

Here 2^Q is the set of all subsets of Q:

$$2^Q = \{ S \mid S \subseteq Q \}$$

NFA's Reading Words

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a NFA. We write

$$(q, \alpha w) \vdash (q', w) \quad \text{if } q' \in \delta(q, \alpha) \text{ with } \alpha \in \Sigma \cup \{\lambda\}$$

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Again we write

$$q \xrightarrow{w} q'$$

whenever $(q, w) \vdash^* (q', \lambda)$.

It means that there is a path from q to q' with transitions labelled $\alpha_1, \ldots, \alpha_n \in (\Sigma \cup \{\lambda\})$ such that $w = \alpha_1 \cdots \alpha_n$.

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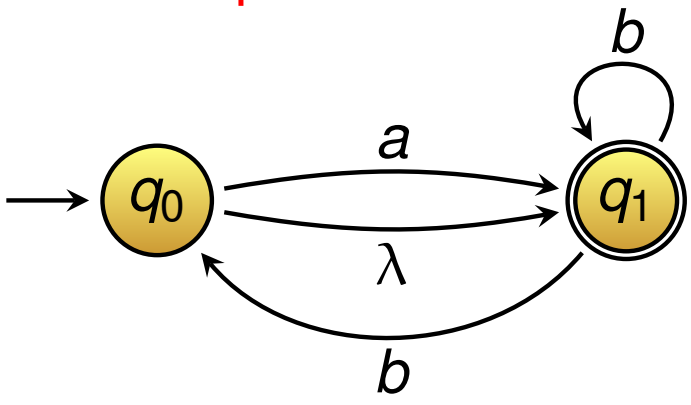

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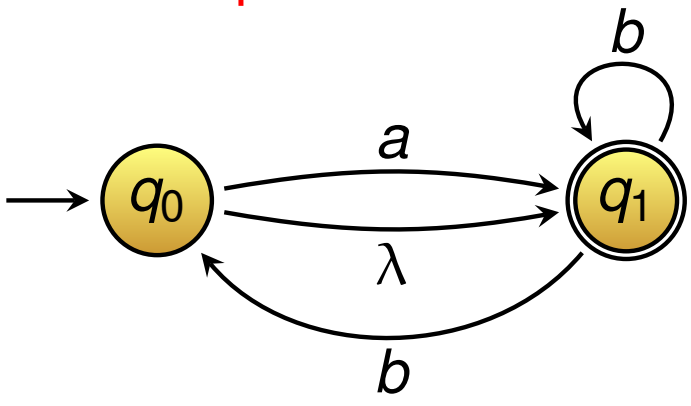

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Let $M = (Q, \Sigma, \delta, q_0, F)$ be a NFA.

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Idea: state of DFA = set of all states the NFA can be in

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Let $M = (Q, \Sigma, \delta, q_0, F)$ be a NFA.

Idea: state of DFA = set of all states the NFA can be in

We construct a DFA $N_D = (Q_D, \Sigma, \delta_D, q_{0D}, F_D)$ where

$$Q_D = 2^Q$$

$$\delta_D(X, a) =$$

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DFA's and NFA's are Equally Expressive

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For every $w \in \Sigma^*$ and $q \in Q$ it holds that

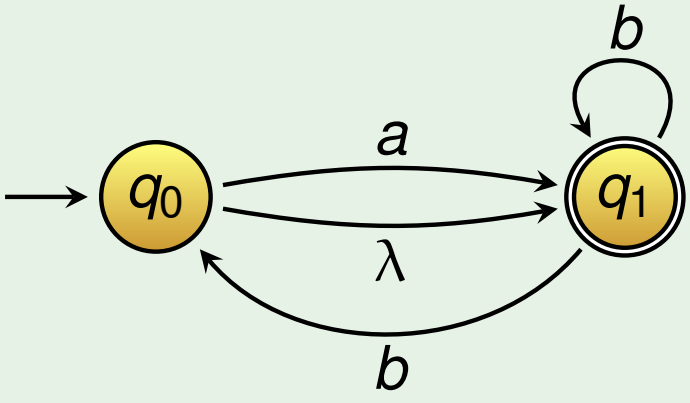
$$(q_0, w) \vdash^* (q, \lambda) \iff (q_{0D}, w) \vdash^* (X, \lambda) \text{ with } q \in X.$$

From this property it follows that $L(N_D) = L(M)$.

Exercise

(Groups of two, 2 minutes)

Given is the following NFA:



Construct a DFA that accepts the same language.

Looking Forward

Read:

- Linz 1.2–1.2, 2.1–2.3

Do the following exercises:

- Linz 1.2: 2, 4, 8, 10
- Linz 2.1: 1, 2d, 3, 7b, 9b,f, 11
- Linz 2.2: 12
- Linz 2.3: 2, 3, 6, 12

Following lecture:

- Alternative descriptions of regular languagse:
  - regular expressions
  - grammars